

FOURTH EDITION

Moon

Net sales

\$20 billion

15

10

IOHN C. HULL

OPTIONS, FUTURES, & OTHER DERIVATIVES

Fourth Edition

John C. Hull

Joseph L. Rotman School of Management,
University of Toronto

Acquisitions Editor: Paul Donnelly
Editorial Assistant: Cheryl Clayton
Editor-in-Chief: PJ Boardman
Marketing Editor: Lori Braumberger
Production Editor: Richard DeLorent

Marketing Editor: Lori Braumberger
Production Editor: Richard DeLorenzo
Permissions Coordinator: Monica Stipanov
Associate Managing Editor: Cynthia Regan
Manufacturing Buyer: Lisa DiMaulo
Manufacturing Supervisor: Paul Smolenski
Manufacturing Manager: Vincent Scelta

Designer: Kevin Kall

Design Manager: Patricia Smythe
Cover Design: Bruce Kenselaar

Cover Illustration: "Global Bull Market"/Beth Phillips Photography

Composition/Illustrator (Interior): Publication Services

Copyright ©2000, 1997, 1993, 1989 by Prentice-Hall, Inc. Upper Saddle River, NJ 07458

All rights reserved. No part of this book may be reproduced, in any form or by any means, without written permission from the publisher.

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Hull, John, 1946-

Options, futures, and other derivatives / John C. Hull.—4th ed.

n. cm

Includes bibliographical references and index.

ISBN 0-13-022444-8

1. Futures. 2. Stock options. 3. Derivative securities.

I. Title.

HG6024.A3H85 1999

332.63'2—dc21 99-26609 CIP

Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada, Inc., Toronto
Prentice-Hall Hispanoamerica, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall of Japan, Inc., Tokyo
Prentice-Hall (Singapore) Pte. Ltd.
Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Preface

This book is appropriate for graduate and advanced undergraduate elective courses in business, economics, and financial engineering. It is also suitable for practitioners who want to acquire a working knowledge of how derivatives can be analyzed.

One of the key decisions that must be made by an author who is writing in the area of derivatives concerns the use of mathematics. If the level of mathematical sophistication is too high, the material is likely to be inaccessible to many students and practitioners. If it is too low, some important issues will inevitably be treated in a rather superficial way. In this book, great care has been taken in the use of mathematics. Nonessential mathematical material has been either eliminated or included in end-of-chapter appendices. Concepts that are likely to be new to many readers have been explained carefully, and many numerical examples have been included.

This book provides a unifying approach to the valuation of all derivatives — not just futures and options. The book assumes that the reader has taken an introductory course in finance and an introductory course in probability and statistics. No prior knowledge of options, futures contracts, swaps, and so on is assumed. It is not therefore necessary for students to take an elective course in investments prior to taking a course based on this book.

Changes in This Edition

This edition contains more material than the third edition. The material in the third edition has been updated and its presentation has been improved in a number of places. The major changes include:

- 1. A new chapter (chapter 14) has been included on value at risk.
- 2. A new chapter (chapter 15) has been included on estimating volatilities and correlations. GARCH models are covered in much more detail than in the third edition.
- Chapter 19 contains much new material and explains the role played by martingales and measures in the valuation of derivatives.
- 4. Chapter 20 on the standard market models for valuing interest rate derivatives has been revised. It now uses the material in chapter 19 to provide a more complete discussion of the models for valuing bond options, caps, and swap options.

