The Theory of Duality & International Trade

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INTRODUCTION

The theory of duality and its application to consumer theory was first introduced by Hotelling in 1932 and later by Shephard and Samuelson in the early 1950s. In recent years duality theory has become more popular and is used in many branches of economics. In fact, over the last decades the duality approach has found its way into intermediate level and advanced microeconomic textbooks.[1] This duality approach is nothing more than a set of techniques or analytical tools that may be used for analyzing existing models. In particular, since economists and others are interested in optimizing behaviour, duality provides a simple and alternative method of analysis.

The application of duality to international trade problems has been present in the journal literature for a number of years. Its incorporation

into trade textbooks has been long overdue and has occurred relatively recently.[2] Unfortunately, the treatments that exist are extremely mathematical (or very terse) and therefore non-accessible to many graduate students and professionals. This situation is indeed unfortunate as to some extent it limits the dissemination of a very useful technique for analyzing international trade issues. One of the main purposes of this book is to simplify some of the mathematical results by the extensive use of geometry so that a wider readership can gain access to this research approach. The book is thus intended to provide a relatively simple exposition of the application of duality theory to certain trade problems. Some mathematics is unavoidable but in general the technical aspects are kept to a minimal level of difficulty. The reader is, however, referred to other sources for the more difficult mathematical proofs.

Duality is an extremely useful analytical tool. For example, many of the instruments of trade policy like tariffs directly affect prices and only indirectly affect quantities. Therefore, the application of duality theory via the indirect utility function or the expenditure function, where prices are the key variables, is often neater and

more appropriate than the use of direct utility functions or demand functions where quantities are the key variables. On another front, duality theory has proven to be useful for the econometric estimation of demand systems that could be generated by utility maximization. In the context of general equilibrium models of international trade, using duality theory as an analytical technique has provided neat and elegant proofs of standard results and insights into how results derived in the standard two country, two good, two factor model of trade can be generalized to higher dimensions. It has also proved useful in the areas of public finance and growth theory.

What exactly is duality? For our purposes the following broad definition given by Russell and Wilkinson (1979) is appropriate. "Duality is defined as the existence of two logical systems characterized by certain interrelationships. The essence of a dual system is a correspondence between concepts in one logical system and concepts in the other which allows us to derive a correspondence between results in one system and results in other" (p.73).

This book is divided into three broad sections. The first two chapters develop the basic analytical

tools that will be used for the remainder of the book. Chapter one discusses consumption theory while the second discusses production theory. These two chapters provide the necessary background and tools for the application of duality to international trade issues.

The next group of four chapters cover the core topics of international trade. Chapter three discusses the existence of a general equilibrium when two countries trade. Chapter four analyzes four of the main theorems that are generally regarded as the "core" of international trade theory. These are the Heckscher-Ohlin, Factor Price Equalization, Rybczynski and Stolper-Samuelson Theorems. Intermediate goods are discussed in chapter five while the final chapter in this group looks at specific factors.

The type of models considered in chapters three to six are perfectly competitive or first-best models. The last part of the book deals with topics in a second-best framework. In particular, chapter seven looks at tariffs and transfers, chapter eight at distortions while chapter ten analyzes an increasing returns to scale model.

Two main topics that I have not covered in this book are uncertainty and exhaustible resources in

international trade. These two topics are very mathematical and excellent treatment exists elsewhere.[3] Finally, the models considered in the book are static models. I have chosen not to include trade and growth since I have already covered this elsewhere. A full treatment of dynamic growth and trade and comparative dynamics would require yet another book.[4]

NOTES

1. These textbooks include Deaton and Muellbauer (1980), Gravelle and Rees (1981), Russell and Wilkinson (1979) and Varian (1984).
2. The two main international trade textbooks that use duality extensively are Dixit and Norman (1980) and Woodland (1982).
3. Suitable references for uncertainty and trade include Batra (1975), Pomery (1979), (1984) and Helpman and Razin (1978). For exhaustible resources the main reference is Kemp and Long (1980), (1984a), (1984b). Non-traded goods has also been excluded. For excellent treatments of this topic the reader is referred to Ethier (1982), Hazari, Sgro and Suh (1980), Komiya (1967) and in a duality framework to Woodland (1982).
4. Growth and Trade is discussed in Hazari (1986),

Sgro (1980) and in a duality framework by Woodland (1982). Also omitted is a treatment of money and the balance of payments. A brief discussion of money and the balance of payments in a duality framework can be found in Dixit and Norman (1980).

CONTENTS

Chapter One

Consumption Theory and Duality

1.1 Introduction

The theory of duality and its application to consumer theory was first introduced by Hotelling in 1932. In recent years duality theory in economics has become more popular. This duality approach refers to a set of techniques, or analytical tools, that may be used for analyzing existing models.[1]

In general, there are several alternative ways of modelling optimizing behaviour. Each method has its own strengths and weaknesses, the choice between them depending on mathematical and economic convenience. The use of duality, among other things, enables the derivation of an indirect utility function and a consumer expenditure function which shows the minimum cost of attaining a given level of utility at an exogenous set of prices.[2]

Only those results of duality theory that are

directly relevant to international trade will be presented in this chapter. In particular, we will concentrate on the derivation of an indirect utility function and an expenditure function. Both these functions are dual to the utility function and hence are equally good characterizations of preferences. A geometrical illustration is presented in section 1.2 followed by a more formal and complete discussion on the relationship between the direct utility function and the indirect utility and expenditure functions.

1.2 Duality of Direct and Indirect Utility Functions: A geometrical presentation

In this section we present a geometrical technique for deriving the indirect utility function.[3] For illustrative purposes only we restrict ourselves to the one consumer, two-commodity case. Traditionally the consumer maximizes a utility function of the form:

$$U(X) = U(X_1, X_2) \tag{1.1}$$

subject to

$$P_1X_1 + P_2X_2 = Y \tag{1.2}$$

where U indicates total utility, P_i and X_i are the prices and quantities of good i respectively, i = 1, 2. Prices and income (Y) are the parameters

and are determined exogenously. For simplicity we set $Y = 1$ so that the budget constraint (1.2) then becomes

$$\bar{P}_1 X_1 + \bar{P}_2 X_2 = 1 \tag{1.3}$$

where $\bar{P}_1 = P_1/Y = P_1/1$, $\bar{P}_2 = P_2/Y = P_2/1$.

The consumer's usual maximization problem is depicted in quadrant 1 of Figure 1.1 where the optimum quantities chosen are (X_1^*, X_2^*) and U^* is the index of the (maximum) level of utility the consumer achieves given fixed prices and income as represented by the budget line AB. That is, $U^* = U(X_1^*, X_2^*)$.

The distance OB in Figure 1.1 represents the maximum amount of good one that the consumer could purchase with his income, assuming $X_2 = 0$. While the distance OA represents the maximum amount of good two that the consumer can purchase with his income assuming $X_1 = 0$. Solving for normalized prices for goods one and two:

$$X_1^{max} = 1/\bar{P}_1 = OB \tag{1.4}$$

$$X_2^{max} = 1/\bar{P}_2 = OA \tag{1.5}$$

The consumer, of course, can achieve utility level U^* with a higher (lower) $\bar{P}_1$ and a lower (higher) $\bar{P}_2$ keeping real income <u>constant</u> and equal