THE STOPPING AND RANGE OF IONS IN SOLIDS

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Volume 1
of
The Stopping and Ranges of Ions in Matter

Edited by J.F. Ziegler

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THE STOPPING AND RANGE OF IONS IN SOLIDS

ABSTRACT

This book presents the theory, applications and computer programs for a comprehensive treatment of the penetration of ions into solids. The theoretical sections are presented in a tutorial manner and require no prior expertise.

The book first presents an historical review of the field. Since the discovery of radioactivity, many major advances in physics resulted in new interest in improving the understanding of ion penetration of matter. Because of this long interest, there is now a rich and varied base of theory to use to predict stopping and range phenomena. The next chapter presents the formalism of the elastic scattering of screened nuclei and the development of nuclear stopping cross-sections. The inelastic scattering processes of an ion penetrating a solid are then presented with a derivation which leads from the basic case of a proton penetrating a free electron gas, to the complex case of the inelastic energy loss of a partially stripped heavy ion in a solid. These various forms of energy loss and scattering are then developed into full descriptions of the methods to calculate the range of ions in solids. The formalism covers the energy loss of ions to the target atoms, the production of ionization, plasmons, phonons and damage in the solid, and the final distribution of the ions. The theoretical results are then transcribed into full computer codes (FORTRAN) so ion penetration phenomena may be calculated for any combination of ions/targets/energies. To aid in conversion of the FORTRAN to any system, the programs are presented both in main-frame FORTRAN and in micro-computer FORTRAN (such as for the IBM-PC). These programs are then used to illustrate the accuracy of the theory by comparing calculations to the data presented in over 1000 papers.

As an Appendix, tables are presented of the solid-state charge distribution of Hartree-Fock solids for all elements.

The stopping and range calculations in this book are considered to be more accurate than those found in Volumes 3-6 of this series. Hence, this volume supersedes these prior volumes.

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THE STOPPING AND RANGE OF IONS IN SOLIDS

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CHAPTER 1 - INTRODUCTION AND HISTORICAL REVIEW

INTRODUCTION

This book covers the physical phenomena associated with the penetration of energetic ions into solids. It is primarily concerned with the quantitative evaluation of how the ions lose energy to the solid and the final distribution of these ions after they stop within the solid. Also considered are the first order effects on the atoms of the solid, particularly the electronic excitation of the atoms, the displacement of lattice atoms by energetic collisions (lattice damage) and the production of plasmons and phonons in the solid by the passing ions. No evaluation is made of thermal effects in the solid, especially redistribution of lattice atoms or implanted ions by thermal or vacancy induced diffusion.

The literature contains a large amount of experimentally determined stopping powers and ion range distributions. They are not, however, so accurate or dense that direct interpolation to unknown systems is usually possible. The main goal of this work is to establish such interpolation formulae based on existing experimental data and unified theoretical concepts.

The theoretical chapters of this book are Chapters 2 and 3 and they are presented in an elementary tutorial style which needs little background. Two special topics in these chapters require extensive explanation and the reader is referred to several review articles for the lengthy details (these are (a) the treatment of relativistic effects for ions with velocities above 10^7 eV/amu, and (b) the interaction of a charged particle with a quantized free electron gas).

After the theoretical presentations, Chapters 4-5 explain how the physics of ion-solid interactions may be combined to calculate the statistically averaged phenomena associated with the penetration of solids by a beam of ions. The results of such calculations are shown and discussed in Chapters 6-7 where the calculations are compared to the extensive data of over 2000 papers in order to evaluate the accuracy of the theory.

Finally, Chapter 8 presents useful computer programs which can be used to calculate the stopping and range of any ion in complex solids, including layered structures of compounds. These programs are extensively notated and cross-referenced with the earlier explanatory chapters.

This book is the final volume in this series on the stopping and range of ions in matter.

