CORE-PLUS MATHEMATICS PROJECT

Course 3
Part A

Contemporary Mathematics in Context

A Unified Approach

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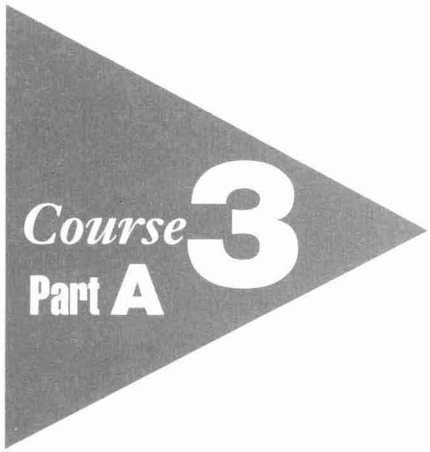

Contemporary Mathematics in Context

A Unified Approach

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Overview of Course 3

Part A

Unit 1 Multiple-Variable Models

Multiple-Variable Models develops student ability to construct and reason with linked quantitative variables and relations involving several variables and several constraints.
Topics include formulas relating several variables by a single equation, systems of equations with several dependent variables or constraints, patterns of change in one or more variables in response to changes in others, solution of systems of equations and inequalities, and linear programming.

Unit 3 Symbol Sense and Algebraic Reasoning

Symbol Sense and Algebraic Reasoning develops student ability to represent and draw inferences about algebraic relations and functions using symbolic expressions and manipulations.
Topics include use of polynomial, exponential, and rational expressions to model relations among quantitative variables, field properties of real numbers and their application to expression of algebraic relations in equivalent forms and to solution of equations and inequalities by methods including factoring and the quadratic formula.

Unit 2 Modeling Public Opinion

Modeling Public Opinion develops student understanding of how public opinion can be measured. The situations analyzed include elections (where there are more than two choices) and sample surveys, including political polling.
Topics include preferential voting, election analysis methods, Arrow's theorem, fairness in social decision making, surveys, sampling, sampling distributions, relationship between a sample and a population, confidence intervals, margin of error, and critical analysis of elections and surveys.

Unit 4 Shapes and Geometric Reasoning

Shapes and Geometric Reasoning introduces students to formal reasoning and deduction in geometric settings.
Topics include inductive and deductive reasoning, counterexamples, the role of assumptions in proof, conclusions concerning supplementary and vertical angles and the angles formed by parallel lines and transversals, conditions insuring similarity and congruence of triangles and their application to quadrilaterals and other shapes, and necessary and sufficient conditions for parallelograms.

Overview of Course 3

Part B

Unit 5 Patterns in Variation

Patterns in Variation extends student understanding of the measurement of variation, develops student ability to use the normal distribution as a model of variation, and introduces students to the probability and statistical inference involved in the control charts used in industry for statistical process control.
Topics include standard deviation and its properties, normal distribution and its relation to standard deviation, statistical process control, control charts, control limits, mutually exclusive events, and the Addition Rule of Probability.

Lesson 1 *Measuring Variation with the Standard Deviation*
Lesson 2 *The Normal Distribution*
Lesson 3 *Statistical Process Control*
Lesson 4 *Looking Back*

Unit 6 Families of Functions

Families of Functions reviews and extends student ability to recognize different function patterns in numerical and graphical data and to interpret and construct appropriate symbolic representations modeling those data patterns.
Topics include review of linear, polynomial, exponential, rational, and periodic functions (including effects of parameters on numeric and graphic patterns) and construction of function rules for function tables and graphs that are transformations of basic types (translation, reflection, stretch).

Lesson 1 *Function Models Revisited*
Lesson 2 *Customizing Models 1: Reflections and Vertical Transformations*
Lesson 3 *Customizing Models 2: Horizontal Transformations*
Lesson 4 *Looking Back*

Unit 7 Discrete Models of Change

Discrete Models of Change extends student ability to represent, analyze, and solve problems in situations involving sequential and recursive change.
Topics include iteration and recursion as tools to model and analyze sequential change in real-world contexts; arithmetic, geometric, and other sequences; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.

