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Mathematical Logic in Latin America

Edited by

A. I. ARRUDA, R. CHUAQUI, N.C.A. DA COSTA

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MATHEMATICAL LOGIC
IN
LATIN AMERICA

Proceedings of the
IV Latin American Symposium on Mathematical Logic
held in Santiago, December 1978

A. I. ARRUDA
Universidade Estadual de Campinas
Brazil

R. CHUAQUI
Universidad Católica de Chile
Santiago, Chile

N. C. A. DA COSTA
Universidade de Sao Paulo
Brazil

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to

ALFRED TARSKI

teacher and friend

PREFACE

This volume constitutes the Proceedings of the Fourth Latin American Symposium on Mathematical Logic held at the Catholic University of Chile, Santiago from December 18 to December 22, 1978. The meeting was sponsored by the Pontifical Catholic University of Chile, the Academy of Sciences of the Institute of Chile, the Association for Symbolic Logic, and the Division of Logic, Methodology and Philosophy of Science of the International Union of History and Philosophy of Science. The Organizing Committee consisted of Ayda I. Arruda, Rolando B. Chuaqui (chairman), Newton C.A. da Costa, Irene Mikenberg, and Angela Bau (Executive Secretary).

Most of the sponsors were represented at the opening session. The Catholic University was represented by its Rector Jorge Swett, its Vice-Rector for Academic Affairs Fernando Martínez, and its Dean of Exact Sciences Rafael Barriga who gave an address. The President of the Chilean Academy of Sciences, Jorge Mardones, also said a few words. Representing the Association for Symbolic Logic, Newton da Costa, Chairman of its Latin American Committee, opened the meeting.

In preparation for the Symposium there was a logic year at the Catholic University. Advanced courses and seminars were given by Ayda I. Arruda (Universidade Estadual de Campinas, Brazil), Jorge E. Bosch (Centro de Altos Estudios en Ciencias Exactas, Buenos Aires, Argentina), Rolando Chuaqui (Universidad Católica de Chile), Newton C.A. da Costa (Universidade de São Paulo, Brazil), and Irene Mikenberg (Universidad Católica de Chile).

Preceding the Symposium, there was a two-week Seminar consisting of short courses. Below are reproduced the Scientific Programs of the Seminar and the Symposium. A look at these programs shows the progress in research in Logic in Latin American in the last few years. (1)

The papers which appear in this volume are the texts, at times considerably expanded and revised, of most of the adresses presented by invitees to the meeting. Also included are two papers by Australian logicians (Bunder and Routley) who could not come because of difficulties in booking space in airlines. Expanded versions of a few short communications are also included.

This volume is dedicated to Professor Alfred Tarski. In his previous visit to Chile and Brazil, he stimulated the development of Logic and encouraged the Organization of the III and IV Symposia. His influence was decisive in getting the sponsorship of the Association for Symbolic Logic. Many of the participants from Latin America and the United States can claim him directly or indirectly as their

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(1) For a history of the previous Latin American Logic Symposia see: A short history of the Latin American Logic Symposia in Non-Classical Logics, Model Theory and Computability, North-Holland Pub. Co. 1977, pp. ix - xvii.

teacher. Although he could not be physically present at the Symposium, he followed the proceedings with great interest.

The Organizing Committee would like to acknowledge the financial support given to the meeting and the publication of these proceedings by the following institutions: the Catholic University of Chile, the Academy of Sciences of the Institute of Chile, the Fundación de Estudios Económicos del Banco Hipotecario de Chile, the Comisión Nacional de Investigaciones Científicas y Tecnológicas, the International Union of History and Philosophy of Science, and the Coca-Cola Export Co.

The editors would like to thank Irene Mikenberg, who was instrumental in the preparing of the camera-ready copy. Most of the typing was done by M. Eliana Cabañas assisted by Rosario Henríquez. The editors wish to express their appreciation.

The editors would also like to thank North-Holland Publishing Co. for the inclusion of this volume in the series Studies in Logic and the Foundations of Mathematics.

The Editors

Instituto de Matemática
Pontificia Universidad Católica de Chile
June 1979.

PROGRAM OF THE SEMINAR

Ayda I. Arruda and Newton C.A. da Costa, (Brazil), *Topics on Paraconsistent and Modal Logic*. (Six lectures).

Jorge Bosch, (Argentina), *Topics in the Philosophy of Science*. (Six lectures).

Luis F. Cabrera, (Chile), *Equivalence Relations and the Continuum Hypothesis*. (Three lectures).

Ulrich Felgner, (West Germany), *The Continuum Hypothesis*. (Two lectures).

Ulrich Felgner, (West Germany), *Applications of the Axiom of Contructibility to Algebra and Topology*. (Ten lectures).

Jerome Malitz, (U.S.A.), *Generalized Quantifiers*. (Four lectures).

