Convection Heat Transfer

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Readers should note that masculine pronouns are used throughout the book for succinctness and that they are intended to refer to both males and females.

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Convection
Heat Transfer

To
Mary Bejan,
to whom I owe
my style

Preface

My main reason for writing a convection textbook is to place the field's past 100 years of exponential growth in perspective. This book is intended for the educator who wants to present his students with more than a review of the generally accepted "classical" methods and conclusions. Through this book I hope to encourage the convection student to question what is known and to think freely and creatively about what is unknown.

There is no such thing as "unanimous agreement" on any topic. The history of scientific progress shows clearly that our present knowledge and understanding—contents of today's textbooks—are the direct result of conflict and controversy. By encouraging our students to question authority, we encourage them to make discoveries on their own. We can all only benefit from the scientific progress that results.

In writing this book, I sought to make available a textbook alternative that offers something new on two other fronts: (1) content, or the selection of topics, and (2) method, or the approach to solving problems in convection heat transfer.

Regarding content, this textbook reflects the relative change in the priorities set by our technological society over the past two decades. Historically, the field of convective heat transfer grew out of great engineering pursuits such as energy conversion (power plant technology), the aircraft, and the exploration of extraterrestrial space. Today, we are forced to face additional challenges, primarily in the areas of "energy" and "ecology." Briefly stated, engineering education today places a strong emphasis on man's need to coexist with the environment. This new emphasis is reflected in the topics assembled in this book. Important areas covered for the first time in a convection textbook are: (1) natural convection on *an equal footing* with forced convection, with application to energy conservation in buildings and to geophysical dynamics, (2) convection through porous media saturated with fluid, with application to geothermal and thermal insulation engineering, and (3) turbulent mixing in free-stream flow, with application to the dispersion of pollutants in the atmosphere and the hydrosphere.

Regarding method, in this book I made a consistent effort to teach problem solving (a *Solutions Manual* is available from the publisher or from me). This book is a textbook to be used for teaching a course, not a handbook. Of course, important engineering results are listed; however, the emphasis is placed on the thinking that leads to these results. A unique feature of this book is that it stresses the importance of correct scale analysis as an eligible and cost-effective method of solution, and as a precondition for more refined methods of solution. It also stresses the need for correct scaling in the graphic reporting of more refined analytical results and of experimental and numerical data. The cost and the "return on investment" associated with a possible method of solution are issues that each student-researcher should examine critically: these issues are stressed throughout the text. The place of computer-aided solutions in convection is the object of an entire chapter contributed jointly by Dimos Poulikakos and Shigeo Kimura. I am very grateful to them for this contribution.

I wrote this book during the academic year 1982–1983, in our mountain-side house on the greenbelt of North Boulder. This project turned out to be a highly rewarding intellectual experience for me, because it forced upon me the rare opportunity to think about an entire field, while continuing my own research on special topics in convection and other areas (specialization usually inhibits the ability to enjoy a bird's-eye-view of anything). It is a cliché in engineering education and research for the author of a new book to end the preface by thanking his family for the "sacrifice" that allowed completion of the work. My experience with writing *Convection Heat Transfer* has been totally different (i.e., much more enjoyable!), to the point that I must thank this book for making me work at home and for triggering so many inspiring conversations with Mary. Convection can be entertaining .

ADRIAN BEJAN

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A. B.

Contents

1

Fundamental Principles

Convective heat transfer or, simply, convection is the study of heat transport processes effected by the flow of fluids. The very word *convection* has its roots in the Latin verb *convehere* [1], which means *to bring together* or *to carry into one place* [2]. Convective heat transfer has grown to the status of a contemporary science because of man's desire to understand and predict the extent to which a fluid flow will act as "carrier" or "conveyor belt" for energy and matter. Convective heat transfer, clearly, is a field at the interface between two older fields—heat transfer and fluid mechanics. For this reason the study of any convective heat transfer problem must rest on a solid understanding of basic heat transfer and fluid mechanics principles. The objective of this first chapter is to review these principles in order to establish a common language to debate the more specific issues addressed in later chapters.