- 5. There are now two chapters on equilibrium and no-arbitrage models of the term structure (chapters 21 and 22). Chapter 21 covers equilibrium models and one-factor no-arbitrage models of the short rate. Chapter 22 covers two-factor models of the short rate, the HJM model, and the LIBOR market (BGM) model.
- Chapter 4 on Interest Rates and Duration has been rewritten to make the material clearer and more relevant.
- Chapter 23 on Credit Risk has been rewritten to reflect developments in this important area.
- 8. More material has been added on volatility smiles and volatility skews (chapter 17).
- 9. The sequencing of the material has been changed slightly. Volatility smiles and alternatives to Black–Scholes now appear before the chapter on exotic options, which in turn appears before the material on interest rate derivatives.
- 10. The notation has been improved and simplified. S_0 and F_0 are used to denote the asset price and the forward price today (that is, at time zero) and the cumbersome "T-t" no longer appears in most parts of the book.
- 11. A glossary of terms has been included.
- 12. Many new problems and questions have been added.

Software

New Excel-based software, DerivaGem, is included with the book. This software is a big improvement over the software included with previous editions. It has been carefully designed to complement the material in the text. Users can calculate options prices, imply volatilities, and calculate Greek letters for European options, American options, exotic options, and interest rate derivatives. Interest rate derivatives can be valued either using Black's model or a no-arbitrage model. The software can be used to display binomial trees (see for example Figure 16.3 and Figure 21.11) and provide many different charts showing the impact of different variables on either option prices or the Greek letters.

The software is described more fully at the end of the book. Updates to the software can be downloaded from my Web site

http://www.mgmt.utoronto.ca/~hull

Slides

Several hundred PowerPoint slides can be downloaded from my Web site. The slides now use only standard fonts. Instructors can adapt the slides to meet their own needs.

Answers to Questions

Solutions to the end-of-chapter problems in the first three editions were available only in the Instructor's Manual. Over the years many people have asked me to make the solutions more generally available. I have hesitated to do this because it would prevent instructors from using the problems as assignment questions.

In this edition I have dealt with this issue by dividing the end-of-chapter problems into two groups: "Questions and Problems" and "Assignment Questions". There

are over 450 Questions and Problems and solutions to these are in a book Options, Futures, & Other Derivatives: Solutions Manual, which is published by Prentice Hall. There are about 80 Assignment Questions. Solutions to these are available only in the Instructor's Manual.

Acknowledgments

Many people have played a part in the production of this book. The academics and practitioners who have made excellent and useful suggestions include Farhang Aslani, Jas Badval, Emilio Barone, Giovanni Barone-Adesi, Alex Bergier, George Blazenko, Laurence Booth, Phelim Boyle, Peter Carr, Don Chance, J.-P. Chateau, Ren-Raw Chen, George Constantinides, Michel Crouhy, Emanuel Derman, Brian Donaldson, Dieter Dorp, Scott Drabin, Jerome Duncan, Steinar Ekern, David Fowler, Louis Gagnon, Dajiang Guo, Jörgen Hallbeck, Ian Hawkins, Michael Hemler, Steve Heston, Bernie Hildebrandt, Kiyoshi Kato, Kevin Kneafsy, Bill Margrabe, Izzy Nelkin, Neil Pearson, Paul Potvin, Shailendra Pandit, Eric Reiner, Richard Rendleman, Gordon Roberts, Chris Robinson, Cheryl Rosen, John Rumsey, Ani Sanyal, Klaus Schurger, Eduardo Schwartz, Michael Selby, Piet Sercu, Duane Stock, Edward Thorpe, Yisong Tian, P.V. Viswanath, George Wang, Jason Wei, Bob Whaley, Alan White, Hailiang Yang, and Victor Zak. I am particularly grateful to Eduardo Schwartz, who read the original manuscript for the first edition and made many comments that led to significant improvements, and to Richard Rendleman and George Constantinides, who made specific suggestions that led to improvements in this edition.

The first three editions of this book were very popular with practitioners and much of the material in the book has been greatly influenced by the informal contacts I have had with practitioners. The students in my elective courses on derivatives at the University of Toronto have also influenced the evolution of the book.

Alan White, a colleague at the University of Toronto (formerly a colleague at York University), deserves a special acknowledgment. Alan and I have been carrying out joint research in the area of derivatives for over 15 years. During that time we have spent countless hours discussing different issues concerning derivatives. Many of the new ideas in this book, and many of the new ways used to explain old ideas, are as much Alan's as mine. Alan read the original version of this book very carefully and made many excellent suggestions for improvement.

