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Handbooks of Modern
Statistical Methods

Handbook of Markov Chain Monte Carlo

Edited by

Steve Brooks

Andrew Gelman

Galin L. Jones

Xiao-Li Meng



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Preface

Over the past 20 years or so, Markov Chain Monte Carlo (MCMC) methods have revolutionized statistical computing. They have impacted the practice of Bayesian statistics profoundly by allowing intricate models to be posited and *used* in an astonishing array of disciplines as diverse as fisheries science and economics. Of course, Bayesians are not the only ones to benefit from using MCMC, and there continues to be increasing use of MCMC in other statistical settings. The practical importance of MCMC has also sparked expansive and deep investigation into fundamental Markov chain theory. As the use of MCMC methods mature, we see deeper theoretical questions addressed, more complex applications undertaken and their use spreading to new fields of study. It seemed to us that it was a good time to try to collect an overview of MCMC research and its applications.

This book is intended to be a reference (not a text) for a broad audience and to be of use both to developers and users of MCMC methodology. There is enough introductory material in the book to help graduate students as well as researchers new to MCMC who wish to become acquainted with the basic theory, algorithms and applications. The book should also be of particular interest to those involved in the development or application of new and advanced MCMC methods. Given the diversity of disciplines that use MCMC, it seemed prudent to have many of the chapters devoted to detailed examples and case studies of realistic scientific problems. Those wanting to see current practice in MCMC will find a wealth of material to choose from here.

Roughly speaking, we can divide the book into two parts. The first part encompasses 12 chapters concerning MCMC foundations, methodology and algorithms. The second part consists of 12 chapters which consider the use of MCMC in practical applications. Within the first part, the authors take such a wide variety of approaches that it seems pointless to try to classify the chapters into subgroups. For example, some chapters attempt to appeal to a broad audience by taking a tutorial approach while other chapters, even if introductory, are either more specialized or present more advanced material. Yet others present original research. In the second part, the focus shifts to applications. Here again, we see a variety of topics, but there are two basic approaches taken by the authors of these chapters. The first is to provide an overview of an application area with the goal of identifying best MCMC practice in the area through extended examples. The second approach is to provide detailed case studies of a given problem while clearly identifying the statistical and MCMC-related issues encountered in the application.

When we were planning this book, we quickly realized that no single source can give a truly comprehensive overview of cutting-edge MCMC research and applications—there is just too much of it and its development is moving too fast. Instead, the editorial goal was to obtain contributions of high quality that may stand the test of time. To this end, all of the contributions (including those written by members of the editorial panel) were submitted to a rigorous peer review process and many underwent several revisions. Some contributions, even after revisions, were deemed unacceptable for publication here, and we certainly welcome constructive feedback on the chapters that did survive our editorial process. We thank all the authors for their efforts and patience in this process, and we ask for understanding from those whose contributions are not included in this book. We believe the breadth and depth of the contributions to this book, including some diverse opinions expressed, imply a continuously bright and dynamic future for MCMC research. We hope

this book inspires further work—theoretical, methodological, and applied—in this exciting and rich area.

Finally, no project of this magnitude could be completed with satisfactory outcome without many individuals' help. We especially want to thank Robert Calver of Chapman & Hall/CRC for his encouragements, guidelines, and particularly his patience during the entire process of editing this book. We also offer our heartfelt thanks to the numerous referees for their insightful and rigorous review, often multiple times. Of course, the ultimate appreciation for all individuals involved in this project comes from your satisfaction with the book or at least a part of it. So we thank you for reading it.

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Steve Brooks is company director of ATASS, a statistical consultancy business based in the United Kingdom. He was formerly professor of Statistics at Cambridge University and received the Royal Statistical Society Guy medal in Bronze in 2005 and the Philip Leverhulme prize in 2004. Like his co-editors, he has served on numerous professional committees both in the United Kingdom and elsewhere, as well as sitting on numerous editorial boards. He is co-author of *Bayesian Analysis for Population Ecology* (Chapman & Hall/CRC, 2009) and co-founder of the National Centre for Statistical Ecology. His research interests include the development and application of computational statistical methodology across a broad range of application areas.

