STRENGTH OF MATERIALS

TIMOSHENKO

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PART I

Elementary Theory and Problems

BY

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PREFACE TO THE SECOND EDITION

In preparing the second edition of this volume, an effort has been made to adapt the book to the teaching requirements of our engineering schools.

With this in view, a portion of the material of a more advanced character which was contained in the previous edition of this volume has been removed and will be included in the new edition of the second volume. At the same time, some portions of the book, which were only briefly discussed in the first edition, have been expanded with the intention of making the book easier to read for the beginner. For this reason, chapter II, dealing with combined stresses, has been entirely rewritten. Also, the portion of the book dealing with shearing force and bending moment diagrams has been expanded, and a considerable amount of material has been added to the discussion of deflection curves by the integration method. A discussion of column theory and its application has been included in chapter VIII, since this subject is usually required in undergraduate courses of strength of materials. Several additions have been made to chapter X dealing with the application of strain energy methods to the solution of statically indetermined problems. In various parts of the book there are many new problems which may be useful for class and home work.

Several changes in the notations have been made to conform to the requirements of American Standard Symbols for Mechanics of Solid Bodies recently adopted by The American Society of Mechanical Engineers.

It is hoped that with the changes made the book will be found more satisfactory for teaching the undergraduate course of strength of materials and that it will furnish a better foundation for the study of the more advanced material discussed in the second volume.

S. TIMOSHENKO

PALO ALTO, CALIFORNIA
June 13, 1940

PREFACE TO THE FIRST EDITION

At the present time, a decided change is taking place in the attitude of designers towards the application of analytical methods in the solution of engineering problems. Design is no longer based principally upon empirical formulas. The importance of analytical methods combined with laboratory experiments in the solution of technical problems is becoming generally accepted.

Types of machines and structures are changing very rapidly, especially in the new fields of industry, and usually time does not permit the accumulation of the necessary empirical data. The size and cost of structures are constantly increasing, which consequently creates a severe demand for greater reliability in structures. The economical factor in design under the present conditions of competition is becoming of growing importance. The construction must be sufficiently strong and reliable, and yet it must be designed with the greatest possible saving in material. Under such conditions, the problem of a designer becomes extremely difficult. Reduction in weight involves an increase in working stresses, which can be safely allowed only on a basis of careful analysis of stress distribution in the structure and experimental investigation of the mechanical properties of the materials employed.

It is the aim of this book to present problems such that the student's attention will be focussed on the practical applications of the subject. If this is attained, and results, in some measure, in increased correlation between the studies of strength of materials and engineering design, an important forward step will have been made.

The book is divided into two volumes. The first volume contains principally material which is usually covered in required courses of strength of materials in our engineering schools. The more advanced portions of the subject are of interest chiefly to graduate students and research engineers, and are incorporated in the second volume of the book. This contains also the new developments of practical importance in the field of strength of materials.

In writing the first volume of strength of materials, attention was given to simplifying all derivations as much as possible so that a student with the usual preparation in mathematics will be able to read it without difficulty. For example, in deriving the theory of the deflection curve, the *area moment method* was extensively used. In this manner, a considerable simplification was made in deriving the deflections of beams for various loading and supporting conditions. In discussing statically indeterminate systems, the *method of superposition* was applied, which proves very useful in treating such problems as continuous beams and frames. For explaining combined stresses and deriving principal stresses, use was made of the *Mohr's circle*, which represents a substantial simplification in the presentation of this portion of the theory.

Using these methods of simplifying the presentation, the author was able to condense the material and to discuss some problems of a more advanced character. For example, in discussing torsion, the twist of rectangular bars and of rolled sections, such as angles, channels, and I beams, is considered. The deformation and stress in helical springs are discussed in detail. In the theory of bending, the case of non-symmetrical cross sections is discussed, the *center of twist* is defined and explained, and the effect of shearing force on the deflection of beams is considered. The general theory of the bending of beams, the materials of which do not follow Hooke's law, is given and is applied in the bending of beams beyond the yielding point. The bending of reinforced concrete beams is given consideration. In discussing combinations of direct and bending stress, the effect of deflections on the bending moment is considered, and the limitation of the method of superposition is explained. In treating combined bending and torsion, the cases of rectangular and elliptical cross sections are discussed, and applications in the design of crankshafts are given. Considerable space in the book is devoted to methods for solving elasticity problems based on the consideration of the strain energy of elastic bodies. These methods are applied in discussing statically indeterminate systems. The stresses produced by impact are also discussed. All these problems of a more advanced character are printed in small type, and may be omitted during the first reading of the book.

The book is illustrated with a number of problems to which solutions are presented. In many cases, the problems are chosen so as to widen the field covered by the text and to illustrate the application of the theory in the solution of design problems. It is hoped that these problems will be of interest for teaching purposes, and also useful for designers.