HISTORICAL SUMMARY

For seventy-five years the stopping of energetic ions in matter has been a subject which has received great theoretical and experimental interest. The theoretical treatment of the stopping of ions in matter is due greatly to the work of Bohr (13a,15a,48a), Bethe (30a,32a,34a), Bloch (33a,33b), Firsov (57a,57b,58a,58b) and Lindhard (53b,54a,63a,64a,68a,68b). It has been reviewed by Bohr (48a), Whaling (58c), Fano (63b), Jackson (62a,75a), Bichsel (70e), Sigmund (75b), Ahlen (80e) and Ziegler, et al. (78a,80a,80b,82b). Soon after the discovery of energetic particle emission from radioactive materials, there was interest in how these corpuscles were slowed down in traversing matter. In 1900, Marie Curie stated (00a) the hypothesis that "les rayons alpha sont des projectiles

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

APRIL 1912.

XLII. *Ionization by Moving Electrified Particles.*
By Sir J. J. THOMSON*.

THE theory developed in this paper is based on the following assumptions:—

1. Cathode or positive rays when they pass through an atom repel or attract the corpuscles in it and thereby give to them kinetic energy.
2. When the energy imparted to a corpuscle is greater than a certain definite value—the value required to ionize the atom—a corpuscle escapes from the atom, and a free corpuscle and positively charged atom are produced.

We must first find under what circumstances a cathode ray moving with a given velocity will lose when it passes by a corpuscle a quantity of energy greater than the amount required to ionize an atom.

In my 'Conduction of Electricity through Gases' it is shown that when a body with a charge E_1 in electrostatic units and mass M_1 is projected with a velocity V towards a body with a charge E_2 and mass M_2 at rest, the energy Q transferred to the latter is given by the equation

$$Q=\frac{4M_1M_2}{(M_1+M_2)^2}T\sin^2\theta,$$

* Communicated by the Author.

Phil. Mag. S. 6. Vol. 23. No. 136. *April* 1912. 2 H

Figure (1-1) A reproduction of the first extended theoretical paper on the energy loss of charged particles in matter.

materiels susceptibles de perdre de leur vitesse en traversant la matiere." Many scientists immediately realized that since these particles could penetrate thin films, such experiments might finally unravel the secrets of the atom. Early attempts to create a particle energy loss theory were inconclusive for there was not yet an accurate proposed model of the atom.

The theoretical treatment for the scattering of two point charges was derived by J. J. Thomson in his classic book on electricity (03a). Much of the traditional particle energy-loss symbolism can be traced to this book which introduced a comprehensive treatment for classical Coulombic scattering between energetic charged particles. This work, however, did not attempt to calculate actual stopping powers.

Enough experimental evidence of radioactive particle interactions with matter was collected in the next decade to make stopping power theory one of the central concerns of those attempting to develop an atomic model. In 1909 Geiger and Marsden (09a) were studying the penetration of alpha-particles through thin foils, and the spread of the trajectories after emerging from the back side. They hoped to determine the distribution of charges within the foil by the angular spread of the transmitted beam. There are conflicting histories as to who made the suggestion that they look for backscattered particles - but the subsequent startling data reversed the current thought on atomic structure. They reported that about .01% of the heavy alpha-particles were scattered back from the target, and from an analysis of the data statistics such backscattered events had to be from isolated single collisions. Two years later Rutherford was able to demonstrate theoretically (11a) that the backscattering was indeed due to a single event, and by analyzing this and electron scattering data he was able to first calculate that the *nucleus* of Al atoms must have a charge of about 22 and about 138 for platinum!

J. J. Thomson, director of the prestigious Cavendish Laboratory, and Niels Bohr, a fresh post-doctoral scientist at Rutherford's Manchester Laboratory, both published almost simultaneously (12a,13a) an analysis of the stopping of charged particles by matter, and they illustrated much of their divergent ideas on the model of an atom. Thomson incredibly ignored in his paper the Rutherford alpha-particle scattering theory (11a) of a year before. But the nuclear atom with a heavy positively-charged core was the basis of Bohr's ideas. (13a,15a).

Bohr's early work is instructive because for the first time a unified theory of stopping was attempted, and we can see in this and similar later works the essential problems of stopping theory:

(a) How does an energetic charged particle (a point charge) lose energy to the quantized electron plasma of a solid (inelastic energy loss)?

(b) How can one incorporate into this interaction simultaneous distortion of the electron plasma caused by the particle (target polarization)?

