

# **EVALUATION AND DECISION MODELS**

## **A Critical Perspective**

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Patrice Perny  
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a critical perspective

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# INTRODUCTION

## 1.1 Motivations

Deciding is a very complex and difficult task. Some people even argue that our ability to make decisions in complex situations is the main feature that distinguishes us from animals (it is also common to say that laughing is the main difference). Nevertheless, when the task is too complex or the interests at stake are too important, it quite often happens that we do not know or we are not sure what to decide and, in many instances, we resort to a decision support technique: an informal one—we toss a coin, we ask an oracle, we visit an astrologer, we consult an expert, we think—or a formal one. Although informal decision support techniques can be of interest, in this book, we will focus on formal ones. Among the latter, we find some well-known decision support techniques: cost-benefit analysis, multiple criteria decision analysis, decision trees, ... But there are many other ones, sometimes not presented as decision support techniques, that help making decisions. Let us cite but a few examples.

- When the director of a school must decide whether a given student will pass or fail, he usually asks each teacher to assess the merits of the student by means of a grade. The director then sums the grades and compares the result to a threshold.
- When a bank must decide whether a given client will obtain a credit or not, a technique, called credit scoring, is often used.
- When the mayor of a city decides to temporarily forbid car traffic in a city because of air pollution, he probably takes the value of some indicators, e.g. the air quality index, into account.
- Groups or committees must also make decisions. In order to do so, they often use voting procedures.

All these formal techniques are what we call (formal) *decision and evaluation models*, i.e. a set of explicit and well-defined rules to collect, assess and process information in order to be able to make recommendations in decision and/or evaluation processes. They are so widespread that almost no one can pretend he is not using or suffering the consequences of one of them. These models—probably because of their formal character—inspire respect and trust: they look scientific. But are they really well founded? Do they perform as well as we want? Can we safely rely on them when we have to make important decisions?

That is why we try to look at formal decision and evaluation models with a critical eye in this book. You guessed it: this book is more than 200 pages long. So, there is probably a lot of criticism. You are right.

None of the evaluation and decision models that we examined are perfect or the best. They all suffer limitations. For each one, we can find situations in which it will perform very poorly. This is not really new: most decision models have had contenders for a long time. Do we want to contend all models at the same time? Definitely not! Our conviction is that there cannot be a best decision or evaluation model—this has been proved in some contexts (e.g. in voting) and seems empirically correct in other contexts—but we are convinced as well that formal evaluation and decision models are useful in many circumstances and here is why:

- Formal models provide explicit and, to a large extent, unambiguous representations of a given problem; they offer a common language for communicating about the problem. They are therefore particularly well suited for facilitating communication among the actors of a decision or evaluation process.
- Formal models require that the decision maker makes a substantial effort to structure his perception or representation of the problem. This effort can only be beneficial as it forces the decision maker to think harder and deeper about his problem.
- Once a formal model has been established, a battery of formal techniques (often implemented on a computer) become available for drawing any kind of conclusion that can be drawn from the model. For example, hundreds of what-if questions can be answered in a flash. This can be of great help if we want to devise robust recommendations.

For all these reasons (complexity, usefulness, importance of the interests at stake, popularity) plus the fact that formal models lend themselves easily to criticism, we think that it is important to deepen our understanding of evaluation and decision models and encourage their users to think more thoroughly about them.

Our aim with this book is to foster reflection and critical thinking among all individuals utilising decision and evaluation models, whether it be for research or applications.

## 1.2 Audience

Most of us are confronted with formal evaluation and decision models. Very often, we use them without even thinking about it. This book is intended for the aware or enlightened practitioner, for anyone who uses decision or evaluation models—for research or for applications—and is willing to question his practice, to have a deeper understanding of what he does. We have tried to keep mathematics and formalism at a very low level so that, hopefully, most of the material will be accessible to the not mathematically-inclined readers. A rich bibliography will allow the interested reader to locate the more technical literature easily.

## 1.3 Structure

There are so many decision and evaluation models that it would be impossible to deal with all of them within a single book. As will become apparent later, most of them rely on similar kinds of principles. We decided to present seven examples of such models. These examples, chosen in a wide variety of domains, will hopefully allow the reader to grasp these principles. Each example is presented in a chapter (Chapters 2 to 8), almost independent of the other chapters. Each of these seven chapters ends with a conclusion, placing what has been discussed in a broader context and indicating links with other chapters. Chapter 9 is somewhat different from the seven previous ones: it does not focus on a decision model but presents a real world application. The aim of this chapter is to emphasise the importance of the decision aiding process (the context of the problem, the position of the actors and their interactions, the role of the analyst, ...), to show that many difficulties arise there as well and that a coherence between the decision aiding process and the formal model is necessary.

Some examples have been chosen because they correspond to decision models that everyone has experienced and can understand easily (student grades and voting). We chose some models because they are not often perceived as decision or evaluation models (student grades, indicators and rule based control). The other examples (cost-benefit analysis, multiple criteria decision support and choice under uncertainty) correspond to well identified and popular evaluation and decision models.