The author takes this opportunity of thanking his friends who have assisted him by suggestions, reading of manuscript and proofs, particularly Messrs. W. M. Coates and L. H. Donnell, teachers of mathematics and mechanics in the Engineering College of the University of Michigan, and Mr. F. L. Everett of the Department of Engineering Research of the University of Michigan. He is indebted also to Mr. F. C. Wilharm for the preparation of drawings, to Mrs. E. D. Webster for the typing of the manuscript, and to the Van Nostrand Company for its care in the publication of the book.

S. TIMOSHENKO

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NOTATIONS

$\sigma_x, \sigma_y, \sigma_z$....Normal stresses on planes perpendicular to x, y and z axes.
σ_n.........Normal stress on plane perpendicular to direction n.
$\sigma_{Y.P.}$.......Normal stress at yield point.
σ_w.........Normal working stress
τ..........Shearing stress
$\tau_{xy}, \tau_{yz}, \tau_{zx}$..Shearing stresses parallel to x, y and z axes on the planes perpendicular to y, z and x axes.
τ_w.........Working stress in shear
δ..........Total elongation, total deflection
ϵ..........Unit elongation
$\epsilon_x, \epsilon_y, \epsilon_z$....Unit elongations in x, y and z directions
γ..........Unit shear, weight per unit volume
E..........Modulus of elasticity in tension and compression
G..........Modulus of elasticity in shear
μ..........Poisson's ratio
Δ..........Volume expansion
K.........Modulus of elasticity of volume
M_t........Torque
M.........Bending moment in a beam
V.........Shearing force in a beam
A.........Cross sectional area
I_y, I_z......Moments of inertia of a plane figure with respect to y and z axes
k_y, k_z......Radii of gyration corresponding to I_y, I_z
I_p.........Polar moment of inertia
Z..........Section modulus
C..........Torsional rigidity
l..........Length of a bar, span of a beam
P, Q......Concentrated forces
t..........Temperature, thickness

α..........Coefficient of thermal expansion, numerical coefficient
U..........Strain energy
w..........Strain energy per unit volume
h..........Depth of a beam, thickness of a plate
q..........Load per unit length
ϕ, θ........Angles
p..........Pressure
D, d.......Diameters
R, r........Radii
W..........Weight, load

CONTENTS

STRENGTH OF MATERIALS

PART I

CHAPTER I

TENSION AND COMPRESSION WITHIN THE ELASTIC LIMIT

1. **Elasticity.**—We assume that a body consists of small particles, or molecules, between which forces are acting. These molecular forces resist the change in the form of the body which external forces tend to produce. If such external forces are applied to the body, its particles are displaced and the mutual displacements continue until equilibrium is established between the external and internal forces. It is said in such a case that the body is in a *state of strain*. During deformation the external forces acting upon the body do work, and this work is transformed completely or partially into the *potential energy of strain*. An example of such an accumulation of potential energy in a strained body is the case of a watch spring. If the forces which produced the deformation of the body are now gradually diminished, the body returns wholly or partly to its initial shape and during this reversed deformation the potential energy of strain, accumulated in the body, may be recovered in the form of external work.

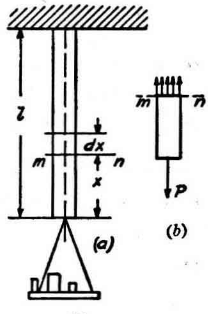

FIG. 1.

Take, for instance, a prismatical bar loaded at the end as shown in Fig. 1. Under the action of this load a certain elongation of the bar will take place. The point of application of the load will then move in a downward direction and positive work will be done by the load during this motion.

When the load is diminished, the elongation of the bar diminishes also, the loaded end of the bar moves up and the potential energy of strain will be transformed into the work of moving the load in the upward direction.

The property of bodies of returning, after unloading, to their initial form is called *elasticity*. It is said that the body is *perfectly elastic* if it recovers its original shape completely after unloading; it is *partially elastic* if the deformation, produced by the external forces, does not disappear completely after unloading. In the case of a perfectly elastic body the work done by the external forces during deformation will be completely transformed into the potential energy of strain. In the case of a partially elastic body, part of the work done by the external forces during deformation will be dissipated in the form of heat, which will be developed in the body during the non-elastic deformation. Experiments show that such structural materials as steel, wood and stone may be considered as perfectly elastic within certain limits, which depend upon the properties of the material. Assuming that the external forces acting upon the structure are known, it is a fundamental problem for the designer to establish such proportions of the members of the structure that it will approach the condition of a perfectly elastic body under all service conditions. Only under such conditions will we have continued reliable service from the structure and no *permanent set* in its members.

2. Hooke's Law.—By direct experiment with the extension of prismatical bars (Fig. 1) it has been established for many structural materials that within certain limits the elongation of the bar is proportional to the tensile force. This simple linear relationship between the force and the elongation which it produces was first formulated by the English scientist Robert Hooke [1] in 1678 and bears his name. Using the notation:

P = force producing extension of bar,
l = length of bar,

[1] Robert Hooke, De Potentia restitutiva, London, 1678.