Andrew Gelman is a professor of statistics and political science and director of the Applied Statistics Center at Columbia University. He has received the Outstanding Statistical Application award from the American Statistical Association, the award for best article published in the *American Political Science Review*, and the Committee of Presidents of Statistical Societies award for outstanding contributions by a person under the age of 40. His books include *Bayesian Data Analysis* (with John Carlin, Hal Stern, and Don Rubin), *Teaching Statistics: A Bag of Tricks* (with Deb Nolan), *Data Analysis Using Regression and Multilevel/Hierarchical Models* (with Jennifer Hill), and, most recently, *Red State, Blue State, Rich State, Poor State: Why Americans Vote the Way They Do* (with David Park, Boris Shor, Joe Bafumi, and Jeronimo Cortina).

Andrew has done research on a wide range of topics, including: why it is rational to vote; why campaign polls are so variable when elections are so predictable; why redistricting is good for democracy; reversals of death sentences; police stops in New York City; the statistical challenges of estimating small effects; the probability that your vote will be decisive; seats and votes in Congress; social network structure; arsenic in Bangladesh; radon in your basement; toxicology; medical imaging; and methods in surveys, experimental design, statistical inference, computation, and graphics.

Galin L. Jones is an associate professor in the School of Statistics at the University of Minnesota. He has served on many professional committees and is currently serving on the editorial board for the *Journal of Computational and Graphical Statistics*. His research interests include Markov chain Monte Carlo, Markov chains in decision theory, and applications of statistical methodology in agricultural, biological, and environmental settings.

Xiao-Li Meng is the Whipple V. N. Jones professor of statistics and chair of the Department of Statistics at Harvard University; previously he taught at the University of Chicago (1991–2001). He was the recipient of the 1997–1998 University of Chicago Faculty Award for Excellence in Graduate Teaching, the 2001 Committee of Presidents of Statistical Societies Award, the 2003 Distinguished Achievement Award and the 2008 Distinguished Service Award from the International Chinese Statistical Association, and the 2010 Medallion Lecturer from the Institute of Mathematical Statistics (IMS). He has served on numerous professional committees, including chairing the 2004 Joint Statistical Meetings and the Committee on Meetings of American Statistical Association (ASA) from 2004 until 2010. He is an elected fellow of the ASA and the IMS. He has also served on editorial boards for *The Annals of Statistics*, *Bayesian Analysis*, *Bernoulli*, *Biometrika*, *Journal of the American Statistical*

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Contents

Preface	xix
Editors	xxi
Contributors	xxiii

Part I Foundations, Methodology, and Algorithms

1. Introduction to Markov Chain Monte Carlo	3
<i>Charles J. Geyer</i>	
1.1 History	3
1.2 Markov Chains	4
1.3 Computer Programs and Markov Chains	5
1.4 Stationarity	5
1.5 Reversibility	6
1.6 Functionals	6
1.7 The Theory of Ordinary Monte Carlo	6
1.8 The Theory of MCMC	8
1.8.1 Multivariate Theory	8
1.8.2 The Autocovariance Function	9
1.9 AR(1) Example	9
1.9.1 A Digression on Toy Problems	10
1.9.2 Supporting Technical Report	11
1.9.3 The Example	11
1.10 Variance Estimation	13
1.10.1 Nonoverlapping Batch Means	13
1.10.2 Initial Sequence Methods	16
1.10.3 Initial Sequence Methods and Batch Means	17
1.11 The Practice of MCMC	17
1.11.1 Black Box MCMC	18
1.11.2 Pseudo-Convergence	18
1.11.3 One Long Run versus Many Short Runs	18
1.11.4 Burn-In	19
1.11.5 Diagnostics	21
1.12 Elementary Theory of MCMC	22
1.12.1 The Metropolis–Hastings Update	22
1.12.2 The Metropolis–Hastings Theorem	23
1.12.3 The Metropolis Update	24
1.12.4 The Gibbs Update	24
1.12.5 Variable-at-a-Time Metropolis–Hastings	25
1.12.6 Gibbs Is a Special Case of Metropolis–Hastings	26
1.12.7 Combining Updates	26
1.12.7.1 Composition	26
1.12.7.2 Palindromic Composition	26
1.12.8 State-Independent Mixing	26
1.12.9 Subsampling	27
1.12.10 Gibbs and Metropolis Revisited	28