(c) How can one extend the point charge-plasma interaction to that for a finite moving atom in a plasma?

(d) How can one estimate the degree of ionization of the moving atom and describe its electrons when it is both ionized and within an electron plasma?

(e) How can one calculate the screened Coulomb scattering of the moving atom with each heavy target nucleus it passes?

(f) How can one include relativistic corrections to all of the above?

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JANUARY 1913.

II. *On the Theory of the Decrease of Velocity of Moving Electrified Particles on passing through Matter.* By N. BOHR, *Dr. phil. Copenhagen* *.

WHEN cathode-rays or α- and β-rays penetrate through matter their velocity decreases. A theory of this phenomena was first given by Sir J. J. Thomson †. In the calculation of this author the cathode- and β-rays are assumed to lose their velocity by collisions with the electrons contained in the atoms of the matter. The form of the law, found by this calculation, connecting the velocity of the particles and the thickness of matter traversed, has been recently shown by Whiddington ‡ to be in good agreement with experiments. Somewhat different conceptions are used in the calculation of Sir J. J. Thomson on the absorption of α-rays, as the latter, on account of their supposed greater dimensions, are assumed to lose their velocity by collisions, not with the single electrons but with the atoms of the matter considered as entities.

According to the theory given by Professor Rutherford § of the scattering of α-rays by matter, the atoms of the matter are supposed to consist of a cluster of electrons kept together by attractive forces from a nucleus. This nucleus, which possesses a positive charge equal to the sum of the negative charges on the electrons, is further supposed to be the seat of the essential part of the mass of the atom, and to have dimensions which are exceedingly small compared with the dimensions of the atom. According to this theory an α-particle consists simply of the nucleus of a helium atom. We see that after such a conception there is no reason to discriminate materially between the collisions of an atom with an α- or β-particle—apart of course from the differences due to the difference in their charge and mass.

An elaborate theory of the absorption and scattering of α-rays, based on Professor Rutherford's conception of the constitution of atoms, was recently published by C. G. Darwin ‖. In the theory of this author the α-particles simply penetrate the atoms and act upon the single electrons contained in them, by forces varying inversely as the square

* Communicated by Prof. E. Rutherford, F.R.S.

† J. J. Thomson, 'Conduction of Electricity through Gases,' pp. 370–382.

‡ R. Whiddington, Proc. Roy. Soc. A. lxxxvi. p. 360 (1912).

§ E. Rutherford, Phil. Mag. xxi. p. 669 (1911).

‖ C. G. Darwin, Phil. Mag. xxiii. p. 907 (1912).

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[SIXTH SERIES.]

OCTOBER 1915.

LX. *On the Decrease of Velocity of Swiftly Moving Electrified Particles in passing through Matter.* By N. BOHR, *Dr. Phil. Copenhagen; p. t. Reader in Mathematical Physics, University of Manchester* *.

THE object of the present paper is to continue some calculations on the decrease of velocity of α and β rays published by the writer in a previous paper in this magazine †. This paper was concerned only with the mean value of the rate of decrease of velocity of the swiftly moving particles, but from a closer comparison with the measurements it appears necessary, especially for β rays, to consider the probability distribution of the loss of velocity suffered by the single particles. This problem has been discussed briefly by K. Herzfeld ‡, but on assumptions as to the mechanism of decrease of velocity essentially different from those used in the following *. Another question which will be considered more fully in the present paper is the effect of the velocity of β rays being comparable with the velocity of light. These calculations are contained in the first three sections. In the two next sections the theory is compared with the measurements. It will be shown that the approximate agreement obtained in the former paper is improved by the closer theoretical discussion, as well as by using the recent more accurate measurements. Section 6 contains some considerations on the ionization produced by α and β rays. A theory for this phenomenon has been given by Sir J. J. Thomson †.

§ 1. *The average value of the rate of decrease of velocity.*

For the sake of clearness it is desirable to give a brief summary of the calculations in the former paper. References to the previous literature on the subject will be found in that paper.

