

LECTURE NOTES
IN PHYSICS

N. Akhmediev
A. Ankiewicz
(Eds.)

Dissipative Solitons



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Preface

The term “soliton” was invented to describe nonlinear solitary wave (localized) solutions of integrable equations such as the Korteweg de Vries and nonlinear Schrödinger equations. However, as localized nonlinear solutions also exist for a wide range of physical situations, this notion was soon extended to cover solitary wave solutions in conservative and non-conservative systems. Non-conservative or dissipative solitons have characteristics that are markedly different from those of conservative systems. The study of their properties has dramatically expanded in the last two decades, making the subject of dissipative solitons an almost independent area of research.

Most previous studies of optical solitons in conservative systems have focussed on the fact that the nonlinearity of the material counteracts the diffraction (spatial systems) or dispersion (temporal systems). This balance results in a localized structure that has been named a “soliton”. A localized structure in a dissipative system requires a continuous energy flow into the system, and, in particular, into the localized structure, in order to keep it “alive”. Hence, a separate balance, namely that between the energy input and energy output, is more important than that between the nonlinearity and dispersion (or diffraction). For dissipative systems, this idea of viewing structure formation as a consequence of energy flow introduces a new paradigm and allows us to understand pulses and forms appearing in many areas of physics, chemistry and biology.

Nicols and Prigogine showed, in the book “Self-organization in nonequilibrium systems” [John Wiley and sons, New York, 1977], that systems far from equilibrium are usually governed by nonlinear equations, and that the input of energy or matter allows for stable structures to form. They suggested that pre-biotic evolution could have involved numerous instabilities leading to the development of complexity. Thus the flow of energy and matter produce order of structure and function. This form of self-organization thus links the animate and the inanimate.

There have been assorted forerunners of this idea, some dating from ancient times. For example, in the first Chinese medical text (500 B.C.), it is explained that the flow of *ch'i* (energy) produces a balance between the *yin* and *yang* organs of the body. So these organs can be viewed as dissipative structures. If the flow stops, then these organs cannot continue with their

correct shapes, and illness occurs. The *I ching* (book of changes) emphasizes that changes and transformations occur because of movement and flow.

The increase in entropy was interpreted by Boltzmann as increasing disorganization. So an isolated system can only evolve to have entropy equal to or higher than the value it previously had. It is then clear that we need non-equilibrium open systems to allow structures to form spontaneously. They need to be able to exchange energy and matter with their environment. It is this imposition of outside energy that allows Bénard cells to form when a liquid is heated from below. Hexagons can form when the naturally-occurring fluctuations are “amplified”, thus leading to convection. Thus, while Newtonian mechanics deals with trajectories of point particles, and the second law of thermodynamics (mid 19th century) introduced irreversibility or directivity of time, the formation of dissipative structures due to energy flow in nonlinear systems can be regarded as a fully new paradigm of dynamics.

The notion of a dissipative soliton is a natural extension of these structures. It can be localized in space and time and have many features known from soliton theory. The cross-fertilization of ideas from both fields must be useful and productive.

This multi-author book is written by experts in this area. The chapters cover the progress in the field of dissipative solitons that has been made recently. It includes a study of dissipative solitons in a variety of media, in optics and reaction-diffusion systems, as well as some mathematical aspects of the theory of dissipative solitons.

There are many properties of dissipative solitons that are common to all these fields, and, unavoidably, there are differences. In one book, all these issues cannot be covered. The choice of material must necessarily be restricted. However, we have tried to make the choice of topics as wide as possible.

We start the book with a chapter discussing basic features of dissipative solitons of the one-dimensional Ginzburg-Landau and Swift-Hohenberg equations. We concentrate on those aspects of the problem that can be studied on the basis of a qualitative analysis of nonlinear dynamical systems.

Spatial dissipative solitons are discussed in two chapters. Firstly, the chapter by Boardman, Velasco and Egan deals with the subject of dissipative solitons in magneto-optics. Then, spatial dissipative solitons in semiconductor materials are discussed in the chapter by Ultanir, Stegeman, Michaelis, Lange and Lederer.