The staff at Prentice Hall have been a continual source of encouragement to me as this project has progressed. I would particularly like to thank Paul Donnelly, my editor, who has always shown a keen interest in the development of this book.

I welcome comments on the book from readers. My e-mail address is

hull@mgmt.utoronto.ca

John C. Hull University of Toronto

Contents

Pretace		XVII
CUADTED 4	laterativation	1
CHAPTER 1.		1
	Forward Contracts	1
	Futures Contracts	4
	Options	5
	Other Derivatives	10
	Types of Traders	11
1.6	Those Big Losses	14
- ,	Summary	15
	Questions and Problems	16
	Assignment Questions	18
CHAPTER 2,	Futures Markets and the Use of Futures for	
	Hedging	19
2.1	Trading Futures Contracts	19
2.2	Specification of the Futures Contract	20
2.3	Operation of Margins	23
2.4	Newspaper Quotes	27
2.5	Convergence of Futures Price to Spot Price	32
2.6	Settlement	33
2.7	Regulation	34
2.8	Hedging Using Futures	35
2.9	Optimal Hedge Ratio	39
2.10	Rolling the Hedge Forward	40
2.11	Accounting and Tax	42

Vi Contents

	Summary	44
	Suggestions for Further Reading	45
	Questions and Problems	46
	Assignment Questions	48
CHAPTER 3.	Forward and Futures Prices	50
3.1	Some Preliminaries	51
3.2	The Forward Price for an Investment Asset	55
3.3	The Effect of Known Income	57
3.4	The Effect of a Known Dividend Yield	58
3.5	Value of a Forward Contract	59
3.6	Forward Prices versus Futures Prices	60
3.7	Stock Index Futures	62
3.8	Foreign Currencies	68
3.9	Futures on Commodities	70
3.10	The Cost of Carry	73
3.11	Delivery Options	73
3.12	Futures Prices and the Expected Future Spot Price	74
	Summary	76
	Suggestions for Further Reading	77
	Questions and Problems	79
	Assignment Questions	81
	Appendix 3A: Assets Providing Dividend Yields	83
	Appendix 3B: Proof That Forward and Futures Prices	
	Are Equal When Interest Rates Are Constant	85
CHAPTER 4.	Interest Rates and Duration	87
4.1	Types of Rates	87
4.2	Zero Rates	88
4.3	Bond Pricing	88
4.4	Determining Zero Rates	90
4.5	Forward Rates	93
4.6	Forward-Rate Agreements	95
4.7	Theories of the Term Structure	97
4.8	Day Count Conventions	98
4.9	Quotations	99
4.10	Interest Rate Futures	101
4.11	Treasury Bond Futures	103
4.12	Eurodollar Futures	107
4.13	Duration	108
4.14	Duration-Based Hedging Strategies	111
4.15	Limitations of Duration	112

		Contents	Vii
	Summary		114
	Suggestions for Further Reading		115
	Ouestions and Problems		116
	Assignment Questions		119
CHAPTER 5.	Swaps		121
5.1	Mechanics of Interest Rate Swaps		121
5.2	The Comparative Advantage Argument		128
5.3	Valuation of Interest Rate Swaps		131
5.4	Currency Swaps		135
	Valuation of Currency Swaps		139
	Other Swaps		141
5.7	Credit Risk		143
	Summary		144
	Suggestions for Further Reading		145
	Questions and Problems		146
	Assignment Questions		148
	Appendix 5A: Construction of Zero-Coupon LIBOR		
	Curve		150
CHAPTER 6.	Options Markets		151
6.1	Underlying Assets		151
6.2	Specification of Stock Options		152
6.3	Newspaper Quotes		156
6.4	Trading		158
6.5	Commissions		159
6.6	Margins		160
6.7	The Options Clearing Corporation		162
6.8	Regulation		163
6.9	Taxation		163
6.10	Warrants, Executive Stock Options, and		
	Convertibles		165
	Summary		166
	Suggestions for Further Reading		166
	Questions and Problems		167
	Assignment Questions		167
CHAPTER 7.	Properties of Stock Option Prices		168
7.1	Factors Affecting Option Prices		168
7.2	Assumptions and Notation		170
7.3	Upper and Lower Bounds for Option Prices		171
7.4	Put-Call Parity		174