Following Sir Ernest Rutherford, we shall assume that the atom consists of a central nucleus carrying a positive charge and surrounded by a cluster of electrons kept together by the attractive forces from the nucleus. The nucleus is the seat of practically the entire mass of the atom and has dimensions exceedingly small compared with the dimensions of the surrounding cluster of electrons. If an α or β particle passes through a sheet of matter it will penetrate

Figure (1-2) Reproductions of the first two papers by Neils Bohr on the stopping of charged particles in matter. Bohr had finished his Ph.D. in 1911. After an unsympathetic visit with J. J. Thomson, Bohr went to work with Rutherford at the Manchester Laboratory and produced these papers. Between these papers he also developed and published his theory of the atom which suggested the quantization of angular momentum.

This is a brief list of the major problems encountered, and scientific interest shifts back and forth between them over the decades because of external scientific tidal forces. Examples might be (a) the development of quantal scattering in the nineteen twenties, (b) the study of nuclear decay and fission in the thirties and forties, (c) the study of nuclear physics in the fifties, (d) the technological applications of ion implantation for material modification in the sixties, and more recently the use of ion beams in material analysis and ion-solid interactions in fusion research. This ebb and flow of interest continues because of the recurrent importance of the problem, and the difficulty of calculating the penetration of energetic atoms in solids from first principles. We briefly review some of the historical milestones in this field below.

One of Bohr's original conclusions was that the energy loss of ions passing through matter could be divided into two components: nuclear stopping (energy loss to the medium's atomic positive cores) and electronic stopping (energy loss to the medium's light electrons). Bohr, in his first papers, correctly deduced that the electronic stopping would be far greater than the nuclear stopping for energetic light ions such as are emitted by radioactive sources. This conclusion was based on recoil kinematics considering only the relative masses and abundances of the target electrons and nuclei.

Bohr further introduced atomic structure into stopping theory by giving target electrons the orbit frequencies obtained from optical spectra and calculating the energy transferred to such harmonic oscillators. He noted that the experimentally reported stopping powers for heavy atom targets indicated that many electrons in these targets must be more tightly bound than the optical data suggested. He also realized that his accounting of the energy loss process was seriously limited by a lack of knowledge of the charge state of the ion inside the matter, i.e., its effective charge in its interaction with the target medium.

A major advance in understanding stopping powers came 20 years later when Bethe (30a,32a,34a) and Bloch (33a,33b) restated the problems from the perspective of quantum mechanics, and derived in the Born approximation the fundamental equations for the stopping of very fast particles in a quantized electron plasma. This theoretical approach remains the basic method for evaluating the energy loss of light particles with velocities of 10 MeV/amu - 2 GeV/amu. This restriction in velocity is because below these velocities the ion projectile may not be fully stripped of its electrons (which is assumed by the theory), and above this velocity there are additional relativistic corrections.

In the late 1930's a renewed interest was taken in energy loss with the discovery of nuclear fission and the energetic heavy particles which resulted from nuclear disintegration. Various theoretical studies were published by Bohr, Lamb, Knipp, Teller and Fermi.

The problem presented by the fission fragment data was how to treat the interaction of a *partially stripped* heavy ion. This is called the 'effective-charge' problem, for it was hoped that if a degree of ionization for the projectile could be estimated, then the traditional stopping power theories could be used. Bohr suggested (40d,41a) that the ion be considered to be stripped of all electrons with velocities lower than the ion velocity, and using the Thomas Fermi atom he could show that

$$Z_1^* = Z_1^{1/3}\, v/v_o \qquad (1-1)$$

where Z_1 is the atomic number of the ion, and Z_1^* is its effective charge in energy loss to the target electrons. v is the ion velocity and v_o is the Bohr velocity ($\sim 2.2\times10^8$ cm/sec).

ZEITSCHRIFT FÜR

PHYSIK

HERAUSGEGEBEN UNTER MITWIRKUNG
DER
DEUTSCHEN PHYSIKALISCHEN GESELLSCHAFT

Bremsformel für Elektronen relativistischer Geschwindigkeit.

Von **H. Bethe**, zurzeit in Rom.

(Eingegangen am 4. Mai 1932.)

