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(Eds.)

Dissipative Solitons

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Preface

The term "soliton" was invented to describe nonlinear solitary wave (localized) solutions of integrable equations such as the Korteveg de Vries and nonlinear Schrödinger equations. However, as localized nonlinear solutions also exist for a wide range of physical situations, this notion was soon extended to cover solitary wave solutions in conservative and non-conservative systems. Non-conservative or dissipative solitons have characteristics that are markedly different from those of conservative systems. The study of their properties has dramatically expanded in the last two decades, making the subject of dissipative solitons an almost independent area of research.

Most previous studies of optical solitons in conservative systems have focussed on the fact that the nonlinearity of the material counteracts the diffraction (spatial systems) or dispersion (temporal systems). This balance results in a localized structure that has been named a "soliton". A localized structure in a dissipative system requires a continuous energy flow into the system, and, in particular, into the localized structure, in order to keep it "alive". Hence, a separate balance, namely that between the energy input and energy output, is more important than than that between the nonlinearity and dispersion (or diffraction). For dissipative systems, this idea of viewing structure formation as a consequence of energy flow introduces a new paradigm and allows us to understand pulses and forms appearing in many areas of physics, chemistry and biology.

Nicols and Prigogine showed, in the book "Self-organization in nonequilibrium systems" [John Wiley and sons, New York, 1977], that systems far from equilibrium are usually governed by nonlinear equations, and that the input of energy or matter allows for stable structures to form. They suggested that pre-biotic evolution could have involved numerous instabilities leading to the development of complexity. Thus the flow of energy and matter produce order of structure and function. This form of self-organization thus links the animate and the inanimate.

There have been assorted forerunners of this idea, some dating from ancient times. For example, in the first Chinese medical text (500 B.C.), it is explained that the flow of *ch'i* (energy) produces a balance between the *yin* and *yang* organs of the body. So these organs can be viewed as dissipative structures. If the flow stops, then these organs cannot continue with their correct shapes, and illness occurs. The *I ching* (book of changes) emphasizes that changes and transformations occur because of movement and flow.

The increase in entropy was interpreted by Boltzmann as increasing disorganization. So an isolated system can only evolve to have entropy equal to or higher than the value it previously had. It is then clear that we need non-equilibrium open systems to allow structures to form spontaneously. They need to be able to exchange energy and matter with their environment. It is this imposition of outside energy that allows Bénard cells to form when a liquid is heated from below. Hexagons can form when the naturally-occurring fluctuations are "amplified", thus leading to convection. Thus, while Newtonian mechanics deals with trajectories of point particles, and the second law of thermodynamics (mid 19th century) introduced irreversibility or directivity of time, the formation of dissipative structures due to energy flow in nonlinear systems can be regarded as a fully new paradigm of dynamics.

The notion of a dissipative soliton is a natural extension of these structures. It can be localized in space and time and have many features known from soliton theory. The cross-fertilization of ideas from both fields must be useful and productive.

This multi-author book is written by experts in this area. The chapters cover the progress in the field of dissipative solitons that has been made recently. It includes a study of dissipative solitons in a variety of media, in optics and reaction-diffusion systems, as well as some mathematical apects of the theory of dissipative solitons.

There are many properties of dissipative solitons that are common to all these fields, and, unavoidably, there are differences. In one book, all these issues cannot be covered. The choice of material must necessarily be restricted. However, we have tried to make the choice of topics as wide as possible.

We start the book with a chapter discussing basic features of dissipative solitons of the one-dimensional Ginzburg-Landau and Swift-Hohenberg equations. We concentrate on those aspects of the problem that can be studied on the basis of a qualitative analysis of nonlinear dynamical systems.

Spatial dissipative solitons are discussed in two chapters. Firstly, the chapter by Boardman, Velasco and Egan deals with the subject of dissipative solitons in magneto-optics. Then, spatial dissipative solitons in semiconductor materials are discussed in the chapter by Ultanir, Stegeman, Michaelis, Lange and Lederer.