The three following chapters are devoted to two-dimensional dissipative solitons in optics. Ackemann and Firth give a review of phenomena related to dissipative solitons in driven optical cavities containing a nonlinear medium (cavity solitons) and similar phenomena (feedback solitons), where a driven nonlinear optical medium is in front of a single feedback mirror. Rozanov presents a review of the features of dissipative solitons in optical systems with nonlinear amplification and absorption, with no driving (holding) radiation, including cases with and without feedback. Taranenko, Sleky and

Weiss review pattern formation and experiments on semiconductor resonator solitons which are aimed at applications. Some close analogies of resonator solitons with the biological structures are also discussed.

Dissipative solitons in the time domain is the topic of Chaps. 7–10. The general properties of dissipative solitons in the time domain, and their particular application in all-optical transmission links, are explained by Peschel, Michaelis, Bakonyi, Onishchukov and Lederer. Laser systems with passive mode-locking are the ideal devices for studying optical dissipative solitons. Short – pulse generation by solid-state passively mode-locked lasers is reviewed by Cundiff. Soto-Crespo and Grelu present theoretical and experimental aspects of multi-soliton generation phenomena in fiber lasers. Various types of mode-locking in fiber lasers via nonlinear mode-locking are discussed in the chapter by Kutz.

Reaction-diffusion systems is a topic that includes a large class of systems from the areas of chemistry, physics, geology and biology. Dissipative solitons in reaction-diffusion systems are covered by Purwins, Bödeker and Liehr.

Dissipative solitons in discrete lattices with gain and loss provide one more facet of the subject, and these solitons have their own distinctive features and properties. Efremidis and Christodoulides present a review of recent results concerning dissipative lattices of Ginzburg-Landau type. The existence of dissipative solitons in various models of nonlinear lattices is examined by Abdullaev.

Bose-Einstein condensates form another type of system that admits dissipative solitons. The chapter by Konotop presents recent results obtained in this “hot” area of research.

The last three chapters are devoted to mathematical aspects of dissipative solitons. The search for closed-form analytic expressions for the solitary waves of nonlinear non-integrable partial differential equations, using non-perturbative techniques, is the topic of the chapter written by Conte and Musette. Stability issues are very important for dissipative solitons. The chapter by Kapitula presents ways of solving this problem for dissipative systems by using perturbation analysis and the Evans function. The chapter by S. R. Choudhury, considers bifurcations and strongly amplitude-modulated pulses of the complex Ginzburg-Landau equation.

Clearly, one book cannot cover all aspects of this rapidly-developing area. We hope that the publication of this book will not only assemble the issues related to dissipative solitons in one place, but will also raise new questions and facilitate further developments in this fascinating area of research.

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Canberra,
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Contents

Dissipative Solitons in the Complex Ginzburg-Landau and Swift-Hohenberg Equations

<i>N. Akhmediev and A. Ankiewicz</i>	1
1 What are Dissipative Solitons?	1
2 Mathematical Model	3
3 Are there Integrable Dissipative Systems?	4
4 Stationary Solitons as Fixed Points	5
5 Stability Analysis	6
6 Energy Flow Across a Soliton	7
7 Pulsating Soliton as a Limit Cycle	8
8 Period Doubling of Pulsating Solitons	9
9 Chaotic Soliton as a Strange Attractor	10
10 Soliton Explosions	11
11 Spectrum of Eigenvalues	12
12 Global Dynamics of the Exploding Soliton	14
13 Conclusion	16
References	17

Dissipative Magneto-Optic Solitons

<i>A.D. Boardman, L. Velasco, and P. Egan</i>	19
1 Introduction	19
2 The Basic Cubic-Quintic Complex Ginzburg-Landau Equation	20
3 Magneto-Optics with Inhomogeneous Magnetisation	22
4 Dissipative Solitons in Voigt Configuration	27
5 Optical Singularities in Dissipative Media	33
References	35

