ESSENTIALS OF ROBUST CONTROL

WITH
JOHN C. DOYLE

ESSENTIALS OF ROBUST CONTROL

KEMIN ZHOU

LOUISIANA STATE UNIVERSITY

with John C. Doyle

CALIFORNIA INSTITUTE OF TECHNOLOGY

Prentice Hall
Upper Saddle River, New Jersey 07458

Library of Congress Catalog in Publicaton Data
Zhou, Kemin.
Essentials of robust control / Kemin Zhou
p. cm.
Includes bibliographical references and index.
ISBN 0-13-525833-2
1. Control theory. 2.H_∞. I. Title.
QA402.3Z475 1998 97-29568
629.8--dc21 CIP

Publisher: Tom Robbins
Associate editor: Alice Dworkin
Editor-in-chief: Marcia Horton
Production editor: AnnMarie Longobardo
Copy editor: Patricia M. Daly
Director of production and manufacturing: David W. Riccardi
Art director: Jayne Conte
Managing editor: Bayani Mendoza de Leon
Cover designer: Bruce Kenselaar
Manufacturing buyer: Julia Meehan
Editorial assistant: Nancy Garcia

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-525833-2

Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada, Inc., Toronto
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall of Japan, Inc., Tokyo
Simon & Schuster Asia Pte. Ltd., Singapore
Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

To

my wife, my son,

my sister, and my brothers

Preface

Robustness of control systems to disturbances and uncertainties has always been the central issue in feedback control. Feedback would not be needed for most control systems if there were no disturbances and uncertainties. Developing multivariable robust control methods has been the focal point in the last two decades in the control community. The state-of-the-art $\mathcal{H}_\infty$ robust control theory is the result of this effort.

This book introduces some essentials of robust and $\mathcal{H}_\infty$ control theory. It grew from another book by this author, John C. Doyle, and Keith Glover, entitled *Robust and Optimal Control*, which has been extensively class-tested in many universities around the world. Unlike that book, which is intended primarily as a comprehensive reference of robust and $\mathcal{H}_\infty$ control theory, this book is intended to be a text for a graduate course in multivariable control. It is also intended to be a reference for practicing control engineers who are interested in applying the state-of-the-art robust control techniques in their applications. With this objective in mind, I have streamlined the presentation, added more than 50 illustrative examples, included many related MATLAB® commands[1] and more than 150 exercise problems, and added some recent developments in the area of robust control such as gap metric, ν-gap metric, model validation, and mixed μ problem. In addition, many proofs are completely rewritten and some advanced topics are either deleted completely or do not get an in-depth treatment.

The prerequisite for reading this book is some basic knowledge of classical control theory and state-space theory. The text contains more material than could be covered in detail in a one-semester or a one-quarter course. Chapter 1 gives a chapter-by-chapter summary of the main results presented in the book, which could be used as a guide for the selection of topics for a specific course. Chapters 2 and 3 can be used as a refresher for some linear algebra facts and some standard linear system theory. A course focusing on $\mathcal{H}_\infty$ control should cover at least most parts of Chapters 4–6, 8, 9, 11–13, and Sections 14.1 and 14.2. An advanced $\mathcal{H}_\infty$ control course should also include the rest of Chapter 14, Chapter 16, and possibly Chapters 10, 7, and 15. A course focusing on robustness and model uncertainty should cover at least Chapters 4, 5, and 8–10. Chapters 17 and 18 can be added to any advanced robust and $\mathcal{H}_\infty$ control course if time permits.

I have tried hard to eliminate obvious mistakes. It is, however, impossible for me to make the book perfect. Readers are encouraged to send corrections, comments, and

[1] MATLAB is a registered trademark of The MathWorks, Inc.

suggestions to me, preferably by electronic mail, at

kemin@ee.lsu.edu

I am also planning to put any corrections, modifications, and extensions on the Internet so that they can be obtained either from the following anonymous ftp:

ftp ee.lsu.edu cd pub/kemin/books/essentials/

or from the author's home page:

http://kilo.ee.lsu.edu/kemin/books/essentials/

This book would not be possible without the work done jointly for the previous book with Professor John C. Doyle and Professor Keith Glover. I thank them for their influence on my research and on this book. Their serious attitudes toward scientific research have been reference models for me. I am especially grateful to John for having me as a research fellow in Caltech, where I had two very enjoyable years and had opportunities to catch a glimpse of his "BIG PICTURE" of control.