Aus der Theorie von Møller[1]) wird der Energieverlust von Elektronen relativistischer Geschwindigkeit beim Durchgang durch Materie abgeleitet. Der Energieverlust pro Zentimeter Weg erreicht bei etwa 96% der Lichtgeschwindigkeit ein Minimum und steigt bei höheren Geschwindigkeiten wieder an; für Elektronen von einigen Milliarden Volt beträgt er etwa 4 Millionen Volt pro Zentimeter Wasser. Eine Tabelle des theoretischen Energieverlustes von Elektronen und Protonen verschiedener Geschwindigkeit wird gegeben.

1. Bis vor kurzem war es nicht möglich, die Streuung von Elektronen sehr hoher Geschwindigkeit quantenmechanisch zu behandeln, weil die bekannten Ansätze für die Wechselwirkungsenergie zweier Elektronen[2]) nur bis zur Größenordnung v^2/c^2 exakt waren (v = Elektronen-, c = Lichtgeschwindigkeit). Inzwischen hat Møller[1]) in einer wichtigen Arbeit die Frage der Wechselwirkungsenergie sehr einfach und befriedigend geklärt, und es ist nunmehr möglich, die Bremsformel für beliebig rasche Elektronen anzugeben, was für die Deutung der Experimente über die korpuskulare Höhenstrahlung von Wichtigkeit ist.

Ein Teilchen der Ladung ez möge sich mit der Geschwindigkeit v durch eine Substanz hindurchbewegen, welche in der Volumeneinheit N Atome der Ordnungszahl Z enthält. Dann verliert das Teilchen pro Zentimeter Weg die Energie

$$-\frac{dT}{dx} = \frac{2\pi e^4 N Z z^2}{m v^2}\left(\lg\frac{2 m v^2 W}{E^2\left(1-\frac{v^2}{c^2}\right)} - \frac{v^2}{c^2}\right), \qquad (1)$$

Bremsvermögen von Atomen mit mehreren Elektronen.

Von **F. Bloch** in Leipzig.

Mit 1 Abbildung. (Eingegangen am 22. Dezember 1932.)

Das Bremsvermögen komplizierterer Atome wird nach der Methode von Thomas-Fermi berechnet. Es ergibt sich ein einfacher Gang mit der Ordnungszahl, der in gutem Einklang mit der Erfahrung steht.

§ 1. Einleitung. Das Problem, die Bremsung rasch bewegter elektrischer Teilchen bei ihrem Durchgang durch Materie zu berechnen, kann im Prinzip als gelöst bezeichnet werden[1]). Man erhält für die pro Zentimeter Wegstrecke an die Atome übertragene Energie (l. c.)

$$\frac{\Delta T}{\Delta z} = N\cdot\frac{4\pi e^2 E^2}{m_0 v^2}\sum_n f_n\left\{\lg\frac{(2)\, m_0 v^2}{\hbar\omega_n} - \frac{1}{2}\lg\left(1-\frac{v^2}{c^2}\right) - \frac{v^2}{2c^2} + \psi(1) - R\left[\psi\left(1+i\frac{eE}{\hbar v}\right)\right]\right\}{}^{2)}. \qquad (1)$$

Figure (1-3) Reproductions of the articles in Zeitschrift fur Physik which indicate a main thrust in stopping theory was in Germany in the early 1930's. These and several similar articles established the quantum mechanical aspects of the stopping of high velocity particles.

Further, he estimated a screening distance, a_{12}, between two colliding atoms which limits the energy transfer between nuclei

$$a_{12} = a_o/(Z_1^{2/3} + Z_2^{2/3})^{1/2} \qquad (1-2)$$

where a_o is the Bohr radius, .59 Å. These two expressions form the basis of much of the stopping theory of the next 30 years. Lamb (40b) considered the same problem as Bohr, and suggested a similar effective charge approximation, but based on the energy rather than the velocity of the ion's electrons. Lamb also got a similar, but less detailed, expression for stopping power assuming Thomas-Fermi atoms. He suggested that the target electron velocity distribution would significantly alter the stopping of the fission fragment.