PROGRAM OF THE SYMPOSIUM

DECEMBER 18.

9,30 - 12,00	Opening Session.
14,00 - 14,50	N.C.A. da Costa, (Brazil), *A Model Theoretical Approach to Vbtos*.
15,15 - 16,05	R. Chuaqui, (Chile), *Foundations of Statistical Methods Using a Semantical Definition of Probability*.
16,30 - 17,20	J.R. Lucas, (England), *Truth, Probability and Set Theory*.

DECEMBER 19.

9,00 - 9,20	M.G. Schwarze, (Chile), *Axiomatizations for* σ*-Additive Measurement Systems*.
9,20 - 9,40	M.S. de Gallego, (Brazil), *The Lattice Structure of 4-Valued Lukasiewicz Algebras*.
9,40 - 10,00	A. Figallo, (Argentina), *The Determinant System for the Free De Morgan Algebras over a Finite Ordered Set*.
10,00 - 10,20	A.M.Sette, (Brazil), *A Functorial Approach to Interpretability*.
10,30 - 11,00	I. Mikenberg, (Chile), *A Closure for Partial Algebras*.

11,15 - 12,05	U. Felgner, (West Germany), *The Model Theory of FC-Groups, Definability and Undecidability.*

DECEMBER 20.

9,00 - 9,50	E.G.K. López-Escobar, (U.S.A.), *Truth-value Semantics for Intuitionistic Logic.*
10,15 - 10,45	A.I. Arruda, (Brazil), *On Paraconsistent Set Theory.*
11,00 - 11,50	W. Reinhardt, (U.S.A.), *Satisfaction Definition and Axioms of Infinity in a Theory of Properties with Necessity Operator.*
14,00 - 14,50	J. Bosch, (Argentina), *Toward a Concept of Scientific Theory Through Special Relativity.*
15,15 - 16,05	O. Chateaubriand, (Brazil), *An Examination of Gödel's Philosophy of Mathematics.*

DECEMBER 21.

9,00 - 9,20	E.H. Alves, (Brazil), *Some Remarks on the Logic of Vagueness.*
9,20 - 9,40	M. Corrada, (Chile), *A Formalization of the Impredicative Theory of Classes Using Zermelo's Aussonderungsaxiom Without Parameters.*
9,45 - 10,35	R. Vaught, (U.S.A.), *Model Theory and Admissible Sets.*
11,00 - 11,50	M. Benda, (U.S.A.), *On Powerful Axioms of Induction.*
14,20 - 14,40	L.F. Cabrera, (Chile), *Universal Sets for Selfdual Classes of Borel Sets.*
15,00 - 15,30	H.P. Sankappanavar, (Brazil), *A Characterization of Principal Congruences of De Morgan Algebras and its Applications.*
15,45 - 16,30	C.C. Pinter, (U.S.A.), *Topological Duality Theory in Algebraic Logic.*

DECEMBER 22.

9,00 - 9,20	L. Flores, (Chile), *Hempel's Nomological Deductive Model.*
9,20 - 9,40	M. Manson, (Chile), *Deontic, Many-valued and Normative Logics.*
9,45 - 10,45	X. Caicedo, (Colombia), *Back-and-forth Systems for Arbitrary Quantifiers.*
11,00 - 11,50	J. Malitz, (U.S.A.), *Compact Fragments of Higher Order Logic.*

CONTENTS

MATHEMATICAL LOGIC IN LATIN AMERICA
A.I. Arruda, R. Chuaqui, N.C.A. da Costa (eds.)

A SURVEY OF PARACONSISTENT LOGIC (*)

Ayda I. Arruda

ABSTRACT. This paper constitutes a first attempt to sistematize the present state of the development of paraconsistent logic, as well as the main topics and open questions related to it. As we want this paper to have mainly an expository character, we will not in general be rigorous, especially when an intuitive presentation is better for a first understanding of the questions under consideration, as, for example, in Section 1. Section 6 is perhaps the only one where the reader will find some original results. The bibliography, though large, is of course not intended to be complete. A general idea of the content of this paper is given by the Index.

INDEX.

1. INFORMAL INTRODUCTION.

Let $\mathcal{L}$ be a language and $\mathbb{F}$ the set of formulas of $\mathcal{L}$; then any non empty subset of $\mathbb{F}$ is said to be a *propositional system of* $\mathcal{L}$. We say that a propositional

(*) This paper was partially written when the author was Visiting Professor at the Catholic University of Chile, with a partial grant of the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Brazil.

system S is *trivial* (or *over-complete*) if S equals to $\mathbb{F}$; otherwise, S is said to to be *non-trivial* (or *not over-complete*).