Dissipative Solitons in Semiconductor Optical Amplifiers

<i>E. Ulanir, G.I. Stegeman, D. Michaelis, C.H. Lange, and F. Lederer</i> ..	37
1 Introduction	37
2 Solitons in Uniformly-Pumped SOAs	39
3 Theory of Solitons in Periodically-Pumped SOAs	41
4 Sample Fabrication	46
5 Experiment Results	46

6	Soliton Interactions	49
7	Conclusion	53
	References	53

Dissipative Solitons in Pattern-Forming Nonlinear Optical Systems: Cavity Solitons and Feedback Solitons

<i>T. Ackemann and W.J. Firth</i>		55
1	Introduction	55
2	History	57
3	Mean-Field Models and Cavity Solitons	58
4	Self-Propelled Cavity Solitons in Semiconductor Microresonators	68
5	Solitons in a Single-Mirror Feedback Arrangement	74
6	Basic Results	79
7	Interaction Behavior	85
8	Applications	88
9	Conclusion	92
	References	93

Solitons in Laser Systems with Saturable Absorption

<i>N.N. Rosanov</i>		101
1	Introduction: Definitions, Examples, History	101
2	Model and Evolution Equation	104
3	Stationary Symmetric Solitons	107
4	Two-Dimensional Laser Solitons	110
5	Numerical Simulations of Asymmetric Solitons	115
6	Energy Flows and Soliton Internal Structure	119
7	Effect of Frequency Detunings and Bio-Solitons	127
	References	129

Spatial Resonator Solitons

<i>V.B. Taranenko, G. Slekyš, and C.O. Weiss</i>		131
1	Introduction: A Multi-Disciplinary View of Pattern Formation and Solitons	131
2	Proof-of-Existence Experiments on Resonator Solitons with Slow Materials	134
3	Semiconductor Resonator Solitons	136
4	Conclusion	158
	References	159

Dynamics of Dissipative Temporal Solitons

<i>U. Peschel, D. Michaelis, Z. Bakonyi, G. Onishchukov, and F. Lederer</i>		161
1	Introduction	161
2	Mathematical Models	164
3	Solutions and Their Stability	170
4	Soliton Experiments	174

5	Soliton Dynamics	175
6	Conclusions	180
	References	180

Soliton Dynamics in Mode-Locked Lasers

<i>S.T. Cundiff</i>	183
1 Mode-Locked Laser Basics	183
2 Soliton Polarization Evolution	185
3 Soliton Explosions	195
4 Carrier-Envelope Phase	200
5 Summary	204
References	205

Temporal Multi-Soliton Complexes Generated by Passively Mode-Locked Lasers

<i>J.M. Soto-Crespo and Ph. Grelu</i>	207
1 Introduction	207
2 Experimental Evidence for Multi-Soliton Formation in Passively Mode-Locked Laser Cavities	210
3 Multi-Soliton Complexes in Various Fiber Laser Configurations	214
4 Multi-Soliton Complexes in Distributed Models	223
5 Non-Distributed Model	231
6 Conclusions	236
References	237

Mode-Locking of Fiber Lasers via Nonlinear Mode-Coupling

<i>J.N. Kutz</i>	241
1 Introduction	241
2 Master Mode-Locking	243
3 Mode-Locking via Nonlinear Mode-Coupling	246
4 Conclusions and Discussion	262
References	263

Dissipative Solitons in Reaction-Diffusion Systems

<i>H.-G. Purwins, H.U. Bödeker, and A.W. Lichr</i>	267
1 Introduction	267
2 Mechanism of Pattern Formation in Reaction-Diffusion Systems	270
3 Numerical Investigations of the Three-Component System	285
4 Analytical Investigations	289
5 Planar Gas-Discharge Systems as Reaction-Diffusion Systems	298