Fermi (40c) considered the same points as Lamb and Bohr, but concentrated upon evaluating the interaction of a charged particle with the dielectric plasma of a solid, and the electric polarization of the medium by the particle. This polarization of the target medium had been suggested first by Swann (38a), and Fermi was able to reduce this difficult problem to a form which could be calculated. He showed that stopping powers were universally proportional to target density, on an equal - mass - traversed basis.

A detailed suggestion for scaling stopping powers was shown by Knipp and Teller (41b), who successfully used the effective charge concept of Bohr and Lamb to scale H stopping values to equivalent He ion stopping powers.

These theoretical studies had only limited success with the fission fragment problem, and the primary practical result was to provide scaling relationships for the heavy ion stopping and ranges. That is, it allowed for interpolation and modest extrapolation from existing data into systems with different ions/targets/energies. Basically, the dominant effects were the target material and the ion's velocity. If the stopping of one ion, say a proton, was known at two velocities in a material, and the stopping of a heavy ion in that material was known at one of these velocities, then its stopping at the second velocity would be a directly proportional. This law was applicable over a wide variety of velocities and ions. In fact it became so widely used that it stimulated several papers of objections to its theoretical shortcomings (63b,72b).

During the 1950's there were fundamental papers on evaluating both the energy transfer from slow particles to quantized electron plasmas, and in the energy loss to target nuclei. The study of particle stopping in a free electron gas is the first step in calculating the energy loss of an ion to a target's electrons. This problem was evaluated in Bohr's earliest papers (13a,15a) where he considered the electrons to be charged harmonic oscillators, with orbit frequencies established by the analysis of optical data. This interaction of a particle with an electron plasma was extended to quantized plasmas and then to Thomas-Fermi atoms by Bethe (30a,32a) and Bloch (33a,33b). An excellent review of relativistic particle stopping powers has recently been made by Ahlen (80e) and the present book will not cover in detail this subject. For energies above 10 MeV/amu you should consult ref. (80a). Fermi then considered how the fast charged particle would polarize a classical electron medium of the target and hence modify the particle/plasma interaction (40c). This work was extended by Fermi and Teller (47b) to a degenerate free electron gas and they found that for slow particles the energy loss would be directly proportional to the particle's velocity. Bohr pointed out (48a) that behind the particle there would be an oscillating wake of charge, and this was evaluated more rigorously by Bohr (48b), and by Neufeld and Ritchie (55a). A full treatment of a charged particle penetrating a quantized electron plasma was presented at about the same

THE

PHYSICAL REVIEW

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MARCH 15, 1940 PHYSICAL REVIEW VOLUME 57

The Ionization Loss of Energy in Gases and in Condensed Materials*

ENRICO FERMI
Pupin Physics Laboratories, Columbia University, New York, New York

(Received January 22, 1940)

It is shown that the loss of energy of a fast charged particle due to the ionization of the material through which it is passing is considerably affected by the density of the material. The effect is due to the alteration of the electric field of the passing particle by the electric polarization of the medium. A theory based on classical electrodynamics shows that by equal mass of material traversed, the loss is larger in a rarefied substance than in a condensed one. The application of these results to cosmic radiation problems is discussed especially in view of the possible explanation on this basis of part of the difference in the absorption of mesotrons in air and in condensed materials that is usually interpreted as evidence for a spontaneous decay of the mesotron.

THE determination of the energy lost by a fast charged particle by ionization and excitation of the atoms through or near which it is passing has been the object of several theoretical investigations. The essential features of the phenomenon are explained as well known

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

OCTOBER 1, 1940 PHYSICAL REVIEW VOLUME 58

Scattering and Stopping of Fission Fragments

N. BOHR
Institute of Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

(Received July 9, 1940)

THE cloud-chamber pictures of tracks of uranium fission fragments in gases obtained by Brostrøm, Bøggild and Lauritsen[1] have revealed a number of interesting differences between such tracks and those of protons and alpha-particles. These differences may be simply shown to be caused by the comparatively high charge and mass of the fission fragments, which imply that nuclear collisions play a much greater part in the phenomenon than is the case for the ordinary light particles.

will have orbital velocities greater than or equal to V.

