APPLIED FINITE MATHEMATICS

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Second Edition

Applied Finite Mathematics

Work-Text

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APPLIED FINITE MATHEMATICS

Finite Mathematics • Finite Mathematics •

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Preface

This book is written for a traditional finite mathematics course for students intending to major in business, economics, and social and biological sciences. The course includes a variety of rich and interesting subjects such as – linear models, linear programming, mathematics of finance, probability theory, stochastic processes and game theory. The purpose of this book is to provide the student with a firm background upon which to build.

The book is designed for a quarter or a semester course in finite mathematics. There are forty five sections in the book, and most of the sections require about one class hour to complete. Each topic is motivated with a concrete example and then generalized to a rule or a formula. The concepts are clearly explained with examples and are closely related to the exercise problems. There are ample problems with applications in real life situations that will generate interest and excitement.

Most textbooks have an enormous amount of content and tend to overwhelm the student. This book has course material for a quarter or a semester course. A great deal of emphasis is placed on core topics. It is felt that linear models and probability theory are the necessary tools for the student to have if he or she is to succeed in the above mentioned fields. The student will find an extensive treatment of these topics in the pages ahead. A thorough knowledge of these topics will prepare the student to apply these concepts in their future educational and professional fields.

Chapter 1 begins with the graphing of straight lines, and shows a quick and easy way to determine an equation of a line when two points, or a point and a slope are given. After learning section 1.3, the student will be able to orally determine the equation of a line in standard form. Once the student learns to find an equation of a line, he or she will apply the concept in application problems that deal with real life situations.

Chapter 2 deals with matrix algebra. The student learns to solve linear systems using both the Gauss-Jordan method and the matrix inverse approach. A special section is devoted to inconsistent and dependent systems and their occurrence in application problems. An application of matrices in cryptography is presented, and the chapter ends with a section on Leontief's economic models. Technology plays a great part here because most of the matrices are manipulated with a calculator. The emphasis is on interpretation and not on computation.

Chapters 3 and 4 involve linear programming using both the graphical approach as well as the simplex method approach. The graphical approach helps the student learn not only how the process works but it also gives him or her an experience in expressing long worded problems in simple mathematical equations and inequalities. Although the simplex method has an algorithmic approach, a special effort is made in explaining the reasoning behind the fundamental theory.

Chapter 5 consists of mathematics of finance. The students are discouraged from learning the seven or eight formulas that are presented in most finite mathematics textbooks. Instead, they are to understand the rationale, and formulate the equations themselves for each situation. The emphasis is placed on identifying the problems. The law of 70 is presented as an additional tool in estimating answers.

Chapters 6, 7 and 8 are the backbone of this course. Chapter 6 prepares the students in mastering counting techniques so that they can apply them in solving probability problems. It is felt that the use of tree diagrams is an easy and natural way to solve many of the probability problems. Tree diagrams not only help students discern the problem, but they also help them consider all possible outcomes. It is for this reason the tree diagrams are used throughout chapters 7 and 8, and in chapter 8 a special section is devoted to problems dealing with tree diagrams.

Chapters 9 and 10 comprise of Markov chains, and game theory. The students find these topics most enjoyable and interesting when they are allowed to manipulate transition matrices with a calculator. The author has tried to find a balance between knowing the underlying theory and finding a solution by raising transition matrices to higher powers.

It is felt that mathematics is learned by doing. Demanding the completion of homework on a regular basis encourages students to make a stronger effort toward that goal. One of the reasons for writing this textbook was to be able to collect homework readily. The work-text with perforated pages allows the instructor to gather and grade homework in an easier manner.