I want to thank my editor from Prentice Hall, Tom Robbins, who originally proposed the idea for this book and has been a constant source of support for me while writing it. Without his support and encouragement, this project would have been a difficult one. It has been my great pleasure to work with him.

I would like to express my sincere gratitude to Professor Bruce A. Francis for giving me many helpful comments and suggestions on this book. Professor Francis has also kindly provided many exercises in the book. I am also grateful to Professor Kang-Zhi Liu and Professor Zheng-Hua Luo, who have made many useful comments and suggestions. I want to thank Professor Glen Vinnicombe for his generous help in the preparation of Chapters 16 and 17. Special thanks go to Professor Jianqing Mao for providing me the opportunity to present much of this material in a series of lectures at Beijing University of Aeronautics and Astronautics in the summer of 1996.

In addition, I would like to thank all those who have helped in many ways in making this book possible, especially Professor Pramod P. Khargonekar, Professor André Tits, Professor Andrew Packard, Professor Jie Chen, Professor Jakob Stoustrup, Professor Hans Henrik Niemann, Professor Malcolm Smith, Professor Tryphon Georgiou, Professor Tongwen Chen, Professor Hitay Özbay, Professor Gary Balas, Professor Carolyn Beck, Professor Dennis S. Bernstein, Professor Mohamed Darouach, Dr. Bobby Bodenheimer, Professor Guoxiang Gu, Dr. Weimin Lu, Dr. John Morris, Dr. Matt Newlin, Professor Li Qiu, Professor Hector P. Rotstein, Professor Andrew Teel, Professor Jagannathan Ramanujam, Dr. Linda G. Bushnell, Xiang Chen, Greg Salomon, Pablo A. Parrilo, and many other people.

I would also like to thank the following agencies for supporting my research: National Science Foundation, Army Research Office (ARO), Air Force of Scientific Research, and the Board of Regents in the State of Louisiana.

Finally, I would like to thank my wife, Jing, and my son, Eric, for their generous support, understanding, and patience during the writing of this book.

Kemin Zhou

Here is how $\mathcal{H}_\infty$ is pronounced in Chinese:

愛趣無窮

It means "The joy of love is endless."

Notation and Symbols

$\mathbb{R}$ and $\mathbb{C}$	fields of real and complex numbers
$\mathbb{F}$	field, either $\mathbb{R}$ or $\mathbb{C}$
$\mathbb{C}_-$ and $\overline{\mathbb{C}}_-$	open and closed left-half plane
$\mathbb{C}_+$ and $\overline{\mathbb{C}}_+$	open and closed right-half plane
$j\mathbb{R}$	imaginary axis
$\in$	belong to
$\subset$	subset
$\cup$	union
$\cap$	intersection
$\Box$	end of proof
$\diamond$	end of remark
$:=$	defined as
$\gtrapprox$ and $\lessapprox$	asymptotically greater and less than
$\gg$ and $\ll$	much greater and less than
$\overline{\alpha}$	complex conjugate of $\alpha \in \mathbb{C}$
$\|\alpha\|$	absolute value of $\alpha \in \mathbb{C}$
$Re(\alpha)$	real part of $\alpha \in \mathbb{C}$
I_n	$n \times n$ identity matrix
$[a_{ij}]$	a matrix with a_{ij} as its ith row and jth column element
$\text{diag}(a_1, \dots, a_n)$	an $n \times n$ diagonal matrix with a_i as its ith diagonal element
A^T and A^*	transpose and complex conjugate transpose of A
A^{-1} and A^+	inverse and pseudoinverse of A
A^{-*}	shorthand for $(A^{-1})^*$
$\det(A)$	determinant of A
$\text{trace}(A)$	trace of A

