



BIOMATHEMATICS

modelling and simulation

editor J C Misra

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Indian Institute of Technology, Kharagpur

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BIOMATHEMATICS

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Dedicated to my parents

Late Ganes Chandra Misra and

Mrs. Pramila Misra

*who served throughout their lives for the
cause of education with a missionary zeal*

THE EDITOR



Dr. Jagadis Chandra Misra, Senior Professor at the Department of Mathematics, Indian Institute of Technology Kharagpur, received his Ph.D. degree in Applied Mathematics from Jadavpur University in 1971 and the highly coveted D.Sc. degree in Applied Mathematics from the University of Calcutta in 1984. For over 35 years he has been engaged in teaching and research at several distinguished institutions in India, Germany and North America. He has been the Chairman of the Department of Mathematics, IIT Kharagpur during the period

1998–2001. As a recipient of the prestigious *Humboldt Fellowship*, he was at the University of Hannover during the period 1975–1977 and also in 1982, where he carried out his research in the field of Biomechanics in collaboration with Professor Oskar Mahrenholtz at the Institute of Mechanics, University of Hannover, in collaboration with Professor Christoph Hartung from the Biomedical Engineering Division of the Medical University at Hannover. He has held the position of Visiting Professor at the University of Calgary in Canada, at the Indian Institute of Technology, Kanpur in India and at the University of Bremen in Germany. In 1984 he received the prestigious *Silver Jubilee Research Award* from IIT Kharagpur, his research publications having been adjudged to be *outstanding*. In 2004 Prof. Misra bagged the highly prestigious *Raja Ram Mohan Roy Prize* for outstanding contributions in the field of Mathematics and the *Rashtriya Gaurav Award* (National Glory) for outstanding academic contributions. He has been the recipient of the coveted *Indian Science Congress Platinum Jubilee Lecture Award* in Mathematical Sciences in 2005.

Professor Misra was elected a *Fellow* of the National Academy of Sciences in 1987, the Institute of Mathematics and its Applications (UK) in 1988 and the Institute of Theoretical Physics in 1988. He was also elected a Fellow of the Royal Society of Medicine, London in 1989 in recognition of the significant impact of his research work in the field of Biomedical Engineering and a Fellow of the Indian National Academy of Engineering, New Delhi in appreciation of the impact of his researches on Engineering and Technology in 1999. He was elected a Member of the International Society of Biorheology New York, GAMM (Germany), Biomechanical Engineering Society (USA) and an Active Member of the New York Academy of Sciences.

Professor Misra published 12 advanced level Books, a monograph and over 150 research papers in international journals in the areas of Biomathematics, Biomechanics, Mathematical Modelling and Theoretical Solid and Fluid Mechanics. His research results have appeared in highly prestigious journals like *Journal of Mathematical Biology* (USA), *Bulletin of Mathematical Biology* (UK), *Journal of Biomechanics* (UK), *Journal of Biomedical Engineering* (UK), *Blood Vessels* (Switzerland), *Rheologica Acta* (Germany), *Biorheology* (UK), *International Journal of Solids and Structures* (UK), *International Journal of Nonlinear Mechanics* (UK), *Fluid Dynamics Research*, *ZAMP* (Switzerland), *Mathematical and Computer Modelling* (USA), *Journal of Mathematical Analysis and Applications* (USA), *Computers and Mathematics with Applications* (USA), etc. His publications have been well cited in scientific literatures and referred to in several text books. He has made pioneering research on mathematical modelling in each of the areas of Cardiovascular Mechanics, Mechanics of Brain Injury, Mechanics of Fracture and Remodelling of Bones and Peristaltic Flows in Physiological Systems. His theoretical findings on the compressibility of vascular tissues is a major breakthrough in the study of arterial biomechanics and were subsequently verified experimentally by Prof. Y. C. Fung at the Bioengineering Laboratory of the University of California, San Diego. The model developed by him for the study of arterial stenosis bears the potential to provide an estimate of the variation of blood viscosity as the height of the stenosis increases. The observations of the study were used by later investigators in the quantification of Doppler colour flow images from a stenosed carotid artery. Misra's theoretical study on the mechanics of cerebral concussion caused due to rotational acceleration of the head has opened up new vistas in neurological research and neurosurgery. On the basis of the study he could make some valuable predictions regarding the threshold of cerebral concussion for humans, in