Before reviewing the foundations of convective heat transfer methodology, it is worth reexamining the historic relationship between fluid mechanics and heat transfer at the interface we call *convection*. Especially during the past 100 years, heat transfer and fluid mechanics have enjoyed a symbiotic relationship in their parallel development, a relationship where one field was stimulated by the curiosity in the other field. Examples of this symbiosis abound in the history of boundary layer theory and natural convection (see Chapters 2 and 4). The field of convection heat transfer grew out of this symbiosis and, if we are to learn anything from history, important advances in convection will continue to result from this symbiosis. Thus, the student and the future researcher would be well advised to devote equal attention to fluid mechanics and heat transfer literature.

MASS CONSERVATION

The first principle to review is undoubtedly the oldest: it has to do with the conservation of mass in a closed system or the "continuity" of mass through a flow (open) system. From engineering thermodynamics, we recall the mass conservation statement for a control volume [3]

$$\frac{\partial M_{cv}}{\partial t} = \sum_{\substack{\text{inlet}\\ \text{ports}}} \dot{m} - \sum_{\substack{\text{outlet}\\ \text{ports}}} \dot{m} \tag{1}$$

where M_{cv} is the mass instantaneously trapped inside the control volume (cv), while the $\dot{m}$'s are the mass flowrates associated with the flow into and out of the control volume. In convective heat transfer we are usually interested in the velocity and temperature distributions in some flow field near a solid wall; hence, the control volume to consider is the infinitesimally small $\Delta x \Delta y$ box drawn around a fixed location (x, y) in a flow field. In Fig. 1.1, as in most of the problems analyzed in this textbook, the flow field is two-dimensional (i.e.,. the same in any plane parallel to the plane of Fig. 1.1); in a three-dimensional flow field, the control volume of interest would be the parallelepiped $\Delta x \Delta y \Delta z$. Taking u and v as the local velocity components at point (x, y), the mass conservation equation (1) requires

$$\frac{\partial}{\partial t}(\rho \Delta x \Delta y) = \rho u \Delta y + \rho v \Delta x - \left[\rho u + \frac{\partial(\rho u)}{\partial x} \Delta x\right] \Delta y$$
$$- \left[\rho v + \frac{\partial(\rho v)}{\partial y} \Delta y\right] \Delta x \tag{2}$$

or, dividing through the constant size of the control volume $(\Delta x \Delta y)$,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{3}$$

In a three-dimensional flow, an analogous argument yields

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{4}$$

where w is the velocity component in the z direction.