The main emphasis is on learning the concepts and not just the formulas and recipes. The approach used in the book encourages students to read a problem, study the information, and then express it in some tangible form, hopefully a mathematical model. Estimating an answer before solving is stressed. The concepts are presented in a manner so that understanding always precedes the computing and number crunching. The students are strongly urged to use a graphing calculator that handles matrices. At De Anza College, most students who take mathematics courses own Texas Instruments' calculator, TI-85 or TI-86. It is felt that a calculator is an essential tool not only in computing, but also in experimenting and exploring. Calculator activities for each chapter are provided in the appendix along with some instructions for the TI-85. My special thanks to both Mr. Chris Avery and Mr. Chris Barker for allowing the use of these calculator activities that were originally developed through their Math Lab.

Finally, I would like to thank my wife, Niki, for supporting my efforts, my daughter, Jessica, for proof reading and making helpful suggestions, and my son, Vijay, for reviewing problems and preparing a complete solution manual. I would like to thank Jim Symons for providing moral support as well as for helpful hints in word processing.

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De Anza College

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Contents

CHAPTER 1

Linear Equations

In this chapter, you will learn to:

1. *Graph a linear equation.*
2. *Find the slope of a line.*
3. *Determine an equation of a line.*
4. *Solve linear systems.*
5. *Do application problems using linear equations.*

1.1 Graphing a Linear Equation

Equations whose graphs are straight lines are called **linear equations.** The following are some examples of linear equations:

$2x - 3y = 6$, $3x = 4y - 7$, $y = 2x - 5$, $2y = 3$, and $x - 2 = 0$.

A line is completely determined by two points, therefore, to graph a linear equation, we need to find the coordinates of two points. This can be accomplished by choosing an arbitrary value for x or y and then solving for the other variable.

◆Example 1 Graph the line: $y = 3x + 2$

Solution: We need to find the coordinates of at least two points.

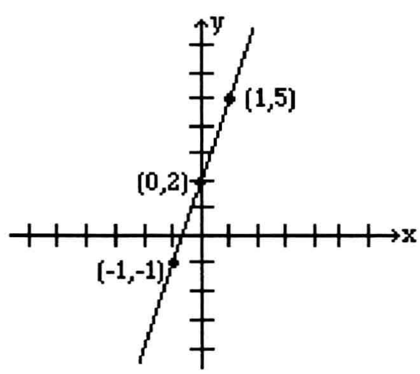
We arbitrarily choose $x = -1$, $x = 0$, and $x = 1$.

If $x = -1$, then $y = 3(-1) + 2$ or -1. Therefore, $(-1, -1)$ is a point on this line.

If $x = 0$, then $y = 3(0) + 2$ or $y = 2$. Hence the point $(0, 2)$.

If $x = 1$, then $y = 5$, and we get the point $(1, 5)$. Below, the results are summarized, and the line is graphed.

x	-1	0	1
y	−1	2	5



◆**Example 2** Graph the line: $2x + y = 4$

Solution: Again, we need to find coordinates of at least two points.

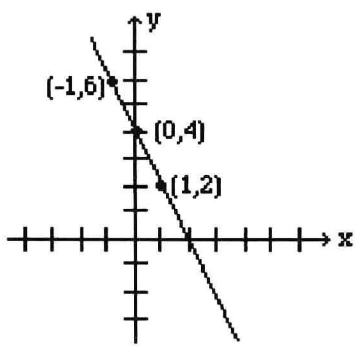
We arbitrarily choose $x = -1$, $x = 0$, and $y = 2$.

If $x = -1$, then $2(-1) + y = 4$ which results in $y = 6$. Therefore, $(-1, 6)$ is a point on this line.

If $x = 0$, then $2(0) + y = 4$, which results in $y = 4$. Hence the point $(0, 4)$.

If $y = 2$, then $2x + 2 = 4$, which yields $x = 1$, and gives the point $(1, 2)$. The table below shows the points, and the line is graphed.

x	-1	0	1
y	6	4	2



The points at which a line crosses the coordinate axes are called the **intercepts**. When graphing a line, intercepts are preferred because they are easy to find. In order to find the x-intercept, we let $y = 0$, and to find the y-intercept, we let $x = 0$.