$\lambda(A)$	eigenvalue of A
$\rho(A)$	spectral radius of A
$\rho_R(A)$	real spectrum radius of A
$\overline{\sigma}(A)$ and $\underline{\sigma}(A)$	the largest and the smallest singular values of A
$\sigma_i(A)$	ith singular value of A
$\kappa(A)$	condition number of A
$\|A\|$	spectral norm of A: $\|A\| = \overline{\sigma}(A)$
Im(A), R(A)	image (or range) space of A
Ker(A), N(A)	kernel (or null) space of A
$\mathcal{X}_-(A)$	stable invariant subspace of A
Ric(H)	the stabilizing solution of an ARE
$g * f$	convolution of g and f
$\angle$	angle
$\langle , \rangle$	inner product
$x \perp y$	orthogonal, $\langle x, y\rangle = 0$
$D_\perp$	orthogonal complement of D
$S^\perp$	orthogonal complement of subspace S, e.g., $\mathcal{H}_2^\perp$
$\mathcal{L}_2(-\infty, \infty)$	time domain square integrable functions
$\mathcal{L}_{2+} := \mathcal{L}_2[0, \infty)$	subspace of $\mathcal{L}_2(-\infty, \infty)$ with functions zero for $t < 0$
$\mathcal{L}_{2-} := \mathcal{L}_2(-\infty, 0]$	subspace of $\mathcal{L}_2(-\infty, \infty)$ with functions zero for $t > 0$
$\mathcal{L}_2(j\mathbb{R})$	square integrable functions on $\mathbb{C}_0$ including at ∞
$\mathcal{H}_2$	subspace of $\mathcal{L}_2(j\mathbb{R})$ with functions analytic in $\mathrm{Re}(s) > 0$
$\mathcal{H}_2^\perp$	subspace of $\mathcal{L}_2(j\mathbb{R})$ with functions analytic in $\mathrm{Re}(s) < 0$
$\mathcal{L}_\infty(j\mathbb{R})$	functions bounded on $\mathrm{Re}(s) = 0$ including at ∞
$\mathcal{H}_\infty$	the set of $\mathcal{L}_\infty(j\mathbb{R})$ functions analytic in $\mathrm{Re}(s) > 0$
$\mathcal{H}_\infty^-$	the set of $\mathcal{L}_\infty(j\mathbb{R})$ functions analytic in $\mathrm{Re}(s) < 0$
prefix **B** and **B**$^{\mathrm{o}}$	*closed* and *open* unit ball, e.g. $\mathbf{B}\boldsymbol{\Delta}$ and $\mathbf{B}^{\mathrm{o}}\boldsymbol{\Delta}$
prefix $\mathcal{R}$	real rational, e.g., $\mathcal{RH}_\infty$ and $\mathcal{RH}_2$, etc.
$\mathcal{R}_p(s)$	rational proper transfer matrices
$G^\sim(s)$	shorthand for $G^T(-s)$
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	shorthand for state space realization $C(sI - A)^{-1}B + D$
$\eta(G(s))$	number of right-half plane poles
$\eta_0(G(s))$	number of imaginary axis poles
$wno(G)$	winding number
$\mathcal{F}_\ell(M, Q)$	lower LFT
$\mathcal{F}_u(M, Q)$	upper LFT
$M \star N$	star product

List of Acronyms

ARE	algebraic Riccati equation
FDLTI	finite dimensional linear time invariant
iff	if and only if
lcf	left coprime factorization
LFT	linear fractional transformation
lhp or LHP	left-half plane $\text{Re}(s) < 0$
LQG	linear quadratic Gaussian
LTI	linear time invariant
MIMO	multi-input multioutput
nlcf	normalized left coprime factorization
NP	nominal performance
nrcf	normalized right coprime factorization
NS	nominal stability
rcf	right coprime factorization
rhp or RHP	right-half plane $\text{Re}(s) > 0$
RP	robust performance
RS	robust stability
SISO	single-input single-output
SSV	structured singular value (μ)
SVD	singular value decomposition