1.13	A Metropolis Example	29
1.14	Checkpointing	34
1.15	Designing MCMC Code	35
1.16	Validating and Debugging MCMC Code	36
1.17	The Metropolis–Hastings–Green Algorithm	37
1.17.1	State-Dependent Mixing	38
1.17.2	Radon–Nikodym Derivatives	39
1.17.3	Measure-Theoretic Metropolis–Hastings	40
1.17.3.1	Metropolis–Hastings–Green Elementary Update	40
1.17.3.2	The MHG Theorem	42
1.17.4	MHG with Jacobians and Augmented State Space	45
1.17.4.1	The MHGJ Theorem	46
	Acknowledgments	47
	References	47
2.	A Short History of MCMC: Subjective Recollections from Incomplete Data	49
	<i>Christian Robert and George Casella</i>	
2.1	Introduction	49
2.2	Before the Revolution	50
2.2.1	The Metropolis et al. (1953) Paper	50
2.2.2	The Hastings (1970) Paper	52
2.3	Seeds of the Revolution	53
2.3.1	Besag and the Fundamental (Missing) Theorem	53
2.3.2	EM and Its Simulated Versions as Precursors	53
2.3.3	Gibbs and Beyond	54
2.4	The Revolution	54
2.4.1	Advances in MCMC Theory	56
2.4.2	Advances in MCMC Applications	57
2.5	After the Revolution	58
2.5.1	A Brief Glimpse at Particle Systems	58
2.5.2	Perfect Sampling	58
2.5.3	Reversible Jump and Variable Dimensions	59
2.5.4	Regeneration and the Central Limit Theorem	59
2.6	Conclusion	60
	Acknowledgments	61
	References	61
3.	Reversible Jump MCMC	67
	<i>Yanan Fan and Scott A. Sisson</i>	
3.1	Introduction	67
3.1.1	From Metropolis–Hastings to Reversible Jump	67
3.1.2	Application Areas	68
3.2	Implementation	71
3.2.1	Mapping Functions and Proposal Distributions	72
3.2.2	Marginalization and Augmentation	73
3.2.3	Centering and Order Methods	74
3.2.4	Multi-Step Proposals	77
3.2.5	Generic Samplers	78

3.3	Post Simulation	80
3.3.1	Label Switching	80
3.3.2	Convergence Assessment	81
3.3.3	Estimating Bayes Factors	82
3.4	Related Multi-Model Sampling Methods	84
3.4.1	Jump Diffusion	84
3.4.2	Product Space Formulations	85
3.4.3	Point Process Formulations	85
3.4.4	Multi-Model Optimization	85
3.4.5	Population MCMC	86
3.4.6	Multi-Model Sequential Monte Carlo	86
3.5	Discussion and Future Directions	86
	Acknowledgments	87
	References	87
4.	Optimal Proposal Distributions and Adaptive MCMC	93
	<i>Jeffrey S. Rosenthal</i>	
4.1	Introduction	93
4.1.1	The Metropolis–Hastings Algorithm	93
4.1.2	Optimal Scaling	93
4.1.3	Adaptive MCMC	94
4.1.4	Comparing Markov Chains	94
4.2	Optimal Scaling of Random-Walk Metropolis	95
4.2.1	Basic Principles	95
4.2.2	Optimal Acceptance Rate as $d \rightarrow \infty$	96
4.2.3	Inhomogeneous Target Distributions	98
4.2.4	Metropolis-Adjusted Langevin Algorithm	99
4.2.5	Numerical Examples	99
	4.2.5.1 Off-Diagonal Covariance	100
	4.2.5.2 Inhomogeneous Covariance	100
4.2.6	Frequently Asked Questions	101
4.3	Adaptive MCMC	102
4.3.1	Ergodicity of Adaptive MCMC	103
4.3.2	Adaptive Metropolis	104
4.3.3	Adaptive Metropolis-within-Gibbs	105
4.3.4	State-Dependent Proposal Scalings	107
4.3.5	Limit Theorems	107
4.3.6	Frequently Asked Questions	108
4.4	Conclusion	109
	References	110
5.	MCMC Using Hamiltonian Dynamics	113
	<i>Radford M. Neal</i>	
5.1	Introduction	113
5.2	Hamiltonian Dynamics	114
5.2.1	Hamilton’s Equations	114
	5.2.1.1 Equations of Motion	114
	5.2.1.2 Potential and Kinetic Energy	115
	5.2.1.3 A One-Dimensional Example	116