Viii Contents

7.5	Early Exercise: Calls on a Non-Dividend-Paying	175
7.6	Stock Fash Eversion Pluts on a New Dividend Poving	175
7.0	Early Exercise: Puts on a Non-Dividend-Paying Stock	176
7.7	Relationship Between American Put and Call	170
	Prices	178
7.8	The Effect of Dividends	179
7.9		180
	Summary	181
	Suggestions for Further Reading	182
	Questions and Problems	183
	Assignment Questions	184
CHAPTER 8	3. Trading Strategies Involving Options	185
8.1	Strategies Involving a Single Option and a Stock	185
8.2	Spreads	187
8.3	Combinations	194
8.4	Other Payoffs	197
	Summary	197
	Suggestions for Further Reading	198
	Questions and Problems	198
	Assignment Questions	199
CHAPTER 9	P. Introduction to Binomial Trees	201
9.1	A One-Step Binomial Model	201
9.2	Risk-Neutral Valuation	205
9.3	Two-Step Binomial Trees	206
9.4	A Put Option Example	209
9.5	· ·	210
9.6		211
9.7	, , , , , , , , , , , , , , , , , , ,	213
9.8		214
	Summary	215
	Suggestions for Further Reading	216
	Questions and Problems	216
	Assignment Questions	217
CHAPTER 1	O. Model of the Behavior of Stock Prices	218
10.1	The Markov Property	218
10.2		219
10.3	The Process for Stock Prices	225

	Content	s IX
10.4	Review of the Model	226
10.5	The Parameters	228
10.6	Ito's Lemma	229
	Summary	231
	Suggestions for Further Reading	232
	Questions and Problems	232
	Assignment Questions	234
	Appendix 10A: Derivation of Ito's Lemma	235
CHAPTER 11.	The Black–Scholes Model	237
11.1	Lognormal Property of Stock Prices	237
11.2	The Distribution of the Rate of Return	239
11.3	Volatility	241
11.4	Concepts Underlying the Black–Scholes–Merton	
	Differential Equation	244
11.5	Derivation of the Black–Scholes-Merton Differential	
	Equation	246
11.6	Risk-Neutral Valuation	248
11.7	Black–Scholes Pricing Formulas	250
11.8	Cumulative Normal Distribution Function	252
11.9	Warrants Issued by a Company on Its Own	
	Stock	253
11.10	Implied Volatilities	255
11.11	The Causes of Volatility	255
11.12	Dividends	257
	Summary	262
	Suggestions for Further Reading	263
	Questions and Problems	264
	Assignment Questions	266
	Appendix 11A: Proof of Black-Scholes-Merton	
	Formula	268
	Appendix 11B: Exact Procedure for Calculating	
	Values of American Calls on Dividend-Paying Stocks	271
	Appendix 11C: Calculation of Cumulative Probability in Bivariate Normal Distribution	272
CHAPTER 12.	Options on Stock Indians Currencies	
CHAPILM 12.	Options on Stock Indices, Currencies, and Futures	273
12.1	Results for a Stock Paying a Continuous Dividend	2,5
12.1	Yield	272
12.2	Option Pricing Formulas	273 275
	Options on Stock Indices	213

