Study Guide to Accompany

INTRODUCTORY ALGEBRA

Michael A. Gallo Charles F. Kiehl

Study Guide to Accompany Introductory Algebra

Michael A. Gallo Monroe Community College

Charles F. Kiehl
State University of New York, Brockport

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PREFACE

This Study Guide has been developed to assist you in your learning of the material presented in our textbook, <u>Introductory Algebra</u>. The Study Guide has been designed to be used as a supplement to the parent text, not a replacement for it. The chapters of the Study Guide conform, section by section, to those of the text.

Each section of the Study Guide is organized in a similar manner to that of the text and includes the following components:

- * A brief review of the corresponding material from the text, including any procedure stated for a given section.
- * Solved examples that consist of selected odd-numbered exercises from the text.
- * Supplementary exercises with answers.

Moreover, each chapter concludes with a chapter test with answers.

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CHAPTER 1 THE STRUCTURE OF ARITHMETIC

1-1 THE NUMBER LINE AND THE NUMBERS OF ARITHMETIC

The numbers of arithmetic consist of the counting numbers, whole numbers, decimal numbers, fractions, and mixed numbers. A number line (a line whose points are named by numbers), can be used to picture the numbers of arithmetic. For example:

* The counting numbers, or natural numbers, consist of the numbers, 1, 2, 3, 4, ..., and are pictured on a number line as follows.



* The whole numbers consist of the counting numbers and zero.



* Decimal numbers and fractional numbers represent parts of a whole, and mixed numbers represent fractional parts greater than one.



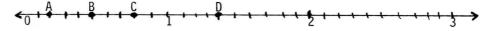
When dealing with fractions, we must be sensitive to the different types of fractions. For example:

- * Fractions such as 1/5, 2/3, 7/10, and 99/100 are called proper fractions since their numerators are less than their denominators.
- * Fractions such as 3/3, 5/2, 17/6, and 100/50 are called improper fractions since their numerators are equal to or greater than their denominators.
- * Fractions such as 1/6, 2/6, 4/6, and 11/6 are called like fractions since they all contain the same denominator.
- * Fractions such as 1/3, 2/5, 18/7, and 12/2 are called unlike fractions since they all contain different denominators.

Whenever a number represents a point on a number line, the number is referred to as the coordinate of the point.

EXAMPLE 1 (#3, PAGE 7)

Given the following number line, find the coordinates of points A, B, C, D.



SOLUTION

Since the interval between 0 and 1 is divided into ten sub-intervals, the coordinate of point A is 2/10, B is 5/10, and C is 8/10. In a similar manner the interval between 1 and 2 is divided into ten sub-intervals. Thus, the coordinate of point D is 1+4/10, or 14/10.

Final Answer: A = 2/10, B = 5/10, C = 8/10, D = 1 4/10

To graph a number on a number line means to locate the point that can be represented by that number.

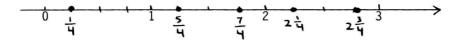
EXAMPLE 2 (# 7, PAGE 7)

Graph the following numbers on a number line.

1/4, 5/4, 7/4, 8/4, 2 1/4, 2 3/4

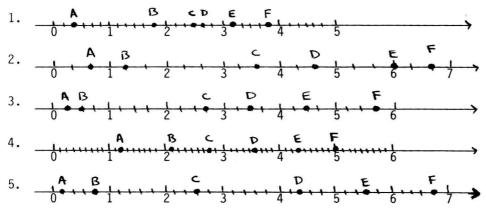
SOLUTION

To graph these numbers we first divide the intervals between the whole numbers into four sub-fractions since the denominators of the fractions are 4. Next, we plot the points on a number line, in their proper location.



SUPPLEMENTARY EXERCISES 1-1

Use the given number lines in 1-5 to find the coordinates of points A, B, C, D, E, and F.



In 6-10, graph the given numbers on a number line.

- 6. 3/8, 1 7/8, 2 2/8, 24/8, 27/8, 4 1/8
- 7. 5/7, 9/7, 2 6/7, 22/7, 4 3/7, 35/7
- 8. 1/2, 3/2, 2 1/2, 6/2, 7/2, 5 1/2
- 9. 1 3/5, 2 1/5, 14/5, 3 2/5, 4 4/5, 27/5
- 10. 3/6, 6/6, 1 5/6, 3 1/6, 26/6, 33/6

ANSWERS

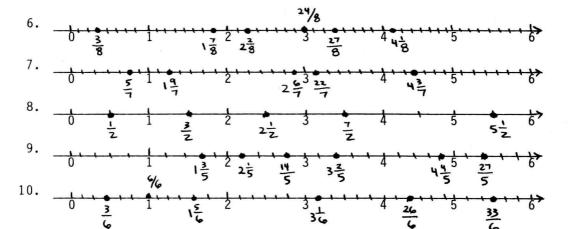
1.
$$A = 2/6$$
, $B = 1.5/6$, $C = 2.3/6$, $D = 2.4/6$, $E = 3.1/6$, $F = 3.5/6$