5.2.2	Properties of Hamiltonian Dynamics	116
5.2.2.1	Reversibility	116
5.2.2.2	Conservation of the Hamiltonian	116
5.2.2.3	Volume Preservation	117
5.2.2.4	Symplecticness	119
5.2.3	Discretizing Hamilton's Equations—The Leapfrog Method	119
5.2.3.1	Euler's Method	119
5.2.3.2	A Modification of Euler's Method	121
5.2.3.3	The Leapfrog Method	121
5.2.3.4	Local and Global Error of Discretization Methods	122
5.3	MCMC from Hamiltonian Dynamics	122
5.3.1	Probability and the Hamiltonian: Canonical Distributions	122
5.3.2	The Hamiltonian Monte Carlo Algorithm	123
5.3.2.1	The Two Steps of the HMC Algorithm	124
5.3.2.2	Proof That HMC Leaves the Canonical Distribution Invariant	126
5.3.2.3	Ergodicity of HMC	127
5.3.3	Illustrations of HMC and Its Benefits	127
5.3.3.1	Trajectories for a Two-Dimensional Problem	127
5.3.3.2	Sampling from a Two-Dimensional Distribution	128
5.3.3.3	The Benefit of Avoiding Random Walks	130
5.3.3.4	Sampling from a 100-Dimensional Distribution	130
5.4	HMC in Practice and Theory	133
5.4.1	Effect of Linear Transformations	133
5.4.2	Tuning HMC	134
5.4.2.1	Preliminary Runs and Trace Plots	134
5.4.2.2	What Stepsize?	135
5.4.2.3	What Trajectory Length?	137
5.4.2.4	Using Multiple Stepsizes	137
5.4.3	Combining HMC with Other MCMC Updates	138
5.4.4	Scaling with Dimensionality	139
5.4.4.1	Creating Distributions of Increasing Dimensionality by Replication	139
5.4.4.2	Scaling of HMC and Random-Walk Metropolis	139
5.4.4.3	Optimal Acceptance Rates	141
5.4.4.4	Exploring the Distribution of Potential Energy	142
5.4.5	HMC for Hierarchical Models	142
5.5	Extensions of and Variations on HMC	144
5.5.1	Discretization by Splitting: Handling Constraints and Other Applications	145
5.5.1.1	Splitting the Hamiltonian	145
5.5.1.2	Splitting to Exploit Partial Analytical Solutions	146
5.5.1.3	Splitting Potential Energies with Variable Computation Costs	146
5.5.1.4	Splitting According to Data Subsets	147
5.5.1.5	Handling Constraints	148
5.5.2	Taking One Step at a Time—The Langevin Method	148
5.5.3	Partial Momentum Refreshment: Another Way to Avoid Random Walks	150

5.5.4	Acceptance Using Windows of States	152
5.5.5	Using Approximations to Compute the Trajectory	155
5.5.6	Short-Cut Trajectories: Adapting the Stepsize without Adaptation	156
5.5.7	Tempering during a Trajectory	157
	Acknowledgment	160
	References	160
6.	Inference from Simulations and Monitoring Convergence	163
	<i>Andrew Gelman and Kenneth Shirley</i>	
6.1	Quick Summary of Recommendations	163
6.2	Key Differences between Point Estimation and MCMC Inference	164
6.3	Inference for Functions of the Parameters vs. Inference for Functions of the Target Distribution	166
6.4	Inference from Noniterative Simulations	167
6.5	Burn-In	168
6.6	Monitoring Convergence Comparing between and within Chains	170
6.7	Inference from Simulations after Approximate Convergence	171
6.8	Summary	172
	Acknowledgments	173
	References	173
7.	Implementing MCMC: Estimating with Confidence	175
	<i>James M. Flegal and Galin L. Jones</i>	
7.1	Introduction	175
7.2	Initial Examination of Output	176
7.3	Point Estimates of θ_π	178
7.3.1	Expectations	178
7.3.2	Quantiles	181
7.4	Interval Estimates of θ_π	182
7.4.1	Expectations	182
7.4.1.1	Overlapping Batch Means	182
7.4.1.2	Parallel Chains	184
7.4.2	Functions of Moments	185
7.4.3	Quantiles	187
7.4.3.1	Subsampling Bootstrap	187
7.4.4	Multivariate Estimation	189
7.5	Estimating Marginal Densities	189
7.6	Terminating the Simulation	192
7.7	Markov Chain Central Limit Theorems	193
7.8	Discussion	194
	Acknowledgments	195
	References	195
8.	Perfection within Reach: Exact MCMC Sampling	199
	<i>Radu V. Craiu and Xiao-Li Meng</i>	
8.1	Intended Readership	199
8.2	Coupling from the Past	199
8.2.1	Moving from Time-Forward to Time-Backward	199