The local mass conservation statement (4) can also be written as

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{5}$$

or

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0 \tag{6}$$

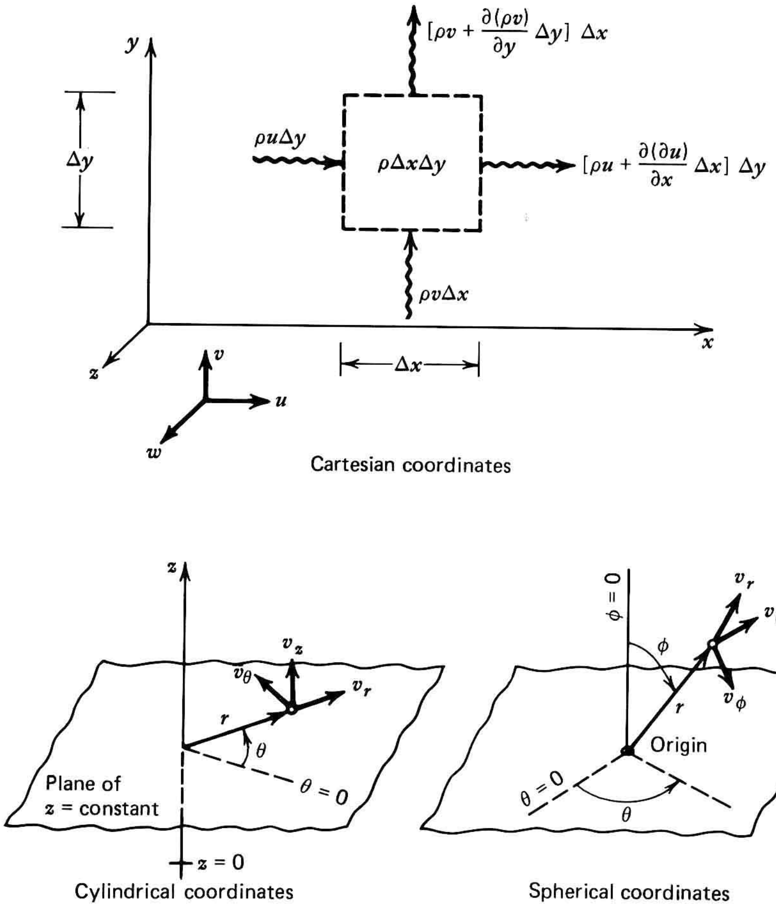

Figure 1.1 Systems of coordinates and mass conservation in a two-dimensional cartesian system.

In this last expression $\mathbf{v}$ is the velocity vector (u, v, w) while D/Dt represents the "material derivative" operator encountered frequently in convective heat and mass transfer,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \tag{7}$$

Of particular interest to the *classroom* treatment of the convection problem is the wide class of flows in which the temporal and spatial variations in density are negligible relative to the local variations in velocity. For this class, the mass conservation statement reads

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{8}$$

The equivalent forms of eq. (8) in cylindrical and spherical coordinates are (Fig. 1.1)

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \tag{9}$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial \phi}\left(v_\phi \sin\phi\right) + \frac{1}{\sin\phi}\frac{\partial v_\theta}{\partial \theta} = 0 \tag{10}$$

It is tempting to regard eqs. (8)–(10) as valid only for incompressible fluids; in fact, their derivation shows that they apply to flows (not fluids) where the density and velocity gradients are such that the $D\rho/Dt$ terms are negligible relative to the $\rho\nabla \cdot \mathbf{v}$ terms in eq. (6). Most of the gas flows encountered in heat exchangers, heated enclosures, and porous media obey the simplified version of the mass conservation principle [eqs. (8)–(10)].

FORCE BALANCES (MOMENTUM EQUATIONS)

From the dynamics of thrust or propulsion systems, we recall that the instantaneous force balance on a control volume requires [4]

$$\frac{\partial}{\partial t}\left(Mv_n\right)_{\text{cv}} = \sum F_n + \sum_{\substack{\text{inlet}\\ \text{ports}}} \left(\dot{m}v_n\right) - \sum_{\substack{\text{outlet}\\ \text{ports}}} \dot{m}v_n \tag{11}$$

where n is the direction chosen for analysis and (v_n, F_n) are the projections of fluid velocity and forces on the n direction. Equation (11) is recognized in the literature as the *momentum principle* or the *momentum theorem*: in essence, eq. (11) is the control volume formulation of Newton's Second Law of Motion where, in addition to terms accounting for *forces* and *mass* $\times$ *acceleration*, we now have the *impact* due to the flow of momentum into the control volume, plus the *reaction* associated with the flow of momentum out of the control volume. In the two-dimensional flow situation of Fig. 1.2, we can write two force balances of type (11), one for the x direction and the other for the y direction.

Consider now the special form taken by eq. (11) when applied to the finite-size control volume $\Delta x \Delta y$ drawn around point (x, y) in Fig. 1.2. Consider first the balance of forces in the x direction. In the top drawing of the $\Delta x \Delta y$ control volume, we see the sense of the impact and reaction forces associated with the flow of momentum through the control volume. In the bottom drawing, we see the more classical forces represented by the normal stress (σ_x), tangential stress (τ_{xy}), and the x body force per unit volume (X).