In an encounter between the fragment and a heavy atom possessing electrons lightly bound and also electrons with velocities greater than V, we may, moreover, assume that only the former electrons, in approximate number V/V_0, will be effective in the stopping. This is true since the faster electrons, just as the electrons carried with the fragment, will be merely adiabatically influenced during the encounter and will therefore

Figure (1-4) Reproductions of the articles in The Physical Review in 1940-1941 which shows that interest had now shifted to the stopping of heavy fission fragments and the development of heavy ion stopping-power scaling theory.

time by Lindhard (54a), Neufeld and Ritchie (55a) and Fano (56c). The Lindhard approach concentrates on non-relativistic particle interactions with a free-electron gas and provides a full general treatment with the following assumptions:

- The free electron gas consists of electrons at zero temperature (single electrons are described by plane waves) on a fixed uniform positive background with overall charge neutrality.
- The initial electron gas is of constant density.
- The interaction of the charged particle is a perturbation on the electron gas.
- All particles are non-relativistic.

The Lindhard approach is widely cited in the literature as it formed part of the first unified theory of ion penetration of solids (63a), and it has been widely used as the basis for calculating the electronic stopping of ions in matter (see, for example, 67a, 70b, 72f, 74a, 75k, 77h, 78a, 79d).

The energy loss to target nuclei is basically the study of screened Coulomb collisions between two colliding atoms. In the 1950's, major advances were made in the elastic energy loss of the ion to target nuclei. Bohr summarized much of the earlier work in 1948 (48a) which used the Thomas Fermi model to estimate the screened Coulomb potential, V(r) between atoms:

$$V(r) = \frac{Z_1 Z_2}{r} \exp(-r/a) \qquad (1-3)$$

where Z_1 and Z_2 are the atomic numbers, r is their separation, and a is a "screening parameter". Once the screening parameter is specified, then the classical scattering and energy transfer can be calculated. Bohr argued that a reasonable approximation might be:

$$a = a_o/(Z_1^{2/3} + Z_2^{\ 2/3})^{1/2}. \qquad (1-4)$$

but this approximation was not derived.

Firsov took a more practical approach and used numerical techniques to derive the interatomic potentials of two colliding Thomas-Fermi atoms (58a,b). After calculating the numeric values of the potentials as a function of the atomic separation he found that the best fit was obtained with the Thomas-Fermi screening function using a screening length:

$$a = a_o/(Z_1^{1/2} + Z_2^{1/2})^{2/3}. \qquad (1-5)$$

Another problem which received wide attention in the 1950's was the degree of ionization of the ion as it goes through materials. As we noted before, Bohr and others suggested that one simple criterion would be to assume that ions lose electrons whose orbital velocities would be less than the ion velocity. He suggested that the ion charge *fraction*, Z^*/Z, would be

$$Z^*/Z = v/(v_o Z_1^{2/3}) \qquad (1-6)$$

This relation comes from the Thomas-Fermi atom which assumes the electronic charge densities of atoms are similar with a common unit of length being proportional to $Z^{-1/3}$. The charge density is proportional to Z^2, and the total binding energy scales as $Z^{7/3}$. Therefore the binding per electron scales as $Z^{4/3}$ and the electron velocities are proportional to $Z^{2/3}$. Lamb

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Passage of Uranium Fission Fragments Through Matter*

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The ranges and rates of energy loss of the fission fragments of uranium are calculated on the basis of a model in which the charge of the fragment is obtained from its energy and its successive ionization potentials. The energy loss cross section for protons of the same velocity is then used to calculate the ranges of the two groups of fragments. For ($Z_1=42$, $A_1=100$) and ($Z_2=50$, $A_2=136$) these are found to be 2.42 cm and 2.08 cm, respectively, for a total assumed kinetic energy of 188 Mev and a final kinetic energy of the lighter fragment of 5 Mev (corresponding to ionization-chamber background). These are in fair agreement with the observed ranges of 2.2 cm and 1.5 cm. The experimental and theoretical range-energy relations are also in fair agreement. The validity of the model is discussed in detail, and it appears that it should be fairly good for fragments above 5 Mev. The initial charges of the fission fragments are found to be 17 and 13, respectively, and these are given as a function of the fragment energy in Table I. The density of ionization is found to decrease along the track, in marked contrast to the behavior for protons and alpha-particles.