8.2.2	Hitting the Limit	200
8.2.3	Challenges for Routine Applications	201
8.3	Coalescence Assessment	201
8.3.1	Illustrating Monotone Coupling	201
8.3.2	Illustrating Brute-Force Coupling	202
8.3.3	General Classes of Monotone Coupling	203
8.3.4	Bounding Chains	204
8.4	Cost-Saving Strategies for Implementing Perfect Sampling	206
8.4.1	Read-Once CFTP	206
8.4.2	Fill's Algorithm	208
8.5	Coupling Methods	210
8.5.1	Splitting Technique	211
8.5.2	Coupling via a Common Proposal	212
8.5.3	Coupling via Discrete Data Augmentation	213
8.5.4	Perfect Slice Sampling	215
8.6	Swindles	217
8.6.1	Efficient Use of Exact Samples via Concatenation	218
8.6.2	Multistage Perfect Sampling	219
8.6.3	Antithetic Perfect Sampling	220
8.6.4	Integrating Exact and Approximate MCMC Algorithms	221
8.7	Where Are the Applications?	223
	Acknowledgments	223
	References	223
9.	Spatial Point Processes	227
	<i>Mark Huber</i>	
9.1	Introduction	227
9.2	Setup	227
9.3	Metropolis–Hastings Reversible Jump Chains	230
9.3.1	Examples	232
9.3.2	Convergence	232
9.4	Continuous-Time Spatial Birth–Death Chains	233
9.4.1	Examples	235
9.4.2	Shifting Moves with Spatial Birth and Death Chains	236
9.4.3	Convergence	236
9.5	Perfect Sampling	236
9.5.1	Acceptance/Rejection Method	236
9.5.2	Dominated Coupling from the Past	238
9.5.3	Examples	242
9.6	Monte Carlo Posterior Draws	243
9.7	Running Time Analysis	245
9.7.1	Running Time of Perfect Simulation Methods	248
	Acknowledgment	251
	References	251
10.	The Data Augmentation Algorithm: Theory and Methodology	253
	<i>James P. Hobert</i>	
10.1	Basic Ideas and Examples	253

10.2 Properties of the DA Markov Chain	261
10.2.1 Basic Regularity Conditions	261
10.2.2 Basic Convergence Properties	263
10.2.3 Geometric Ergodicity	264
10.2.4 Central Limit Theorems	267
10.3 Choosing the Monte Carlo Sample Size	269
10.3.1 Classical Monte Carlo	269
10.3.2 Three Markov Chains Closely Related to X	270
10.3.3 Minorization, Regeneration and an Alternative CLT	272
10.3.4 Simulating the Split Chain	275
10.3.5 A General Method for Constructing the Minorization Condition	277
10.4 Improving the DA Algorithm	279
10.4.1 The PX-DA and Marginal Augmentation Algorithms	280
10.4.2 The Operator Associated with a Reversible Markov Chain	284
10.4.3 A Theoretical Comparison of the DA and PX-DA Algorithms	286
10.4.4 Is There a Best PX-DA Algorithm?	288
Acknowledgments	291
References	291
11. Importance Sampling, Simulated Tempering, and Umbrella Sampling	295
<i>Charles J. Geyer</i>	
11.1 Importance Sampling	295
11.2 Simulated Tempering	297
11.2.1 Parallel Tempering Update	299
11.2.2 Serial Tempering Update	300
11.2.3 Effectiveness of Tempering	300
11.2.4 Tuning Serial Tempering	301
11.2.5 Umbrella Sampling	302
11.3 Bayes Factors and Normalizing Constants	303
11.3.1 Theory	303
11.3.2 Practice	305
11.3.2.1 Setup	305
11.3.2.2 Trial and Error	307
11.3.2.3 Monte Carlo Approximation	308
11.3.3 Discussion	309
Acknowledgments	310
References	310
12. Likelihood-Free MCMC	313
<i>Scott A. Sisson and Yanan Fan</i>	
12.1 Introduction	313
12.2 Review of Likelihood-Free Theory and Methods	314
12.2.1 Likelihood-Free Basics	314
12.2.2 The Nature of the Posterior Approximation	315
12.2.3 A Simple Example	316
12.3 Likelihood-Free MCMC Samplers	317
12.3.1 Marginal Space Samplers	319
12.3.2 Error-Distribution Augmented Samplers	320