X Contents

285
291
293
294
295
296
297
298
302
303
305
303
307
307 307
307
307 308
307 308 308
307 308 308 310
307 308 308 310 319
307 308 308 310 319 322
307 308 308 310 319 322 326
307 308 308 310 319 322 326 326
307 308 308 310 319 322 326 326 329
307 308 308 310 319 322 326 329 329
307 308 308 310 319 322 326 329 329 330
307 308 308 310 319 322 326 326 329 330 331
307 308 308 310 319 322 326 329 330 331 334
307 308 308 310 319 322 326 329 329 330 331 334 335
307 308 308 310 319 322 326 326 329 330 331 334 335 336
307 308 308 310 319 322 326 326 329 330 331 334 335 336 337

		Contents	хi
CHAPTER 14.	Value at Risk		342
14.1	Daily Volatilities		342
14.2			343
14.3	•		345
14.4	How Interest Rates Are Handled		346
14.5	When the Linear Model Can Be Used		350
14.6	A Quadratic Model		352
14.7	Monte Carlo Simulation		355
14.8	Historical Simulation		356
14.9	Stress Testing and Back-Testing		357
14.10	Principal Components Analysis		357
	Summary		361
	Suggestions for Further Reading		362
	Questions and Problems		362
	Assignment Questions		364
	Appendix 14A: Use of the Cornish–Fisher Expansion to Estimate VaR		366
CHAPTER 15.	Estimating Volatilities and Correlations		368
	-		
15.1	,		368
15.2	The Exponentially Weighted Moving Average Model		370
15.3	The GARCH (1,1) Model		372
15.4	Choosing Between the Models		374
15.5	Maximum Likelihood Methods		374
15.6	Using GARCH (1,1) to Forecast Future Volatility		379
15.7	Correlations		382
	Summary		384
	Suggestions for Further Reading		385
	Questions and Problems		386
	Assignment Questions		387
CHAPTER 16.	Numerical Procedures		388
16.1	Binomial Trees		388
16.2	Using the Binomial Tree for Options on Indices,		
	Currencies, and Futures Contracts		395
16.3	Binomial Model for a Dividend-Paying Stock		398
16.4	Extensions of the Basic Tree Approach		401
16.5	Alternative Procedures for Constructing Trees		403

XII Contents

16.6	Monte Carlo Simulation	406
16.7	Variance Reduction Procedures	411
16.8	Finite Difference Methods	415
16.9	Analytic Approximation to American Option	
	Prices	425
	Summary	425
	Suggestions for Further Reading	426
	Questions and Problems	428
	Assignment Questions	431
	Appendix 16A: Analytic Approximation to American	
	Option Prices	432
CHAPTER 17.	Volatility Smiles and Alternatives to	
	Black–Scholes	435
17.1	Preliminaries	435
17.2	Foreign Currency Options	436
	Equity Options	438
	The Volatility Term Structure	440
	Volatility Matrices	441
17.6	Relaxing the Assumptions in Black–Scholes	442
17.7	Alternative Models for Stock	
	Options	443
17.8	Pricing Models Involving Jumps	445
17.9	Stochastic Volatility Models	446
17.10	Empirical Research	448
	Summary	450
	Suggestions for Further Reading	450
	Questions and Problems	453
	Assignment Questions	454
	Appendix 17A: Pricing Formulas for Alternative	
	Models	455
CHAPTER 18.	Exotic Options	458
18.1	Types of Exotic Options	458
	Path-Dependent Derivatives	471
18.3	Lookback Options	475
18.4	Barrier Options	477
18.5	Options on Two Correlated Assets	482
18.6	Implied Trees	485
18.7	Hedging Issues	487
18.8	Static Options Replication	487