2.
$$A = 2/3$$
, $B = 1 1/3$, $C = 3 2/3$, $D = 4 2/3$, $E = 6$, $F = 6 2/3$

3.
$$A = 1/4$$
, $B = 2/4$, $C = 2 3/4$, $D = 3 2/4$, $E = 4 2/4$, $F = 5 3/4$

4.
$$A = 1 \frac{2}{9}$$
, $B = 2 \frac{1}{9}$, $C = 2 \frac{7}{9}$, $D = 3 \frac{5}{9}$, $E = 4 \frac{3}{9}$, $F = 5$

5.
$$A = 1/5$$
, $B = 4/5$, $C = 2 3/5$, $D = 4 2/5$, $E = 5 3/5$, $F = 6 4/5$



1-2 THE RATIONAL NUMBERS

The numbers of arithmetic can all be expressed as fractions. For example:

* Natural numbers and whole numbers can be written in fractional form.

$$0 = \frac{0}{1}$$
, $7 = \frac{7}{1}$, $23 = \frac{23}{1}$

* Decimal numbers can be written in fractional form.

$$0.6 = \frac{6}{10}$$
, $0.85 = \frac{85}{100}$, $3.9 = 3\frac{9}{10}$ or $\frac{39}{10}$

* Clearly, fractions can be written in fractional form.

* Mixed numbers can be written in fractional form.

$$1\frac{7}{8} = \frac{15}{8}$$
, $2\frac{9}{10} = \frac{29}{10}$, $5\frac{7}{8} = \frac{47}{8}$

In arithmetic, any number that can be written in fractional form, a/b, where "a" is a whole number and "b" is a natural number is called a rational number. collectively, the numbers of arithmetic are rational numbers.

Since the numbers of arithmetic are rational numbers we can convert them to equivalent forms. That is, we can convert fractions to equivalent fractions, decimals to fractions, fractions to decimals, mixed numbers to (improper) fractions, and improper fractions to mixed numbers.

EXAMPLE 1 (#1, PAGE 17)

Convert (or reduce) 3/9 to an equivalent fraction that is expressed in lowest terms.

EXAMPLE 2 (#5, PAGE 17)

Reduce 18/30 to lowest terms.

EXAMPLE 3 (#13, PAGE 17)

Convert 0.75 to an equivalent fraction and reduce.

$$\frac{3}{9} = \frac{(3)(1)}{(3)(3)} = \frac{1}{3}$$

(Cancel the common $\frac{3}{9} = \frac{\cancel{(2)}(1)}{\cancel{(2)}(3)} = \frac{1}{3}$ factor 3 from both the numerator and denominator.)

Final Answer: $\frac{1}{2}$

SOLUTION

$$\frac{18}{30} = \frac{9}{15} = \frac{3}{5}$$

Cancel 2 from both numerator and denominator.

Cancel 3 from both numerator and denominator.

Final Answer: 3/5

SOLUTION

$$0.75 = \frac{75}{100}$$
 (The denominator is 100 since 0.75 contains two decimal places.)

$$\frac{75}{100} = \frac{3}{4}$$
 (Cancel 25 from both the numerator and denominator.)

Final Answer: 3/4

EXAMPLE 4 (#17, PAGE 17)

Convert 5.12 to an equivalent fraction and reduce.

SOLUTION

$$5.12 = 5\frac{12}{100}$$
 (Convert to a mixed number.)

$$5\frac{12}{100} = \frac{512}{100}$$
 (Convert to an improper fraction.)

$$\frac{512}{100} = \frac{128}{25}$$
 (Cancel 4 from both the numerator and denominator.)

Final Answer: 128/25

EXAMPLE 5 (#23, PAGE 17)

Convert 7/20 to an equivalent decimal number.

SOLUTION

$$\frac{7}{20} = \frac{20/7.00}{60}$$
 (Divide the denominator into the numerator.)
$$\frac{100}{0}$$

Final Answer: 0.35

EXAMPLE 6 (#33, PAGE 18)

Convert 11 6/7 to an equivalent fraction.

SOLUTION

$$11\frac{6}{7} \Rightarrow 11 \times 7 = 77$$
 (Multiply the denominator by the whole number.)
 $11\frac{6}{7} \Rightarrow 77 + 6 = 83$ (Add the numerator to the product.)
 $11\frac{6}{7} = \frac{83}{7}$ (Place the sum over the denominator.)

Final Answer: 83/7

EXAMPLE 7 (#41, PAGE 18)

Convert 11/5 to an equivalent mixed number.