Lesson 1 *Modeling Sequential Change Using Recursion*
Lesson 2 *A Discrete View of Linear, Exponential, and Polynomial Models*
Lesson 3 *Iterating Functions*
Lesson 4 *Looking Back*

Capstone Looking Back at Course 3

Making the Best of It: Optimal Forms and Strategies is a thematic, two-week project-oriented activity that enables students to pull together and apply the important mathematical concepts and methods developed throughout the course.

Preface

The first three courses in the *Contemporary Mathematics in Context* series provide a common core of broadly useful mathematics for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. The series builds upon the theme of *mathematics as sense-making*. Through investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and which, in turn, enables them to make sense out of new situations and problems.

Each course in the *Contemporary Mathematics in Context* curriculum shares the following mathematical and instructional features.

- ***Multiple Connected Strands*** Each year the curriculum features four strands of mathematics, unified by fundamental themes, by common topics, and by habits of mind or ways of thinking. Developing mathematics each year along multiple strands helps students develop diverse mathematical insights and nurtures their differing strengths and talents.
- ***Mathematical Modeling*** The curriculum emphasizes mathematical modeling and modeling concepts including data collection, representation, interpretation, prediction, and simulation. The modeling perspective permits students to experience mathematics as a means of making sense of data and problems that arise in diverse contexts within and across cultures.
- ***Access*** The curriculum is designed so that core topics are accessible to a wide range of students. Differences in student performance and interest can be accommodated by the depth and level of abstraction to which common topics are pursued, by the nature and degree of difficulty of applications, and by providing opportunities for student choice on homework tasks and projects.
- ***Technology*** Numerical, graphics, and programming and link capabilities such as those found on many graphics calculators are assumed and capitalized on. This use of technology permits the curriculum and instruction to emphasize multiple representations (numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking and problem solving are central.
- ***Active Learning*** Instruction and assessment practices are designed to promote mathematical thinking through the use of engaging problem situations. Both collaborative groups and individual work are used as students explore, conjecture, verify, apply, evaluate, and communicate mathematical ideas.

Unified Mathematics

Each course of *Contemporary Mathematics in Context* features important mathematics drawn from four "strands."

The Algebra and Functions strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear,

exponential, power, and periodic functions, as well as combinations of these various types. Attention is also given to modeling with systems of equations, both linear and nonlinear, and to symbolic reasoning.

The primary goal of the Geometry and Trigonometry strand is to develop visual thinking and ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. Specific activities include describing patterns with regard to shape, size, and location; representing patterns with drawings or coordinates; predicting changes and invariants in shapes and patterns; and organizing geometric facts and relationships through deductive reasoning.

The primary role of the Statistics and Probability strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

The Discrete Mathematics strand develops student ability to model and solve problems involving sequential change, decision-making in finite settings, and relationships among a finite number of elements. Topics include matrices, vertex-edge graphs, recursion, voting methods, and systematic counting methods (combinatorics). Key themes are existence (Is there a solution?), optimization (What is the best solution?), and algorithmic problem-solving (Can you efficiently construct a solution?).

These four strands are connected within units by fundamental ideas such as symmetry, matrices, recursion, functions, data analysis and curve-fitting. The strands also are connected across units by mathematical habits of mind such as visual thinking, recursive thinking, searching for and describing patterns, making and checking conjectures, reasoning with multiple representations, inventing mathematics, and providing convincing arguments. The strands are unified further by the fundamental themes of data, representation, shape, and change. Important mathematical ideas are continually revisited through this attention to connections within and across strands, enabling students to develop a robust understanding of mathematics.

Active Learning and Teaching

The manner in which mathematical ideas are developed can be as important as the mathematics to which students are introduced. *Contemporary Mathematics in Context* features multi-day lessons centered on big ideas. Lessons are organized around a four-phase cycle of classroom activities, described below, designed to engage students in investigating and making sense of problem situations, in constructing important mathematical concepts and methods, and in communicating orally and in writing their thinking and the results of their efforts. Most classroom activities are designed to be completed by students working together collaboratively in heterogeneous groupings of two to four students.

