

# Real-Time Rendering

Third Edition



Tomas Akenine-Möller  
Eric Haines  
Naty Hoffman

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# Real-Time Rendering

## Third Edition

Dedicated to Eva, Felix, and Elina  
T. A-M.

Dedicated to Cathy, Ryan, and Evan  
E. H.

Dedicated to Dorit, Karen, and Daniel  
N. H.

# Preface

How much has happened in six years! Luckily for us, there has not been a paradigm shift (yet), but the amount of creative energy that has gone into the field these past years is incredible. To keep up with this ever-growing field, we are fortunate this time to be three authors instead of two. Naty Hoffman joins us this edition. He has years of experience in the field and has brought a fresh perspective to the book.

Since the second edition in 2002, researchers have produced hundreds of articles in the area of interactive computer graphics. At one point Naty filtered through conference proceedings, saving articles he thought were worth at least a mention. From just the research conferences alone the list grew to over 350 references; this did not include those in journals, book series like *GPU Gems* and *ShaderX*, or web articles. We realized we had a challenge ahead of us. Even a thorough survey of each area would make for a book of nothing but surveys. Such a volume would be exhaustive, but exhausting, and ultimately unsatisfying. Instead, we have focused on theory, algorithms, and architectures that we felt are key in understanding the field. We survey the literature as warranted, but with the goal of pointing you at the most recent work in an area and at resources for learning more.

This book is about algorithms that create synthetic images fast enough that the viewer can interact with a virtual environment. We have focused on three-dimensional rendering and, to a limited extent, on user interaction. Modeling, animation, and many other areas are important to the process of making a real-time application, but these topics are beyond the scope of this book.

We expect you to have some basic understanding of computer graphics before reading this book, as well as computer science and programming. Some of the later chapters in particular are meant for implementers of various complex algorithms. If some section does lose you, skim on through or look at the references. One of the most valuable services we feel we can provide is to have you realize what others have discovered and that you do not yet know, and to give you ways to learn more someday.

We make a point of referencing relevant material wherever possible, as well as providing a summary of further reading and resources at the end of

most chapters. Time invested in reading papers and books on a particular topic will almost always be paid back in the amount of implementation effort saved later.

Because the field is evolving so rapidly, we maintain a website related to this book at: <http://www.realtimerendering.com>. The site contains links to tutorials, demonstration programs, code samples, software libraries, book corrections, and more. This book's reference section is available there, with links to the referenced papers.

Our true goal and guiding light while writing this book was simple. We wanted to write a book that we wished we had owned when we had started out, a book that was both unified yet crammed with details not found in introductory texts. We hope that you will find this book, our view of the world, of some use in your travels.

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In answering a seemingly endless stream of questions, fact-checking numerous passages, and providing many screenshots, Natalya Tatarchuk of ATI/AMD went well beyond the call of duty in helping us out. In addition to responding to our usual requests for information and clarification, Wolfgang Engel was extremely helpful in providing us with articles from the upcoming *ShaderX*<sup>6</sup> book and copies of the difficult-to-obtain *ShaderX*<sup>2</sup> books.<sup>1</sup> Ignacio Castaño at NVIDIA provided us with valuable support and contacts, going so far as to rework a refractory demo so we could get just the right screenshot.

The chapter reviewers provided an invaluable service to us. They suggested numerous improvements and provided additional insights, helping us immeasurably. In alphabetical order they are: Michael Ashikhmin, Dan Baker, Willem de Boer, Ben Diamand, Ben Discoe, Amir Ebrahimi,

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<sup>1</sup>Check our website; he and we are attempting to clear permissions and make this two-volume book [307, 308] available for free on the web.

Christer Ericson, Michael Gleicher, Manny Ko, Wallace Lages, Thomas Larsson, Grégory Massal, Ville Miettinen, Mike Ramsey, Scott Schaefer, Vincent Scheib, Peter Shirley, K.R. Subramanian, Mauricio Vives, and Hector Yee.

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Tomas Akenine-Möller

Eric Haines

Naty Hoffman

March 2008

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Finally, and most importantly, our deepest thanks go to our families for giving us the huge amounts of quiet time we have needed to complete this edition. Honestly, we never thought it would take this long!

Tomas Akenine-Möller  
Eric Haines  
May 2002

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Eric Haines  
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# Chapter 1

## Introduction

Real-time rendering is concerned with making images rapidly on the computer. It is the most highly interactive area of computer graphics. An image appears on the screen, the viewer acts or reacts, and this feedback affects what is generated next. This cycle of reaction and rendering happens at a rapid enough rate that the viewer does not see individual images, but rather becomes immersed in a dynamic process.

The rate at which images are displayed is measured in frames per second (fps) or Hertz (Hz). At one frame per second, there is little sense of interactivity; the user is painfully aware of the arrival of each new image. At around 6 fps, a sense of interactivity starts to grow. An application displaying at 15 fps is certainly real-time; the user focuses on action and reaction. There is a useful limit, however. From about 72 fps and up, differences in the display rate are effectively undetectable.

Watching images flicker by at 60 fps might be acceptable, but an even higher rate is important for minimizing response time. As little as 15 milliseconds of temporal delay can slow and interfere with interaction [1329]. There is more to real-time rendering than interactivity. If speed was the only criterion, any application that rapidly responded to user commands and drew anything on the screen would qualify. Rendering in real-time normally means three-dimensional rendering.

Interactivity and some sense of connection to three-dimensional space are sufficient conditions for real-time rendering, but a third element has become a part of its definition: graphics acceleration hardware. While hardware dedicated to three-dimensional graphics has been available on professional workstations for many years, it is only relatively recently that the use of such accelerators at the consumer level has become possible. Many consider the introduction of the 3Dfx Voodoo 1 in 1996 the real beginning of this era [297]. With the recent rapid advances in this market, add-on three-dimensional graphics accelerators are as standard for home computers as a pair of speakers. While it is not absolutely required for real-