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Velocity-Range Relation for Fission Fragments

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Considerations indicated in an earlier note as regards the rate of velocity loss of fission fragments along the range are developed in greater detail and a comparison is given between the calculations and more recent experiments. Especially is a more precise estimate given for the charge effective in electronic encounters which are determining for the stopping effect over the first part of range, and for the screening distance in nuclear collisions which are responsible for the ultimate stopping. In the estimate of the effect of electronic interactions, use is made of a comparison with the stopping of α-particles of the same velocities. In this connection, however, a certain correction is necessary due to an intrinsic difference in the stopping formulae to be applied in the two cases. Moreover, fission fragment tracks show, in contrast to α-rays, a considerable range straggling originating in the end part of the range. It is shown that in this respect also the calculation agrees closely with the experimental data.

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On the Energy Loss of Heavy Ions

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AND

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The energy loss of heavy ions is due to collisions with electrons and with nuclei. The first process is essentially determined by the ionic charge, which in turn depends on the ratio of the velocities of the most loosely bound electron within the ion, and of the ion. The former velocity is calculated from the Thomas-Fermi model while the ratio of the two velocities is adjusted to empirical data. The nuclear contribution to the stopping cross section is calculated by the known classical method. Though approximations could not be avoided, the procedure lends itself to the systematization of experimental data on intermediate and heavy ions.

Figure (1-5) Reproductions of the articles in The Physical Review in 1940-1941 which shows that interest had now shifted to the stopping of heavy fission fragments and the development of heavy ion stopping-power scaling theory.

had proposed (40b) the electron binding energy was the important stripping criterion, while Bohr suggested it was the electron velocity. A definitive clarification was made by Northcliffe (60c) who reduced a wide variety of experimental data by dividing each ion/target/energy experimental stopping power by the stopping power of protons in the same target and at the same *velocity*. In perturbation theory this ratio should scale as $(Z^*)^2$ where Z^* is the number of electrons left on the ion. He found a large amount of data could be accurately described using the relation:

$$Z^*/Z = 1 - a \exp\left[\frac{b}{Z^{2/3}} \cdot \frac{v}{v_o}\right] \qquad (1-7)$$

where a and b are fitting constants. The expression expands to be the Bohr relationship.

By the end of the 1950's the status may be summarized as:

(a) A good treatment of the energy loss of a charged particle to a quantized electron plasma. The theory includes both polarization of the medium about the charge, and discussions of extensions of particle interactions with electron plasmas to electrons in atomic matter.

(b) A good calculation of interatomic potentials and the energy transferred during a scattering collision between two atoms.

(c) A good evaluation of the effective charge of heavy ions in solids for the intermediate velocity range ($3v_o < E < 30\ v_o$)

Problems left to be solved included:

(d) How to extend the electron plasma point-charge interaction theory to the interaction with a finite sized ion?

(e) How to derive fundamentally the effective charge of a moving ion (where effective charge is defined as a combination of ion charge state plus target polarization)?

(f) And finally, how do you modify all of the above to use more realistic Hartree-Fock atoms rather than Thomas-Fermi statistical atoms?

In 1963 the first unified approach to stopping and range theory was made by Lindhard, Scharff and Schiott (63a) and their approach is commonly called the LSS-theory. This work brought together all the pieces, and bridging approximations were made so that calculations of stopping and range distributions could, for the first time, be made within a single model. This remarkable achievement was the result of over a decade of study by Lindhard and collaborators (53b, 54a, 63a, 64a, 68a, 68b), with the later publications deriving in detail some of the major equations of LSS theory. LSS theory was the peak of stopping and range theory based on statistical atoms. With this theory it was possible to predict the range of ions in solids within a factor of two - a remarkable achievement considering it was applicable over the entire range of atomic species and energies up to the stopping power maximum (70a, 70f, 75e, 75f, 75g, 77a). Since it was based on Thomas-Fermi atoms it was most accurate for atoms with many electrons in the intermediate range where they are neither fully stripped nor almost neutral. The theory naturally shows no shell effects.