SOLUTION

$$\frac{\frac{11}{5} \Rightarrow 5/\overline{11}}{\frac{10}{1}} \text{ (Divide the denominator into the numerator.)} \qquad \begin{array}{c} 2 \leftarrow \text{quotient} \\ \frac{2}{10} + \frac{10}{1} \\ \frac{10}{1} + \frac{10}{1} \\ \frac{$$

(The quotient is the whole number part, the remainder is placed over the divisor for the fractional part.)

 $\frac{11}{5} = 2\frac{1}{5}$

Final Answer: 2 1/5

The numbers of arithmetic can also be compared using the inequality symbols, < (less than), > (greater than), and ≠ (not equal).

EXAMPLE 8 (#57, PAGE 18)

Compare 3/8 and 1 3/4.

SOLUTION

To compare fractions express the fractions as like fractions and then compare their numerators.

 $\frac{3}{8} = \frac{3}{8}$ and $1\frac{3}{4} = \frac{7}{4} = \frac{14}{8}$ $\frac{3}{8} < \frac{14}{8}$ (since 3 < 14)

Final answer: 3/8 < 1 3/4

EXAMPLE 9 (#59, PAGE 18)

Compare 0.9 and 0.256.

SOLUTION

To compare decimals express the decimals as similar decimals and then compare the numbers, ignoring their decimal points.

$$0.9 = 0.900$$
 and $0.256 = 0.256$
 $0.900 > 0.256$ (since $900 > 256$)

Final Answer: 0.9 > 0.256

SUPPLEMENTARY EXERCISES 1-2

In 1-5, reduce to lowest terms.

1. 4/12 2. 21/35 3. 30/75 4. 28/70 5. 90/135

In 6-10, convert to an equivalent fraction and reduce.

6. 0.68 7. 2.03 8. 4.25 0.94 10. 1.17

In 11-15, convert to an equivalent decimal number. Round to the nearest hundredth if the deimal does not terminate.

11. 3/4 12. 5/1613. 2/3 14, 1/5 15. 6/7

In 16-20, convert to an equivalent improper fraction.

16. 4 2/9 17. 1 3/10 18. 13 1/3 19. 5 7/8 20. 3 2/5 In 21-25, convert to an equivalent mixed number.

.

14/3

21.

22. 22/7

23. 31/4

24. 25/6

25. 36/11

In 26-30, compare the given fractions.

26. 4/7, 5/9

27. 21/4, 5 1/3

28. 13/19, 5/8

29. 2 2/5, 13/6

30. 3/11, 4/13

In 31-35, compare the given decimals.

31. 0.83, 0.799

32. 0.1, 0.004

33. 0.65, 0.701

34. **0.521**, **0.805**

35. 0.08, 0.2

ANSWERS

1. 1/3

2. 3/5

3. 2/5

4. 2/5

5. 2/3

6. 17/25

7. 2 3/100

8. 4 1/4

9. 47/50

10. 1 17/100

11. 0.75

12. 0.31

13. 0.67

14. 0.2

15. 0.86

16. 38/9

17. 13/10

18. 40/3

19. 47/8

20. 17/5

21. 4 2/3

22. 3 1/7

23. 7 3/4

24. 4 1/6

25. 3 3/11

26. 4/7 > 5/9

27. 21/4 < 5 1/3

28. 13/19 > 5/8

29. 2 2/5 > 13/6

30. 3/11 < 4/13

31. 0.83 > 0.799

32. 0.1 > 0.004

33. 0.65 < 0.701

34. 0.521 < 0.805 35. 0.08 < 0.2

1-3 OPERATIONS ON RATIONAL NUMBERS

ADDITION AND SUBTRACTION

Addition and subtraction of fractions can only be performed with like fractions. Similarily, addition and subtraction of decimals can only be performed with similar decimals.

EXAMPLE 1 (#1, PAGE 27)

Add 3/5 and 5/7.

SOLUTION

$$\frac{3}{5} + \frac{5}{7}$$
= $\frac{21}{35} + \frac{25}{35}$ (Convert to like fractions.)
= $\frac{46}{35}$ (Add numerators and place sum over common denominator.)

Final Answer: 46/35

EXAMPLE 2 (#17, PAGE 27)

Subtract 5 2/3 - 2 3/4.

SOLUTION

$$5\frac{2}{3} - 2\frac{3}{4}$$

= $5\frac{8}{12} - 2\frac{9}{12}$ (Convert to like fractions.)
= $4\frac{20}{12} - 2\frac{9}{12}$ (Express 5 8/12 as 4 20/12.)
= $2\frac{11}{12}$ (Subtract.)