Let * be a unary operator defined on the set $\mathbb{F}$ of formulas of the language $\mathcal{L}$ (i. e., if A is a formula of $\mathcal{L}$, then *A is also a formula of $\mathcal{L}$). We say that a propositional system S is **-inconsistent* if, and only if, there is at least a formula A of $\mathbb{F}$ such that both A and *A belong to S; otherwise, S is said to be **-consistent*.

For example, the classical propositional calculus, taking * as negation, is *-consistent and non-trivial; taking * as the possibility symbol $\Diamond$, S5 is $\Diamond$-inconsistent and non-trivial. Of course, in these examples we identify the system under consideration with the set of their theses.

A propositional system S of $\mathcal{L}$ is said to be a *theory* if it has an underlying logic $\boldsymbol{L}$; that is to say, S is closed under the rules of the logic $\boldsymbol{L}$. Since any theory T is also a propositional system, then the properties of *-consistency, triviality, etc., can be applied to T .

In general, we will consider only theories where the * operator denotes negation. Whenever the theory under consideration has more than one negation, and it is perfectly clear which negation we are dealing with, we simplify the terminology talking simply of inconsistent or consistent theories. Nevertheless, when it is convenient to make explicit one of these negations, for example $\neg$, we shall employ the expressions $\neg$-consistency and $\neg$-inconsistency.

A *logic* $\boldsymbol{L}$ is said to be *paraconsistent* if it can be employed as underlying logic of inconsistent but non-trivial theories; a *theory* is said to be *paraconsistent* if its underlying logic is a paraconsistent logic. It is convenient to observe that in a paraconsistent logic the principle of contradiction, $\neg(A \& \neg A)$, as some other connected theses, is not necessarily invalid; nonetheless, from A and $\neg A$ is not, in general, possible to deduce every formula. Clearly, most of extant logics, for instance, classical and intuitionistic ones, are not paraconsistent.

There are two basic categories of paraconsistent logic. Firstly, *weak paraconsistent logics* — logics which can be used not only as underlying logics of paraconsistent theories, but also of consistent ones (for example, the systems C_n, $1 \leqslant n \leqslant \omega$, of da Costa, presented in Section 5). Secondly, *strong paraconsistent logics* — logics which can be used as underlying logics of paraconsistent theories, but not of consistent ones (for example, the system *V2* of Arruda 1977, and the systems **DM** and **DL** of Routley and Meyer 1976).

It is still convenient to insist that if in the underlying logic of a theory T there are two sorts of negation, T may be inconsistent in connection with one negation but consistent relatively to the other. This is precisely what happens with certain paraconsistent theories based on the systems C_n , $1 \leqslant n \leqslant \omega$, in which there is a classical and a non-classical negation.

The basic problem of paraconsistent theories is that of building reasonable systems of paraconsistent logic, and of studying their properties. As we shall see later, there exist strong paraconsistent logics, and very important paraconsistent theories.

2. PARADOXES, ANTINOMIES, AND HEGEL'S THESIS.

In what follows we shall not try to give an analysis of the meanings of the words *paradox* and *antinomy*. Our aim is simply to give to these terms precise meanings which are important for the development of our paper.

A *formal paradox* in a theory T is the derivation in T (or a metalogical proof that such derivation is possible) of two theorems of the forms A and ¬A, where ¬ is the symbol of negation. Since we are identifying theories with sets of formulas, a theorem in a theory T is simply an element of T .

A *formal antinomy* in a theory T is a metalogical proof that T is trivial.

Of course, if T is a theory based on a system of paraconsistent logic, T may be such that it is possible to derive a formal paradox in T, though T may not be antinomical (that is, we can not prove a formal antinomy in T).

Trivial theories are without logical and metalogical importance. Consequently, any antinomical theory, since it is trivial, lacks logical and metalogical interest. On the contrary, paradoxical theories which are not trivial (or not antinomical) are relevant and deserve to be studied.

As we shall see, there are important paraconsistent theories which are paradoxical and apparently non-trivial. For example, there are strong set theories, even stronger than the usual ones, which are paradoxical but apparently non-trivial (see Section 6).

If & denotes conjunction, a formula of the form A & ¬A is called a contradiction. In several systems of paraconsistent logic, if A and ¬A are both theorems, then A & ¬A is also a theorem. So the same occurs in a theory T having one of these systems as underlying logic. Then we may say that a theory is paradoxical if it contains at least one theorem of the form A & ¬A. Whether a paradox in a theory T is also an antinomy, depends on the underlying logic of T. For instance, in classical logic, a paradox is also an antinomy, and conversely. Indeed, it cannot be true in connection with most systems of paraconsistent logic.

We have just fixed the meaning of formal paradox, and formal antinomy, but for the aims of this paper we need also to define what we understand by an *informal paradox* , and by an *informal antinomy*, or simply, a *paradox* and an *antinomy*.