The launch phase promotes class discussion of a situ-

ation and of related questions to think about, setting the context for the student work to follow. In the second or explore phase, students investigate more focused problems and questions related to the launch situation. This investigative work is followed by a class discussion in which students summarize mathematical ideas developed in their groups, providing an opportunity to construct a shared understanding of important concepts, methods, and approaches. Finally, students are given a task to complete on their own, assessing their initial understanding of the concepts and methods.

Each lesson also includes tasks to engage students in Modeling with, Organizing, Reflecting on, and Extending their mathematical understanding. These MORE tasks are central to the learning goals of each lesson and are intended primarily as individual work outside of class. Selection of tasks for use with a class should be based on student performance and the availability of time and technology. Students can exercise some choice of tasks to pursue, and at times they can be given the opportunity to pose their own problems and questions to investigate.

Multiple Approaches to Assessment

Assessing what students know and are able to do is an integral part of *Contemporary Mathematics in Context*. Initially, as students pursue the investigations that make up the curriculum, the teacher is able to informally assess student performance in terms of process, content, disposition, or other factors. At the end of each investigation, the "Checkpoint" and accompanying class discussion provide an opportunity for the teacher to assess the levels of understanding that the various groups of students have reached. Finally, the "On Your Own" problem situation, as well as the tasks in the MORE sets, provides further opportunities to assess the level of understanding of each individual student. Quizzes, in-class exams, take-home assessment activities, and extended projects are included in the teacher resource materials.

Acknowledgments

Development and evaluation of the student text materials, teacher materials, assessments, and calculator software for *Contemporary Mathematics in Context* were funded through a grant from the National Science Foundation to the Core-Plus Mathematics Project (CPMP). We are indebted to the support, understanding, and input of Midge Cozzens, Director of the NSF Division of Elementary, Secondary, and Informal Education, and of our program officers James Sandefur, Eric Robinson, and John Bradley.

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Computing tools are fundamental to the use of *Contemporary Mathematics in Context*. Appreciation is expressed to Texas Instruments and, in particular, Dave Santucci for collaborating with us by providing classroom sets of graphing calculators to field-test schools.

As seen on page v, CPMP has been a collaborative effort that has drawn on the talents and energies of teams of mathematics educators at several institutions. This diversity of experiences and ideas has been a particular strength of the project. Special thanks is owed to the support staff at these institutions, particularly at Western Michigan University.

From the outset, our work has been guided by the advice of an international advisory board consisting of Diane Briars (Pittsburgh Public Schools), Jeremy Kilpatrick (University of Georgia), Kenneth Ruthven (University of Cambridge), David A. Smith (Duke University), and Edna Vasquez (Detroit Renaissance High School). Preliminary versions of the curriculum materials also benefited from careful reviews by the following mathematicians and mathematics educators: Alverna Champion (Grand Valley State University), Cherie Cornick (Wayne County Alliance for Mathematics and Science), Edgar Edwards (formerly of the Virginia State Department of Education), Richard Scheaffer (University of Florida), Martha Siegel (Towson University), Edward Silver (University of Pittsburgh), and Lee Stiff (North Carolina State University).

Our gratitude is expressed to the teachers and students in our 35 evaluation sites listed on pages vi and vii. Their experiences using pilot- and field-test versions of *Contemporary Mathematics in Context* provided constructive feedback and improvements. We learned a lot together about making mathematics meaningful and accessible to a wide range of students.

A very special thank you is extended to Barbara Janson for her interest and encouragement in publishing a core mathematical sciences curriculum that breaks new ground in terms of content, instructional practices, and student assessment. Finally, we want to acknowledge Eric Karnowski for his thoughtful and careful editorial work and express our appreciation to the staff of Everyday Learning who contributed to the publication of this program.

To the Student

Contemporary Mathematics in Context, Course 3 builds on the mathematical concepts, methods, and habits of mind developed in Courses 1 and 2. With this text, you will continue to learn mathematics by doing mathematics, not by studying "worked out" examples. You will investigate important mathematical ideas and ways of thinking as you try to understand and make sense of realistic situations. Because real-world situations and problems often involve data, shape, change, or chance, you will learn fundamental concepts and methods from several strands of mathematics. In particular, you will develop an understanding of broadly useful ideas from algebra and functions, from statistics and probability, from geometry and trigonometry, and from discrete mathematics. You also will see connections among these strands—how they weave together to form the fabric of mathematics.