A = cross sectional area of bar,
δ = total elongation of bar,
E = elastic constant of the material, called its *Modulus of Elasticity*,

Hooke's experimental law may be given by the following equation:

$$\delta = \frac{Pl}{AE}. \qquad (1)$$

The elongation of the bar is proportional to the tensile force and to the length of the bar and inversely proportional to the cross sectional area and to the modulus of elasticity. In making tensile tests precautions are usually taken to secure central application of the tensile force. In this manner any bending of the bar will be prevented. Excluding from consideration those portions of the bar in the vicinity of the applied forces,[2] it may be assumed that during tension all longitudinal fibers of the prismatical bar have the same elongation and the cross sections of the bar originally plane and perpendicular to the axis of the bar remain so after extension.

In discussing the magnitude of internal forces let us imagine the bar cut into two parts by a cross section *mn* and let us consider the equilibrium of the lower portion of the bar (Fig. 1, *b*). At the lower end of this portion the tensile force P is applied. On the upper end there are acting the forces representing the action of the particles of the upper portion of the strained bar on the particles of the lower portion. These forces are continuously distributed over the cross section. A familiar example of such a continuous distribution of forces over a surface is that of a hydrostatic pressure or of a steam pressure. In handling such continuously distributed forces *the intensity of force*, i.e., the force per unit area, is of a great importance. In our case of axial tension, in which all fibers have the same elongation, the

[2] The more complicated stress distribution near the points of application of the forces will be discussed later in Part II.

distribution of forces over the cross section *mn* will be *uniform.* Taking into account that the sum of these forces, from the condition of equilibrium (Fig. 1, *b*), must be equal to P and denoting the force per unit of cross sectional area by σ, we obtain

$$\sigma = \frac{P}{A}. \tag{2}$$

This force per unit area is called *stress.* In the following, the force will be measured in pounds and the area in square inches so that the stress will be measured in pounds per square inch. The elongation of the bar per unit length is determined by the equation

$$\epsilon = \frac{\delta}{l} \tag{3}$$

and is called the *unit elongation* or the *tensile strain.* Using eqs. (2) and (3), Hooke's law may be represented in the following form:

$$\epsilon = \frac{\sigma}{E}, \tag{4}$$

and the unit elongation is easily calculated provided the stress and the modulus of elasticity of the material are known. The unit elongation ϵ is a pure number representing the ratio of two lengths (see eq. 3); therefore, from eq. (4), it may be concluded that the modulus of elasticity is to be measured in the same units as the stress σ, i.e., in pounds per square inch. In Table I, which follows, the average values of the modulus E for several materials are given in the first column.[3]

Equations (1)–(4) may be used also in the case of the compression of prismatical bars. Then δ will denote the total longitudinal contraction, ϵ the *compressive strain* and σ the *compressive stress.* The modulus of elasticity for compression is for most structural materials the same as for tension. In calculations, tensile stress and tensile strain are considered as positive, and compressive stress and strain as negative.

[3] More details on the mechanical properties of materials are given in Part II.

TABLE 1

MECHANICAL PROPERTIES OF MATERIALS

Materials	E lbs./in.^{2}	Yield Point lbs./in.^{2}	Ultimate Strength lbs./in.^{2}
Structural carbon steel 0.15 to 0.25% carbon	30×10^6	30×10^3–40×10^3	55×10^3–65×10^3
Nickel steel 3 to 3.5% nickel	29×10^6	40×10^3–50×10^3	78×10^3–100×10^3
Duraluminum	10×10^6	35×10^3–45×10^3	54×10^3–65×10^3
Copper, cold rolled	16×10^6		28×10^3–40×10^3
Glass	10×10^6		3.5×10^3
Pine, with the grain	1.5×10^6		8×10^3–20×10^3
Concrete, in compression	4×10^6		3×10^3

Problems

1. Determine the total elongation of a steel bar 25 in. long, if the tensile stress is equal to 15×10^3 lbs. per sq. in.

Answer.

$$\delta = \epsilon \times l = \frac{25}{2{,}000} = \frac{1}{80} \text{ in.}$$

2. Determine the tensile force on a cylindrical steel bar of one inch diameter, if the unit elongation is equal to $.7 \times 10^{-3}$.

Solution. The tensile stress in the bar, from eq. (4), is

$$\sigma = \epsilon \cdot E = 21 \times 10^3 \text{ lbs. per sq. in.}$$

The tensile force, from eq. (2), is

$$P = \sigma \cdot A = 21 \times 10^3 \times \frac{\pi}{4} = 16{,}500 \text{ lbs.}$$

3. What is the ratio of the moduli of elasticity of the materials of two bars of the same size if under the action of equal tensile forces the unit elongations of the bars are in the ratio 1 : 15/8. Determine these elongations if one of the bars is of steel, the other of copper and the tensile stress is 10,000 lbs. per sq. inch.

Solution. The moduli are inversely proportional to the unit elongations. For steel

$$\epsilon = \frac{10{,}000}{30 \times 10^6} = \frac{1}{3{,}000},$$

for copper

$$\epsilon = \frac{1}{1{,}600}.$$