Vortex Flow

Akira Ogawa

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Vortex Flow
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PREFACE

The subject of vortices is one of general interest in mechanical engineering, chemical engineering, and also in powder technology.

Interest in vortices also has a respectable history. For example, Leonardo da Vinci depicted the very interesting drawings of various kinds of vortex and eddy flows. The fundamental concept of vortex flow which was related to the idea of circulation was developed much later in the form of the lift force of the airplane wing in the theory of Kutta (1910) and Joukowski (1906).

The biggest vortex flows on earth are the cyclones in the Indian Ocean, typhoons in Japan, and also hurricanes. In fiction, there is a most interesting description of the sea wheel by Edgar Allen Poe (1808–1849) in his short story ''A Descent into the Maelstrom''. The phenomena of vortex flows is also common in daily life although we often don't notice them. For example, vortex flow in the bath tub, Benard-Karman vortex band as a noise which is produced when a strong wind blows across an electric wire, and a vortex ring produced when tobacco smoke is emitted from the smoker's mouth.

Engineering applications of vortex flows include: cyclone dust collectors for separating solid particles from three-dimensional turbulent rotational gas flows by centrifugal force (see ''Separation of Particles from Air and Gases'', Vols I and II, CRC Press, 1984) and also hydro-cyclones; the vortex tubes for heat separation in three-dimensional turbulent air flow; combustion chambers in the Diesel engine; mixing chambers; and also other technical fields.

However, the internal structure of the mechanisms of vortex flow is rather complicated. The simplest model is the Thomson-Rankine combined vortex model which was defined in a book called *Applied Mechanics* by Rankine (1876). This vortex model is composed of forced and free vortices. This combined vortex model was first proposed by Professor Thomson (Belfast) who studied the hydraulic vortex turbine (see ''Hydraulics and its Applications'' by Gibson, 1912). In addition, there are many kinds of vortex models which can be used in various technical applications. But, for the present, the more general vortex model which can be applied to air and liquids has not been derived. A key reason for this is that even if many more kinds of detecting methods are developed and further improved, it is very difficult to measure the distributions of three-dimensional Reynolds stresses in turbulent rotational flow.

In this book, the fundamental concepts and applications of vortex flows are described in detail. The phenomena of vortex breakdown, the detailed mathematical descriptions of the stability criterion, and also the thickness of the boundary layer are excluded. For readers who are interested in these and other more advanced studies of vortex flows, they may consult the many references cited at the end of each chapter.

Akira Ogawa

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