Contents

Preface vii

Notation and Symbols xv

List of Acronyms xvii

1 Introduction 1

1.1 What Is This Book About? 1
1.2 Highlights of This Book 3
1.3 Notes and References 9
1.4 Problems 10

2 Linear Algebra 11

2.1 Linear Subspaces 11
2.2 Eigenvalues and Eigenvectors 12
2.3 Matrix Inversion Formulas 13
2.4 Invariant Subspaces 15
2.5 Vector Norms and Matrix Norms 16
2.6 Singular Value Decomposition 19
2.7 Semidefinite Matrices 23
2.8 Notes and References 24
2.9 Problems 24

3 Linear Systems 27

3.1 Descriptions of Linear Dynamical Systems 27
3.2 Controllability and Observability 28
3.3 Observers and Observer-Based Controllers 31
3.4 Operations on Systems 34
3.5 State-Space Realizations for Transfer Matrices 35
3.6 Multivariable System Poles and Zeros 38
3.7 Notes and References 41
3.8 Problems 42

4 $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Spaces **45**
4.1 Hilbert Spaces 45
4.2 $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Spaces 47
4.3 Computing $\mathcal{L}_2$ and $\mathcal{H}_2$ Norms 53
4.4 Computing $\mathcal{L}_\infty$ and $\mathcal{H}_\infty$ Norms 55
4.5 Notes and References 61
4.6 Problems 62

5 Internal Stability **65**
5.1 Feedback Structure 65
5.2 Well-Posedness of Feedback Loop 66
5.3 Internal Stability 68
5.4 Coprime Factorization over $\mathcal{RH}_\infty$ 71
5.5 Notes and References 77
5.6 Problems 77

6 Performance Specifications and Limitations **81**
6.1 Feedback Properties 81
6.2 Weighted $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Performance 85
6.3 Selection of Weighting Functions 89
6.4 Bode's Gain and Phase Relation 94
6.5 Bode's Sensitivity Integral 98
6.6 Analyticity Constraints 100
6.7 Notes and References 102
6.8 Problems 102

7 Balanced Model Reduction **105**
7.1 Lyapunov Equations 106
7.2 Balanced Realizations 107
7.3 Model Reduction by Balanced Truncation 117
7.4 Frequency-Weighted Balanced Model Reduction 124
7.5 Notes and References 126
7.6 Problems 127

8 Uncertainty and Robustness **129**
8.1 Model Uncertainty 129
8.2 Small Gain Theorem 137
8.3 Stability under Unstructured Uncertainties 141
8.4 Robust Performance 147
8.5 Skewed Specifications 150
8.6 Classical Control for MIMO Systems 154
8.7 Notes and References 157
8.8 Problems 158

9 Linear Fractional Transformation 165
9.1 Linear Fractional Transformations . . . 165
9.2 Basic Principle . . . 173
9.3 Redheffer Star Products . . . 178
9.4 Notes and References . . . 180
9.5 Problems . . . 181

10 μ and μ Synthesis 183
10.1 General Framework for System Robustness . . . 184
10.2 Structured Singular Value . . . 187
10.3 Structured Robust Stability and Performance . . . 200
10.4 Overview of μ Synthesis . . . 213
10.5 Notes and References . . . 216
10.6 Problems . . . 217

11 Controller Parameterization 221
11.1 Existence of Stabilizing Controllers . . . 222
11.2 Parameterization of All Stabilizing Controllers . . . 224
11.3 Coprime Factorization Approach . . . 228
11.4 Notes and References . . . 231
11.5 Problems . . . 231

12 Algebraic Riccati Equations 233
12.1 Stabilizing Solution and Riccati Operator . . . 234
12.2 Inner Functions . . . 245
12.3 Notes and References . . . 246
12.4 Problems . . . 246

13 $\mathcal{H}_2$ Optimal Control 253
13.1 Introduction to Regulator Problem . . . 253
13.2 Standard LQR Problem . . . 255
13.3 Extended LQR Problem . . . 258
13.4 Guaranteed Stability Margins of LQR . . . 259
13.5 Standard $\mathcal{H}_2$ Problem . . . 261
13.6 Stability Margins of $\mathcal{H}_2$ Controllers . . . 265
13.7 Notes and References . . . 267
13.8 Problems . . . 267