Following Quine 1966, we can define paradox as follows: "A *paradox* is any conclusion that at first sounds absurd but that has an argument to sustain it ." The paradoxes may be classified as *veridical* and *falsidical*.

A *veridical paradox* is a conclusion that sounds absurd but is in fact true. For example, the conclusion that a person is 4n years old at his n th birthday ; this is surpring but true if we talk of a person who was born at 29 th February. The 'barber paradox' is also veridical if we take it as meaning that no village contains a barber who shaves only those persons who do not shave themselves.

A *falsidical paradox* is a paradox where there is a fallacy in the argument. As an example, we consider the paradox about the division by zero, in the version of A. de Morgan (mentioned in Quine 1966): Let us suppose that $x = 1$; then, $x^2 = x$. So $x^2 - 1 = x - 1$. Dividing both sides by $x - 1$, we obtain that $x + 1 = 1$, that is, since $x = 1$, that $2 = 1$.

The classification of the paradoxes as veridical and falsidical is not sufficient to cover all the known paradoxes; we need also to consider antinomies.

Still following Quine 1966, an *antinomy* is an argument that produces a self-contradiction by accepted ways of reasoming. As examples of antinomies we can mention those of Grelling, Epimenides, and Russell.

In general, to solve a paradox consists in showing that the paradoxical conclusion is true or that the argument which sustain it is a fallacy. The first solution correspond to veridical paradoxes, and the second, to falsidical. However, we cannot solve an antinomy except by rejecting some well accepted principles.

It is worth-while to observe that most formal paradoxes and antinomies may be considered in the metalogical level as informal antinomies. Thus, the formal antinomy of Russell, in Frege's system, may be considered as an antinomy at the metalinguistic level: Frege's system, apparently codifying valid logical postulates, and valid rules of inference, is nonetheless contradictory.

Of course, one may wonder what is the relation between paraconsistent logic, and paradoxes and antinomies. Since most of the antinomies are essentially contradictions,perhaps some of them are really true from the point of view of paraconsistent logic, and we do not have to surmount them. This is the question we shall consider next.

Hegel's thesis (or *Heraclitus-Hegel's thesis*) is the statement that there are true contradictions (cf. Petrov 1974). Sometimes, Hegel's thesis is also formulated as to imply that consistency is a sufficient but not necessary condition for the existence of abstract objects; concerning the existence of concrete objects, consistency is neither necessary nor sufficient. Clearly, Hegel's thesis can only be supported with the help of a paraconsistent logic.

We shall show that at the abstract level Hegel's thesis is in fact true: there are paraconsistent theories (like the set theories described in Section 6) in which certain objects have inconsistent properties; for example, they belong and simultaneously do not belong to the same class. Therefore, one of the main achievements of paraconsistent logic is to have proved that Hegel's thesis is true at the formal and abstract level. This means that an antinomy from the point of view of classical logic may be — surprising enough — a veridical paradox from the standpoint of paraconsistent logic. (Such result seems to signify that in the field of logic there is a certain kind of relativism, an issue which we will not discuss here.)

Now, what can we say about the validity of Hegel's thesis in connection with concrete objects? It seems to us that paraconsistent logic is unable to settle such problem. Only special sciences and epistemology can establish the truth or falsity of Hegel's thesis at the level of real, concrete objects; or, equivalently, if the real world is consistent or not.

Anyway, we believe that Petrov is right in his interpretation of existing antinomies: they do not prove that Hegel's thesis is true for concrete objects, but at least they give us some hints on the plausibility of it. According to Petrov 1971, p. 388:

"No elimination of fallacy in scientific knowledge has negative consequences for the adequacy or the completeness of knowledge.

"Certain ... antinomies (as the classical ones originated in quantum physics by the wave and corpuscular aspects of elementary particles) , however, can in principle be eliminated only with the aid of theories and methods the acceptance of which encroaches too much upon the adequacy or the completeness of knowledge.

"The conclusion is therefore plausible that certain ... antinomies are not fallacies as standard logic wants us to believe, but are peculiar objective truths."

Therefore, we can conclude that the fundamental significance of paraconsistent logic in connection with the common antinomies is that we are now able to accept most of them as veridical paradoxes, at least at the abstract level. Therefore, from now on we should not try to exclude antinomies *a priori* , because contradictions are forbidden by logic. Only *a posteriori* elimination of antinomies is legitimate depending on logical, scientific, and epistemological reasons.

Finally, it is worthwhile to remark that, among contemporary philosophers, Wittgenstein maintained original views about contradiction and logic: "Indeed, even at this stage, I predict a time when there will be mathematical investigations of calculi containing contradictions, and people will actually be proud of having emancipated themselves from contradiction." (Wittgenstein 1964, p. 332.) "If a contradiction were now acctually found in arithmetic, that would only prove that arithmetic with *such* a contradiction in it could render a very good service."