Final Answer: 2 11/12

EXAMPLE 3 (#25, PAGE 27)

Add 3.26 + 9.6 + 34 + 3.0938.

SOLUTION

$$\begin{array}{c}
3.2600 \\
9.6000 \\
34.0000 \\
+ 3.0938 \\
\hline
49.9538
\end{array}$$
(Make decimals similar and line up the decimal points.)

Final Answer: 49.9538

EXAMPLE 4 (#27, PAGE 27)

Subtract 6.2 - 3.9.

SOLUTION

$$\begin{array}{c}
6.2 \\
-3.9 \\
-3.9
\end{array}
\Rightarrow
\begin{array}{c}
5.12 \\
\cancel{8.2} \\
-3.9 \\
2.3
\end{array}$$

Final Answer: 2.3

MULTIPLICATION AND DIVISION

Multiplication of fractions is accomplished by multiplying the numerators together and multiplying the denominators together. The product of the numerators placed over the product of the denominators represents the product of the fraction. Division of fractions is accomplished by multiplying the first fraction by the reciprocal of the second fraction.

EXAMPLE 5 (#9, PAGE 27)

Multiply 3/8 by 2/15.

SOLUTION

Method 1
 Method 2

$$\frac{3}{8} \cdot \frac{2}{15}$$
 $\frac{3}{8} \cdot \frac{2}{15}$

 = $\frac{(3)(2)}{(8)(15)}$
 = $\frac{(3)(2)}{(8)(15)}$
 (Cancel common factors.)

 = $\frac{6}{120}$
 (Multiply.)
 = $\frac{1}{20}$
 (Multiply.)

Final Answer: 1/20

EXAMPLE 6 (#23, PAGE 27)

Divide 1 1/2 by 1 2/3.

SOLUTION

$$1\frac{1}{2} \div 1\frac{2}{3}$$

$$= \frac{3}{2} \div \frac{5}{3}$$
 (Convert to fractions.)
$$= \frac{3}{2} \cdot \frac{3}{5}$$
 (Change to multiplication.)
$$= \frac{9}{10}$$
 (Multiply.)

Final Answer: 9/10

To multiply decimals we multiply as we would whole numbers and then include in the answer as many decimal places as there are contained in the numbers being multiplied.

EXAMPLE 7 (#29, PAGE 27) Multiply 0.0006 by 66.5. 0.0006 x 66.5 0036 0036 0.03990 (5 decimal places)

Final Answer: 0.03990

To divide decimals we first convert the divisor to a whole number by moving the decimal point all the way over to the right, and then move the decimal point of the dividend an equal number of places to the right. Once this is done, we then divide as we would whole numbers.

EXAMPLE 8 (#32, PAGE 27)

SOLUTION

Divide 27.2 by 0.32.

0.32/
$$\overline{27.2}$$

= 32/ $\overline{2720.}$ (Move decimal points in both divisor and dividend.)

= $85.$
= $32/\overline{2720.}$ (Divide.)
$$\frac{256}{160}$$

$$\frac{160}{0}$$

Final Answer: 85

SUPLEMENTARY EXERCISES 1-3

In 1-5, add the given fractions. Reduce.

In 1-5, and the given fractions. Reduce

1. 2/3 + 1/5 2. 3/8 + 7/6 3. 5/9 + 2/3 4. 4/7 + 7/4 5. 1/5 + 3/8 In 6-10, subtract the given fractions. Reduce.

6. 3/4 - 1/3 7. 5/7 - 2/3 8. 7/8 - 3/4 9. 5/6 - 4/9 10. 6/14 - 3/7 In 11-15, add the given mixed numbers. Reduce.

11. $2\frac{1}{3} + 3\frac{5}{8}$ 12. $1\frac{7}{8} + 3\frac{2}{6}$ 13. $5\frac{3}{4} + 4\frac{2}{3}$ 14. $7\frac{1}{6} + 3\frac{5}{12}$ 15. 12 + $8\frac{2}{3}$

In 16-20, subtract the given mixed numbers. Reduce.

16. $3\frac{2}{3} - 1\frac{1}{6}$ 17. $5\frac{4}{5} - 3\frac{2}{3}$ 18. $6\frac{1}{8} - 3\frac{3}{4}$ 19. $5\frac{7}{9} - 1\frac{2}{8}$ 20. $4 - 1\frac{3}{5}$ In 21-25, add the given decimals.

31. 2/3 x 4/7 32. 5/6 • 3/4 33. 7/8 • 6/9 34. 12/18 • 7/9 35. 11/28 • 7/22 In 36-40, divide the given fractions. Reduce.

36. 4/5 ÷ 3/5 37. 9/12 ÷ 2/3 38. 7/12 ÷ 14/6 39. 7/9 ÷ 2/8 40. 1/3 ÷ 9/15

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