14 $\mathcal{H}_\infty$ Control 269
14.1 Problem Formulation . . . 269
14.2 A Simplified $\mathcal{H}_\infty$ Control Problem . . . 270
14.3 Optimality and Limiting Behavior . . . 282
14.4 Minimum Entropy Controller . . . 286
14.5 An Optimal Controller . . . 286

14.6 General $\mathcal{H}_\infty$ Solutions 288
14.7 Relaxing Assumptions 291
14.8 $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Integral Control 294
14.9 $\mathcal{H}_\infty$ Filtering 297
14.10Notes and References 299
14.11Problems 300

15 Controller Reduction **305**
15.1 $\mathcal{H}_\infty$ Controller Reductions 306
15.2 Notes and References 312
15.3 Problems 313

16 $\mathcal{H}_\infty$ Loop Shaping **315**
16.1 Robust Stabilization of Coprime Factors 315
16.2 Loop-Shaping Design 325
16.3 Justification for $\mathcal{H}_\infty$ Loop Shaping 328
16.4 Further Guidelines for Loop Shaping 334
16.5 Notes and References 341
16.6 Problems 342

17 Gap Metric and ν-Gap Metric **349**
17.1 Gap Metric 350
17.2 ν-Gap Metric 357
17.3 Geometric Interpretation of ν-Gap Metric 370
17.4 Extended Loop-Shaping Design 373
17.5 Controller Order Reduction 375
17.6 Notes and References 375
17.7 Problems 375

18 Miscellaneous Topics **377**
18.1 Model Validation 377
18.2 Mixed μ Analysis and Synthesis 381
18.3 Notes and References 389
18.4 Problems 390

Bibliography **391**

Index **407**

Chapter 1

Introduction

This chapter gives a brief description of the problems considered in this book and the key results presented in each chapter.

1.1 What Is This Book About?

This book is about basic robust and $\mathcal{H}_\infty$ control theory. We consider a control system with possibly multiple sources of uncertainties, noises, and disturbances as shown in Figure 1.1.

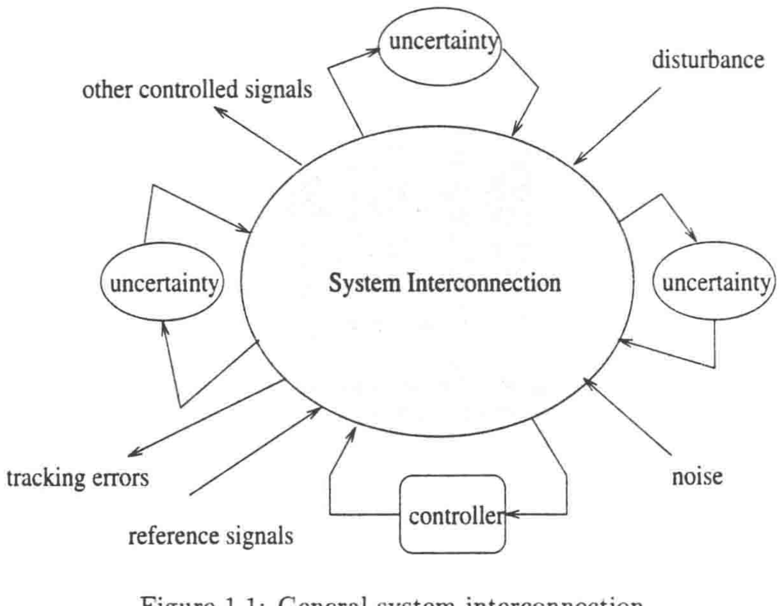

Figure 1.1: General system interconnection

We consider mainly two types of problems:

- Analysis problems: Given a controller, determine if the controlled signals (including tracking errors, control signals, etc.) satisfy the desired properties for all admissible noises, disturbances, and model uncertainties.
- Synthesis problems: Design a controller so that the controlled signals satisfy the desired properties for all admissible noises, disturbances, and model uncertainties.