◆**Example 3** Find the intercepts of the line: $2x - 3y = 6$, and graph.

Solution: To find the x-intercept, we let $y = 0$ in our equation, and solve for x.

$$2x - 3(0) = 6$$

$$2x - 0 = 6$$

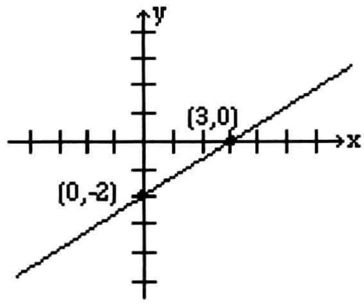
$$2x = 6$$

$$x = 3$$

Therefore, the x-intercept is 3.

Similarly by letting $x = 0$, we obtain the y-intercept which is -2.

Note: If the x-intercept is 3, and the y-intercept is –2, then the corresponding points are (3, 0) and (0, –2), respectively.



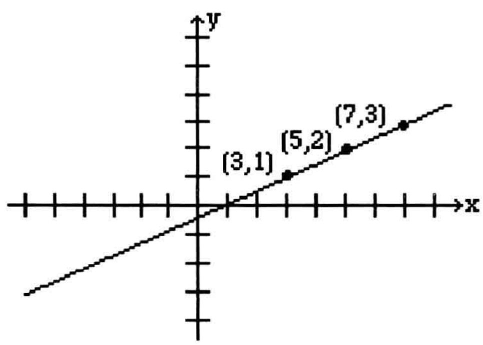
In higher math, equations of lines are sometimes written in parametric form. For example, x = 3 + 2t, y = 1 + t. The letter t is called the parameter or the dummy variable. Parametric lines can be graphed by finding values for x and y by substituting numerical values for t.

◆ ***Example 4*** Graph the line given by the parametric equations: x = 3 + 2t, y = 1 + t

Solution: Let t = 0, 1 and 2, and then for each value of t find the corresponding values for x and y.

The results are given in the table below.

t	0	1	2
x	3	5	7
y	1	2	3



Horizontal and Vertical Lines

When an equation of a line has only one variable, the resulting graph is a horizontal or a vertical line.

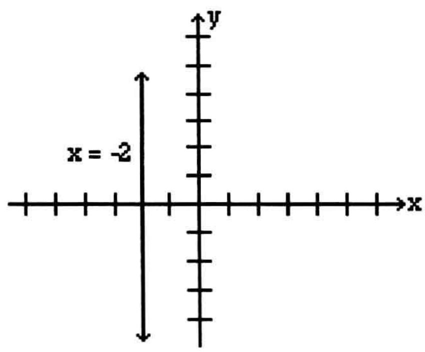
The graph of the line x = a, where a is a constant, is a vertical line that passes through the point (a, 0). Every point on this line has the x-coordinate a, regardless of the y-coordinate.

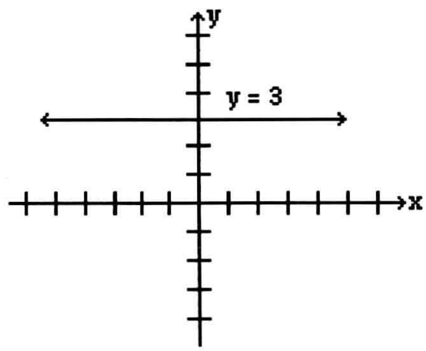
The graph of the line y = b, where b is a constant, is a horizontal line that passes through the point (0, b). Every point on this line has the y-coordinate b, regardless of the x-coordinate.

◆Example 5 Graph the lines: $x = -2$, and $y = 3$.

Solution: The graph of the line $x = -2$ is a vertical line that has the x-coordinate -2 no matter what the y-coordinate is. Therefore, the graph is a vertical line passing through $(-2, 0)$.

The graph of the line $y = 3$, is a horizontal line that has the y-coordinate 3 regardless of what the x-coordinate is. Therefore, the graph is a horizontal line that passes through $(0, 3)$.