The three following chapters are devoted to two-dimensional dissipative solitons in optics. Ackemann and Firth give a review of phenomena related to dissipative solitons in driven optical cavities containing a nonlinear medium (cavity solitons) and similar phenomena (feedback solitons), where a driven nonlinear optical medium is in front of a single feedback mirror. Rozanov presents a review of the features of dissipative solitons in optical systems with nonlinear amplification and absorption, with no driving (holding) radiation, including cases with and without feedback. Taranenko, Slekys and

Weiss review pattern formation and experiments on semiconductor resonator solitons which are aimed at applications. Some close analogies of resonator solitons with the biological structures are also discussed.

Dissipative solitons in the time domain is the topic of Chaps. 7–10. The general properties of dissipative solitons in the time domain, and their particular application in all-optical transmission links, are explained by Peschel, Michaelis, Bakonyi, Onishchukov and Lederer. Laser systems with passive mode-locking are the ideal devices for studying optical dissipative solitons. Short – pulse generation by solid-state passively mode-locked lasers is reviewed by Cundiff. Soto-Crespo and Grelu present theoretical and experimental aspects of multi-soliton generation phenomena in fiber lasers. Various types of mode-locking in fiber lasers via nonlinear mode-locking are discussed in the chapter by Kutz.

Reaction-diffusion systems is a topic that includes a large class of systems from the areas of chemistry, physics, geology and biology. Dissipative solitons in reaction-diffusion systems are covered by Purwins, Bödeker and Liehr.

Dissipative solitons in discrete lattices with gain and loss provide one more facet of the subject, and these solitons have their own distinctive features and properties. Efremidis and Christodoulides present a review of recent results concerning dissipative lattices of Ginzburg-Landau type. The existence of dissipative solitons in various models of nonlinear lattices is examined by Abdullaev.

Bose-Einstein condensates form another type of system that admits dissipative solitons. The chapter by Konotop presents recents results obtained in this "hot" area of research.

The last three chapters are devoted to mathematical aspects of dissipative solitons. The search for closed-form analytic expressions for the solitary waves of nonlinear non-integrable partial differential equations, using non-perturbative techniques, is the topic of the chapter written by Conte and Musette. Stability issues are very important for dissipative solitons. The chapter by Kapitula presents ways of solving this problem for dissipative systems by using perturbation analysis and the Evans function. The chapter by S. R. Choudhury, considers bifurcations and strongly amplitude-modulated pulses of the complex Ginzburg-Landau equation.

Clearly, one book cannot cover all aspects of this rapidly-developing area. We hope that the publication of this book will not only assemble the issues related to dissipative solitons in one place, but will also raise new questions and facilitate further developments in this fascinating area of research.

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Dissipative Solitons in the Complex Ginzburg-Landau and Swift-Hohenberg Equations

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Abstract. We explain the meaning of dissipative solitons and place them in a framework which shows their use in various scientific fields. Indeed, dissipative solitons form a new paradigm for the investigation of phenomena involving stable structures in nonlinear systems far from equilibrium. We consider those aspects of the problem that can be studied on the basis of a qualitative analysis of nonlinear systems.

1 What are Dissipative Solitons?

A dissipative soliton is a localized structure which exists for an extended period of time, even though parts of the structure experience gain and loss of energy and/or mass. This "structure" could be a profile of light intensity, temperature, magnetic field, etc. These solitons exist in "open" systems which are far from equilibrium. Thus energy and matter can flow into the system through its boundaries. The structure exists indefinitely in time, as long as the parameters in the system stay constant. It may evolve (i.e. change its shape periodically or otherwise) but it disappears when the source of energy or matter is switched off, or if the parameters of the system move outside the possible range of existence of the soliton.

In contrast to solitons in conservative systems, solitons in systems far from equilibrium are dynamical objects that have non-trivial internal energy flows. Since they are produced by dissipative systems, they depend strongly on an energy supply from an external source. Even if it is a stationary object, a dissipative soliton continuously re-distributes energy between its parts. A pump of energy is essential, and this means that the structures are defined by the rules of the system (gain, loss, dispersion, nonlinearity, etc.), rather than by the initial conditions [1]. Stationary solitons (pulses, fronts, etc.) can form where the overall gain and loss are balanced. A wide range of initial conditions can thus evolve into a dissipative soliton. These structures will typically appear in biochemical, optical and thermal systems, as they are "generic" and do not require particular formations to create them.

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