Most of our analysis and synthesis will be done on a unified linear fractional transformation (LFT) framework. To that end, we shall show that the system shown in Figure 1.1 can be put in the general diagram in Figure 1.2, where P is the interconnection matrix, K is the controller, Δ is the set of all possible uncertainty, w is a vector signal including noises, disturbances, and reference signals, z is a vector signal including all controlled signals and tracking errors, u is the control signal, and y is the measurement.

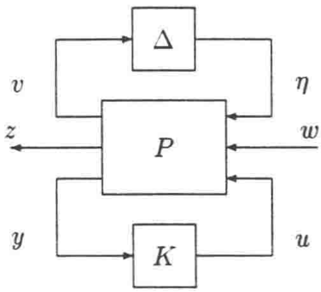

Figure 1.2: General LFT framework

The block diagram in Figure 1.2 represents the following equations:

$$\begin{aligned} \begin{bmatrix} v \\ z \\ y \end{bmatrix} &= P \begin{bmatrix} \eta \\ w \\ u \end{bmatrix} \\ \eta &= \Delta v \\ u &= Ky. \end{aligned}$$

Let the transfer matrix from w to z be denoted by T_{zw} and assume that the admissible uncertainty Δ satisfies $\overline{\sigma}(\Delta) < 1/\gamma_u$ for some $\gamma_u > 0$. Then our analysis problem is to answer if the closed-loop system is stable for all admissible Δ and $\|T_{zw}\|_\infty \leq \gamma_p$ for some prespecified $\gamma_p > 0$, where $\|T_{zw}\|_\infty$ is the $\mathcal{H}_\infty$ norm defined as $\|T_{zw}\|_\infty = \sup_\omega \overline{\sigma}\left(T_{zw}(j\omega)\right)$. The synthesis problem is to design a controller K so that the aforementioned robust stability and performance conditions are satisfied.

In the simplest form, we have either $\Delta = 0$ or $w = 0$. The former becomes the well-known $\mathcal{H}_\infty$ control problem and the later becomes the robust stability problem. The two problems are equivalent when Δ is a single-block unstructured uncertainty through the application of the small gain theorem (see Chapter 8). This robust stability consequence was probably the main motivation for the development of $\mathcal{H}_\infty$ methods.

The analysis and synthesis for systems with multiple-block Δ can be reduced in most cases to an equivalent $\mathcal{H}_\infty$ problem with suitable scalings. Thus a solution to the $\mathcal{H}_\infty$ control problem is the key to all robustness problems considered in this book. In the next section, we shall give a chapter-by-chapter summary of the main results presented in this book.

We refer readers to the book *Robust and Optimal Control* by K. Zhou, J. C. Doyle, and K. Glover [1996] for a brief historical review of $\mathcal{H}_\infty$ and robust control and for some detailed treatment of some advanced topics.

1.2 Highlights of This Book

The key results in each chapter are highlighted in this section. Readers should consult the corresponding chapters for the exact statements and conditions.

Chapter 2 reviews some basic linear algebra facts.

Chapter 3 reviews system theoretical concepts: controllability, observability, stabilizability, detectability, pole placement, observer theory, system poles and zeros, and state-space realizations.

Chapter 4 introduces the $\mathcal{H}_2$ spaces and the $\mathcal{H}_\infty$ spaces. State-space methods of computing real rational $\mathcal{H}_2$ and $\mathcal{H}_\infty$ transfer matrix norms are presented. For example, let

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \in \mathcal{RH}_\infty.$$

Then

$$\|G\|_2^2 = \operatorname{trace}(B^*QB) = \operatorname{trace}(CPC^*)$$

and

$$\|G\|_\infty = \max\{\gamma : \; H \text{ has an eigenvalue on the imaginary axis}\},$$

where P and Q are the controllability and observability Gramians and

$$H = \begin{bmatrix} A & BB^*/\gamma^2 \\ -C^*C & -A^* \end{bmatrix}.$$