Note: Most students feel that the coordinates of points must always be integers. This is not true, and in real life situations, not always possible. Do not be intimidated if your points include numbers that are fractions or decimals.

GRAPHING A LINEAR EQUATION

Work the following problems.

1) Is the point (2, 3) on the line $5x - 2y = 4$?	2) Is the point (1, – 2) on the line $6x - y = 4$?
3) For the line $3x - y = 12$, complete the following ordered pairs. (2,) (, 6) (0,) (, 0)	4) For the line $4x + 3y = 24$, complete the following ordered pairs. (3,) (, 4) (0,) (, 0)
5) Graph $y = 2x + 3$	6) Graph $y = -3x + 5$
7) Graph $y = 4x - 3$	8) Graph $x - 2y = 8$
9) Graph $2x + y = 4$	10) Graph $2x - 3y = 6$

Name:_______________________________

11) Graph $2x + 4 = 0$	12) Graph $2y - 6 = 0$
13) Graph the following three equations on the same set of coordinate axes. $y = x + 1$ $y = 2x + 1$ $y = -x + 1$	14) Graph the following three equations on the same set of coordinate axes. $y = 2x + 1$ $y = 2x$ $y = 2x - 1$
15) Graph the line using the parametric equations $x = 1 + 2t,\ y = 3 + t$	16) Graph the line using the parametric equations $x = 2 - 3t,\ y = 1 + 2t$

1.2 Slope of a Line

In this section, you will learn to:

1. *Find the slope of a line if two points are given.*
2. *Graph the line if a point and the slope are given.*
3. *Find the slope of the line that is written in the form $y = mx + b$.*
4. *Find the slope of the line that is written in the form $Ax + By = c$.*

In the last section, we learned to graph a line by choosing two points on the line. A graph of a line can also be determined if one point and the "steepness" of the line is known. The precise number that refers to the steepness or inclination of a line is called the **slope** of the line.

From previous math courses, many of you remember slope as the "rise over run," or "the vertical change over the horizontal change" and have often seen it expressed as:

$$\frac{rise}{run},\ \frac{vertical\ \ change}{horizontal\ \ change},\ \frac{\Delta y}{\Delta x}\ \text{etc.}$$

We give a precise definition.

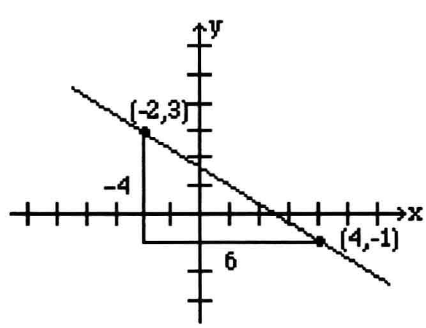
Definition: If (x_1, y_1) and (x_2, y_2) are two different points on a line, then the slope of the line is

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

◆Example 1 Find the slope of the line that passes through the points (-2, 3) and (4, -1), and graph the line.

Solution: Let $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (4, -1)$ then the slope

$$m = \frac{-1-3}{4-(-2)} = -\frac{4}{6} = -\frac{2}{3}$$



To give the reader a better understanding, both the vertical change, –4, and the horizontal change, 6, are shown in the above figure.

When two points are given, it does not matter which point is denoted as (x_1, y_1) and which (x_2, y_2). The value for the slope will be the same. For example, if we choose $(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (-2, 3)$, we will get the same value for the slope as we obtained earlier. The steps involved are as follows.

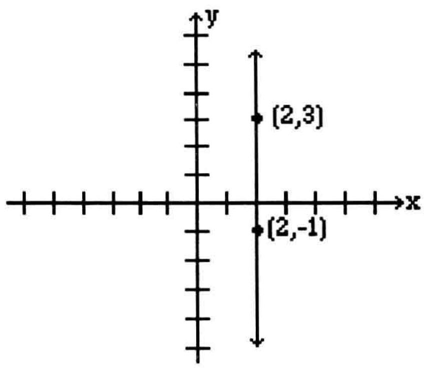
$$m = \frac{3-(-1)}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$$

The student should further observe that if a line rises when going from left to right, then it has a positive slope; and if it falls going from left to right, it has a negative slope.