6	Conclusion	305
	References	305
Discrete Ginzburg-Landau Solitons		
	<i>N.K. Efremidis and D.N. Christodoulides</i>	309
1	Introduction	309
2	Formulation	311
3	Linear Properties	311
4	Nonlinear Properties	317
	References	324
Discrete Dissipative Solitons		
	<i>F.Kh. Abdullaev</i>	327
1	Introduction	327
2	Model and Basic Equations	328
3	Exact Localized Solutions of DCGL Equation	337
4	Conclusion	339
	References	340
Nonlinear Schrödinger Equation with Dissipation: Two Models for Bose-Einstein Condensates		
	<i>V.V. Konotop</i>	343
1	Introduction	343
2	From a Three-Dimensional Gross-Pitaevskii Equation to the One-Dimensional Nonlinear Schrödinger Equation	344
3	Periodic Solutions	347
4	Management of Matter Waves Using the Feshbach Resonance	355
5	BEC in an Optical Lattice Controlled by the Feshbach Resonance	362
6	Modulation Instability of BEC in a Parabolic Trap	366
7	Concluding Remarks	369
	References	370
Solitary Waves of Nonlinear Nonintegrable Equations		
	<i>R. Conte and M. Musette</i>	373
1	Introduction	373
2	The Known Solutions of the Examples	376
3	Investigation of the Amount of Integrability	381
4	Selection of Possibly Single-Valued Dependent Variables	384
5	On the Cost of Obtaining Closed Form Expressions	385
6	First Class of Methods: Truncations	386
7	Second Class of Methods: First-Order Sub-Equation	398
8	Conclusion	402

9	Appendix. Classical Results on First-Order Autonomous Equations	404
	References	404

Stability Analysis of Pulses via the Evans Function:

Dissipative Systems

	<i>T. Kapitula</i>	407
1	Introduction	407
2	Basic Example	410
3	Construction of the Evans Function	413
4	The Linearization of the Nonlinear Schrödinger Equation	416
5	Dissipative Perturbations	421
	References	427

Bifurcations and Strongly Amplitude-Modulated Pulses of the Complex Ginzburg-Landau Equation

	<i>S.R. Choudhury</i>	429
1	Introduction	429
2	Background Properties of Coherent Structures of the CGLE Equation	430
3	Numerical Pulse Solutions	434
4	Bifurcations in the CGLE and Various Theoretical Approaches	438
5	Summary	442
	References	442

	Index	445
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Dissipative Solitons in the Complex Ginzburg-Landau and Swift-Hohenberg Equations

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Abstract. We explain the meaning of dissipative solitons and place them in a framework which shows their use in various scientific fields. Indeed, dissipative solitons form a new paradigm for the investigation of phenomena involving stable structures in nonlinear systems far from equilibrium. We consider those aspects of the problem that can be studied on the basis of a qualitative analysis of nonlinear systems.

1 What are Dissipative Solitons?

A dissipative soliton is a localized structure which exists for an extended period of time, even though parts of the structure experience gain and loss of energy and/or mass. This “structure” could be a profile of light intensity, temperature, magnetic field, etc. These solitons exist in “open” systems which are far from equilibrium. Thus energy and matter can flow into the system through its boundaries. The structure exists indefinitely in time, as long as the parameters in the system stay constant. It may evolve (i.e. change its shape periodically or otherwise) but it disappears when the source of energy or matter is switched off, or if the parameters of the system move outside the possible range of existence of the soliton.

In contrast to solitons in conservative systems, solitons in systems far from equilibrium are dynamical objects that have non-trivial internal energy flows. Since they are produced by dissipative systems, they depend strongly on an energy supply from an external source. Even if it is a stationary object, a dissipative soliton continuously re-distributes energy between its parts. A pump of energy is essential, and this means that the structures are defined by the rules of the system (gain, loss, dispersion, nonlinearity, etc.), rather than by the initial conditions [1]. Stationary solitons (pulses, fronts, etc.) can form where the overall gain and loss are balanced. A wide range of initial conditions can thus evolve into a dissipative soliton. These structures will typically appear in biochemical, optical and thermal systems, as they are “generic” and do not require particular formations to create them.