	Conten	its	Xiii
	Summary		489
	Suggestions for Further Reading		491
	Questions and Problems		492
	Assignment Questions		494
	Appendix 18A: Calculation of the First Two Moments		
	of Arithmetic Averages and Baskets	,	496
CHAPTER 19.	Extensions of the Theoretical Framework for Pricing Derivatives: Martingales and		
	Measures		498
19.1	The Market Price of Risk		498
19.2	Derivitives Dependent on Several State Variables	,	503
19.3	Derivatives Dependent on Commodity Prices		506
19.4	Martingales and Measures		507
19.5	Alternative Choices for the Numeraire		510
19.6	Extension to Multiple Independent Factors		513
19.7	Applications		514
19.8	Change of Numeraire		517
19.9	Quantos		518
19.10	Siegel's Paradox		521
	Summary		521
	Suggestions for Further Reading		522
	Questions and Problems		523
	Assignment Questions Appendix 19A. Generalization of the's Lorenza		525
	Appendix 19A: Generalization of Ito's Lemma Appendix 19B: Derivation of the General Differential		526
	Equation Satisfied by Derivatives		527
CHAPTER 20.	Interest Rate Derivatives: The Standard		
CHAFTER 20.	Market Models		530
20.1	Black's Model		
20.1	Bond Options		531 533
20.2	Interest Rate Caps		537
20.4	European Swap Options		543
20.5	Generalizations		547
20.6	Convexity Adjustments		547 547
20.7	Timing Adjustments		552
20.8	When Is an Adjustment Necessary?		555
20.9	Accrual Swaps		556
20.10	Spread Options		557
20.11	Hedging Interest Rate Derivatives		557
	Summary		558
	Suggestions for Further Reading		559

XiV Contents

	Questions and Problems	559
	Assignment Questions	561
	Appendix 20A: Proof of the Convexity Adjustment	
	Formula	563
CHAPTER 21.	Interest Rate Derivatives: Models of the	
	Short Rate	564
21.1	Equilibrium Models	564
21.2	One-Factor Equilibrium Models	565
21.3	The Rendleman and Bartter Model	566
21.4	The Vasicek Model	567
21.5	The Cox, Ingersoll, and Ross Model	570
21.6	Two-Factor Equilibrium Models	571
21.7	No-Arbitrage Models	571
21.8	The Ho and Lee Model	572
21.9	The Hull and White Model	574
21.10	Options on Coupon-Bearing Bonds	577
21.11	Interest Rate Trees	578
21.12	A General Tree-Building Procedure	580
21.13	Nonstationary Models	591
21.14	Calibration	593
21.15	Hedging Using a One-Factor Model	594
21.16	Forward Rates and Futures Rates	595
	Summary	596
	Suggestions for Further Reading	596
	Questions and Problems	598
	Assignment Questions	599
CHAPTER 22.	Interest Rate Derivatives: More Advanced	
	Models	601
22.1	Two-Factor Models of the Short	
	Rate	601
22.2	The Heath, Jarrow, and Morton Approach	604
22.3	The LIBOR Market Model	609
22.4	Mortgage-Backed Securities	615
	Summary	618
	Suggestions for Further Reading	618
	Questions and Problems	620
	Assignment Questions	620
	Appendix 22A: The $A(t,T)$, σ_P , and $\theta(t)$ Functions in	
	the Two-Factor Hull–White Model	621

		Contents	ΧV
CHAPTER 23.	Credit Risk		623
23.1	The Probability of Default and Expected Losses		624
23.2	Adjusting the Prices of Derivatives to Reflect		
	Counterparty Default Risk		632
23.3	Credit Value at Risk		641
23.4	Credit Derivatives		644
23.5	Valuation of Convertible Bonds		646
	Summary		648
	Suggestions for Further Reading		649
	Questions and Problems		650
	Assignment Questions		652
	Appendix 23A: Manipulation of the Matrices of		
	Credit Rating Changes		654
	Glossary of Notation		655
	Glossary of Terms		658
	DerivaGem Software		672
	Major Exchanges Trading Futures and Options		676
	Table for $N(x)$ when $x \le 0$		678
	Table for $N(x)$ when $x \ge 0$		679
	Author Index		680
	Subject Index		683