◆ *Example 2* Find the slope of the line that passes through the points (2, 3) and (2, -1), and graph.

Solution: Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (2, -1)$ then the slope

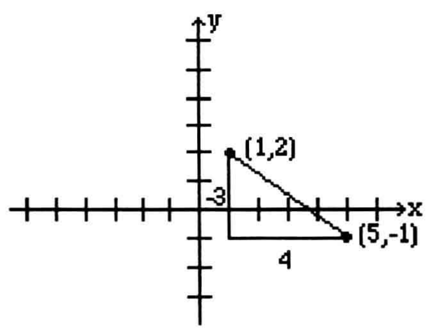
$$m = \frac{-1-3}{2-2} = -\frac{4}{0} = \text{undefined.}$$



Note: The slope of a vertical line is undefined.

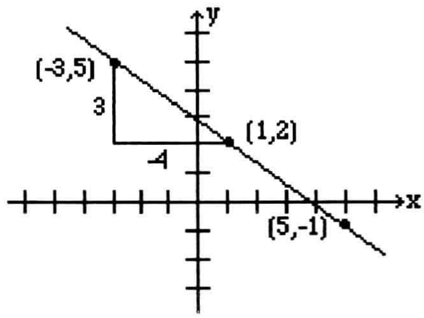
◆ *Example 3* Graph the line that passes through the point (1, 2) and has slope $-\frac{3}{4}$.

Solution: Slope equals $\frac{\text{rise}}{\text{run}}$. The fact that the slope is $\frac{-3}{4}$, means that for every rise of –3 units (fall of 3 units) there is a run of 4. So if from the given point (1, 2) we go down 3 units and go right 4 units, we reach the point (5, –1). The following graph is obtained by connecting these two points.



Alternatively, since $\frac{3}{-4}$ represents the same number, the line can be drawn by starting at the point (1,2) and choosing a rise of 3 units followed by a run of –4 units. So from the point

(1, 2), we go up 3 units, and to the left 4, thus reaching the point (–3, 5) which is also on the same line. See figure below.



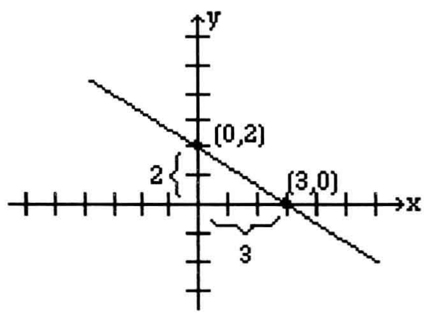
◆Example 4 Find the slope of the line 2x + 3y = 6.

Solution: In order to find the slope of this line, we will choose any two points on this line.

Again, the selection of x and y intercepts seems to be a good choice. The x-intercept is (3, 0), and the y-intercept is (0, 2). Therefore, the slope is

$$m = \frac{2-0}{0-3} = -\frac{2}{3}.$$

The graph below shows the line and the intercepts:x and y.



◆Example 5 Find the slope of the line y = 3x + 2.

Solution: We again find two points on the line. Say (0, 2) and (1, 5).

Therefore, the slope is $m = \frac{5-2}{1-0} = \frac{3}{1} = 3$.

Look at the slopes and the y-intercepts of the following lines.

The line	slope	y-intercept
y = 3x + 2	3	2
y = – 2x + 5	– 2	5
y = 3/2 x – 4	3/2	– 4

It is no coincidence that when an equation of the line is solved for y, the coefficient of the x term represents the slope, and the constant term represents the y-intercept.

In other words, for the line y = mx + b, m is the slope, and b is the y-intercept.