## 1.4 Outline

Chapter 2 is devoted to the problem of voting. After showing the analogy between voting and multiple criteria decision support, we present a sequence of twelve short examples, each one illustrating a problem that arises with a particular voting method. We begin with simple methods based on pairwise comparisons and we end up with the Borda method. Although the goal of

this book is not to overwhelm the reader with theory, we informally present two theorems (Arrow and Gibbard-Satterthwaite) that in one way or another explain why we encountered so many difficulties in our twelve examples.

Then we turn to the way voters' preferences are modelled. We present many different models, each one trying to outdo the previous one but suffering its own weaknesses. Finally, we explore some issues that are often neglected: who is going to vote? Who are the candidates? These questions are difficult and we show that they are important. The construction of the set of voters and the set of candidates, as well as the choice of a voting method must be considered as part of the voting process.

After examining voting, we turn in Chapter 3 to another very familiar topic for the reader: students' marks or grades. Marks are used for different purposes (e.g. ranking the students, deciding whether a student is allowed to begin the next level of study, deciding whether a student gets a degree, ...). Students are assessed in a huge variety of ways in different countries and schools. This seems to indicate that assessing students might not be trivial. We use this familiar topic to discuss operations such as evaluating a performance and aggregating evaluations.

In Chapter 4, three particular indicators are considered: the Human Development Index (used by the United Nations), the ATMO index (an air pollution indicator used by the French government) and the decathlon score. We present a few examples illustrating some problems occurring with indicators. We assert that some difficulties are the consequences of the fact that the role of an indicator is often manifold and not well defined. An indicator is a measure but, often, it is also a tool for controlling or managing (in a broad sense).

Cost-benefit analysis (CBA) is a decision aiding method that is extremely popular among economists. Following the CBA approach, a project should only be undertaken when its benefits outweigh its costs. First we present the principles of CBA and its theoretical foundations. Then, using an example in transportation studies, we illustrate some difficulties encountered with CBA. Finally, we clarify some of the hypotheses at the heart of CBA and criticise the relevance of these hypotheses in some decision aiding processes.

In Chapter 6, using a well documented example, we present some difficulties that arise when one wants to choose from or rank a set of alternatives considered from different viewpoints. We examine several aggregation methods that lead to a value function on the set of alternatives, namely the weighted sum, the sum of utilities (direct and indirect assessment) and AHP (the Analytic Hierarchy Process). Then we turn to the so called outranking methods. Some of these methods can be used even when the data are not very rich or precise. The price we pay for this is that results provided by these methods are not rich either, in the sense that conclusions that can be drawn regarding a decision are not clear-cut.

Chapter 7 is dedicated to the study of automatic decision systems. These systems concern the execution of repetitive decision tasks and the great majority of them are based on more or less explicit decision rules aimed towards

reflecting the usual decision policy of humans. The goal of this section is to show the interest of some formal tools (e.g. fuzzy sets) to model decision rules but also to clarify some problems arising when simulating the rules. Three examples are presented: the first one concerns the control of an automatic watering system while the others are about the control of a food process. The first two examples describe decision systems based on explicit decision rules; the third one addresses the case of implicit decision rules.

The goal of Chapter 8 is to raise some questions about the modelling of uncertainty. We present a real-life problem concerning the planning of electricity production. This problem is characterised by many different uncertainties: for example, the price of oil or the electricity demand in 20 years time. This problem is classically described by using a decision tree and solved with an expected utility approach. After recalling some well known criticisms directed against this approach, we present the approach that has been used by the team that “solved” this problem. Some of the drawbacks of this approach are discussed as well. The relevance of probabilities is criticised and other modelling tools, such as belief functions, fuzzy set theory and possibility theory, are briefly mentioned.

Convinced that there is more to decision aiding than just number crunching, we devote the last chapter to the description of a real world decision aiding process that took place in a large Italian company a few years ago. It concerns the evaluation of offers following a call for tenders for a GIS (Geographical Information System) acquisition. Some important elements such as the participating actors, the problem formulation, the construction of the criteria, etc. deserve greater consideration. One should ideally never consider these elements separately from the aggregation process because they can impact the whole decision process and even the way the aggregation procedure behaves.

## 1.5 Who are the authors ?

The authors of this book are European academics working in six different universities, in France and in Belgium. They teach in engineering, business, mathematics, computer science and psychology schools. Their background is quite varied as well: mathematics, economics, engineering, law and geology but they are all active in decision support and more particularly in multiple criteria decision support. Among their special interests are preference modelling, fuzzy logic, aggregation techniques, social choice theory, artificial intelligence, problem structuring, measurement theory, operations research, ... Besides their interest in multiple criteria decision support, they share a common view on this field. Five of the six authors of the present volume presented their thoughts on the past and the objectives of future research in multiple criteria decision support in the *Manifesto of the new MCDA era* (Bouyssou et al. 1993).