Because real-world situations and problems are often open-ended, you will find that there may be more than one correct approach and more than one correct solution. Therefore, you will frequently be asked to explain your ideas. You also will increasingly be asked to provide more general arguments or proofs for mathematical statements. This text will provide you with help and practice in reasoning and communicating clearly about mathematics.

Because the solution of real-world problems often involves teamwork, you will continue to often work collaboratively with a partner or in small groups as you investigate realistic and interesting situations. As in Courses 1 and 2, you will find that 2 or 4 students working collaboratively on a problem can often accomplish more than any one of you would working individually. Because technology is commonly used in solving real-world problems, you will continue to use a graphing calculator or computer as a tool to help you understand and make sense of situations and problems you encounter.

As in Courses 1 and 2, you're going to learn a lot of useful mathematics—and it's going to make sense to you. You're going to strengthen your skills in working cooperatively and communicating with others as well. You're also going to strengthen your skills in using technology tools intelligently and effectively. You'll have plenty of opportunities to be creative, too, so let your imagination lead you and enjoy.

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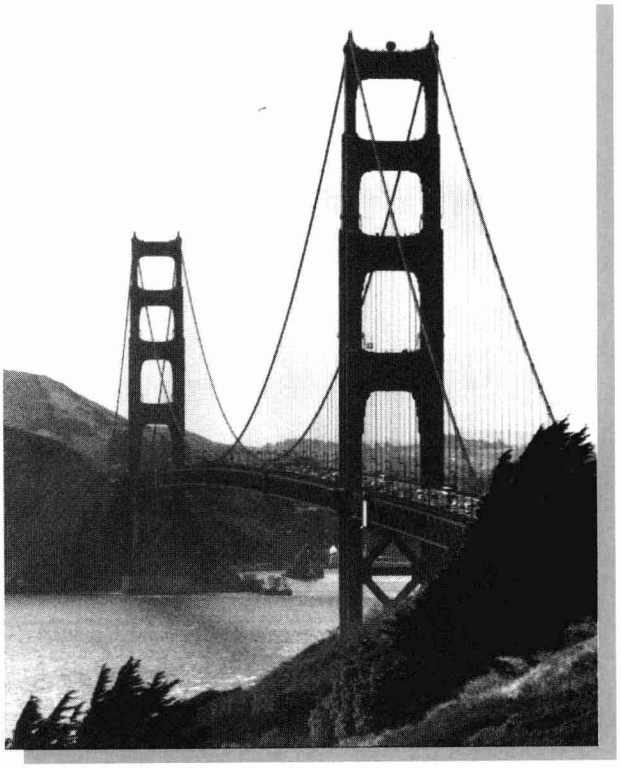
Multiple-Variable Models

Unit 1



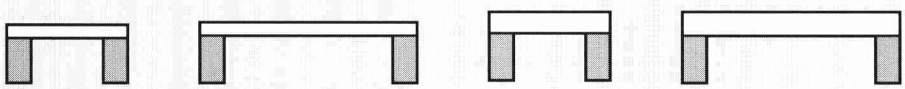
Lesson 1

Linked Variables

Karate is a very impressive form of the martial arts. You may have seen live or video exhibitions of highly trained men and women breaking bricks and boards with chops from their hands, feet, or even heads. Some of you may have even attempted a karate chop and discovered that, without proper technique and training, it can hurt.

Think About This Situation

Karate chops break bricks and boards by applying carefully aimed bursts of energy. Different targets require different amounts of energy. Think about the four target boards pictured here:



a Which board do you think would require the greatest energy to break?

b The target boards differ in length and thickness. How would you expect those two variables to affect required breaking energy?

c Breaking energy E depends on board length L and thickness T. What sort of equation might be used to express E as a function of L and T?

d What other variables would you consider in judging the energy required to break a board? How would you expect those variables to be related to each other and to E, L, and T?