The authors are very active in theoretical research on the foundations of decision aiding, mainly from an axiomatic point of view, but have been involved in a variety of applications ranging from software evaluation to location of a

nuclear repository, through the rehabilitation of a sewer network or the location of high-voltage lines.

In spite of the large number of co-authors, this book is not a collection of papers. It is a joint work.

## 1.6 Conventions

To refer to a decision maker, a voter or an individual whose sex is not determined, we decided not to use the politically correct “he/she” but just “he” in order to make the text easy to read. The fact that all of the authors are male has nothing to do with this choice. The same applies for “his/her”.

None of the authors is a native English speaker. Therefore, even if we did our best to write in correct English, the reader should not be surprised to find some mistakes or inelegant expressions. We beg the reader’s leniency for any incorrectness that might remain.

The adopted spelling is the British and not the American one.

## 1.7 Acknowledgements

We are ggreatly indebted to our ~~colleague~~ friend Philippe Fortemps  
\cite{Fortemps99}.

Without him and his knowledge of Late- this paragraph.%\newline  
x, this book would look like

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## CHOOSING ON THE BASIS OF SEVERAL OPINIONS: THE EXAMPLE OF VOTING

Voting is easy! You've voted hundreds of times in committees, in presidential elections, for the senate, ... Is there much to say about voting? Well, just think about the way heads of state or members of parliament are elected in Australia, France, the UK, ...

**United Kingdom's members of parliament** The territory of the UK is divided into about 650 constituencies. One representative is elected in each constituency. Each voter chooses one of the candidates in his constituency. The winner is the candidate that is chosen by more voters than any other one. Note that the winner does not have to win an overall majority of votes.

**France's members of parliament** As in the UK, the French territory is divided into single-seat constituencies. In a constituency, each voter chooses one of the candidates. If one candidate receives more than 50 % of the votes, he is elected. Otherwise a second stage is organised. During the second stage, all candidates that were chosen by more than 12.5 % of the registered voters may compete. Once more, each voter chooses one of the candidates. The winner is the candidate that received the most votes.

**France's president** Each voter chooses one of the candidates. If one candidate has been chosen by more than 50 % of the voters, he is elected. Otherwise a second stage is organised. During the second stage, only two candidates remain: those with the highest scores. Once again, each voter chooses one of the candidates. The winner is the candidate that has been chosen by more voters than the other one.

**Australia's members of parliament** The territory is divided into single-seat constituencies called divisions. In a division, each voter is asked to rank all candidates: he puts a 1 next to his preferred candidate, a 2 next to



his second preferred candidate, then a 3, and so on until his least preferred candidate. Then the ballot papers are sorted according to the first preference votes. If a candidate has more than 50 % of the ballot papers, he is elected. Otherwise, the candidate that received fewer papers than any other is eliminated and the corresponding ballot papers are transferred to the candidates that got a 2 on these papers. Once more, if a candidate has more than 50 % of the ballot papers, he is elected. Otherwise, the candidate that received fewer papers than any other is eliminated and the corresponding ballot papers are transferred to the candidates that got a 3 on these papers, etc. In the worst case, this process ends when all but two candidates are eliminated, because, unless they are tied, one of the candidates necessarily has more than 50 % of the papers. Note that, as far as we know, it seems that the case of a tie is seldom considered in electoral laws.

**Canada's members of parliament and prime minister** Every five years, the Canadian parliament is elected as follows. The territory is divided into about 270 constituencies called counties. In each county, each party can present one candidate. Each voter chooses one candidate. The winner in a county is the candidate that is chosen by more voters than any other one. He is thus the county's representative in the parliament. The leader of the party that has the most representatives becomes prime minister.

Those interested in voting methods and the way they are applied in various countries will find valuable information in Farrell (1997) and Nurmi (1987). The diversity of the methods applied in practice probably reflects some underlying complexity and, in fact, if you take a closer look at voting, you will be amazed by the incredible complexity of the subject. In spite of its apparent simplicity, thousands of papers have been devoted to the problem of voting (Kelly 1991) and our guess is that many more are to come.

Our aim in this chapter is, on the one hand, to show that many difficult and interesting problems arise in voting and, on the other hand, to convince the reader that a formal study of voting might be enlightening. This chapter is organised as follows. In Section 1, we make the following basic assumption: each voter's preferences can accurately be represented by a ranking of all candidates from best to worse, without ties. Then we show some problems occurring when aggregating the rankings, using classical voting systems such as those applied in France or the United Kingdom. We do this through the use of small and classical examples. In Section 2, we consider other preference models than the linear ranking of Section 1. Some models are poorer in information but more realistic. Some are richer and less realistic. In most cases, the aggregation remains a difficult task. In Section 3, we change the focus and try to examine voting in a much broader context. Voting is not instantaneous. It is not just counting the votes and performing some mathematical operation to find the winner. It is a process that begins when somebody decides that a vote should occur (or even earlier) and ends when the winner begins his mandate (or even