terms of angular acceleration. He was the first to account for the effect of material damping of osseous tissues on bone remodelling induced due to the surgical procedure of intra-medullary nailing. Misra's study on the effect of a magnetic field on the flow of a second-grade electrically conducting fluid serves as a very important step towards the perception of MHD flow of blood in atherosclerotic vessels. It throws sufficient light on the quantitative features of blood flow in constricted arteries.

Professor Misra has been a Member of Expert Committees of the National Science Foundation of USA, Imperial Press of UK and World scientific of Singapore. He has also been a Member of the expert committees of the Council of Scientific and Industrial Research, New Delhi, of the Indira Gandhi National Open University as well as a member of the Technical Advisory Committee of the Indian Statistical Institute, Calcutta.

Professor Misra is an Associate Editor of the *International Journal of Innovative Computing, Information and Control* (Japan) and a Member of the Editorial Board of the *International Journal of Scientific Computing*.

On various occasions he delivered invited lectures at the Courant Institute of Mathematical Sciences, New York and at the Cornell Medical Center, New York, Universities of California at Los Angeles and San Diego in USA, Imperial College London, Universities of Cambridge, Oxford, Manchester and Glasgow in UK, University of Hamburg-Harburg and University of Kassel in Germany, University of Paris and Ecole Polytechnique in France, Graz University in Austria, Delft University in the Netherlands, Universities of Tokyo, Osaka and Kobe in Japan, National University of Singapore and Hong Kong University of Science and Technology. He delivered invited plenary lectures and chaired plenary sessions at the Fourth International Congress of Biorheology held in Tokyo and International Conferences on Computational and Applied Mathematics held in Belgium in 2002, Applied Mathematics and Mathematical Physics held at Sylhet in 2003, and Information Technology held in Kathmandu in 2003. Professor Misra delivered the prestigious *Bhatnagar Memorial Lecture* in 2001 and the *Indian Science Congress Platinum Jubilee Lecture* in Mathematical Sciences in 2005. In 2006, Prof. Misra was elected the President of Mathematical Sciences Section (including Statistics) of the Indian Science Congress.

Professor Misra built up an active school of research at IIT Kharagpur and guided 25 research scholars towards their Ph.D. degrees.

PREFACE

This really is the golden age of Mathematics. It has been said that half the Mathematics ever created has been in the last 100 years and that half the mathematicians who have ever lived are alive today. We have seen such achievements as the resolution of the four-colour problem and Fermat's last theorem, with the latter being a special manifestation of a much more general result!

This book consists of chapters that deal with important topics in Biomathematics. A glance through any modern textbook or journal in the fields of ecology, genetics, physiology or biochemistry reveals that there has been an increasing use of mathematics, which ranges from the solution of complicated differential equation in population studies to the use of transfer functions in the analysis of eye-tracking mechanisms. This volume deals with Applied Mathematics in Biology and Medicine and is concerned with applied mathematical models and computer simulation in the areas of Molecular and Cellular Biology, Biological Soft Tissues and Structures as well as Bioengineering.

In this volume an attempt has been made to cover biological background and mathematical techniques whenever required. The aim has been to formulate various mathematical models on a fairly general platform, making the biological assumptions quite explicit and to perform the analysis in relatively rigorous terms. I hope, the choice and treatment of the problems will enable the readers to understand and evaluate detailed analyses of specific models and applications in the literature.

The purpose of bringing out this volume on Biomathematics dealing with interdisciplinary topics has been twofold. The objectives are to promote research in applied mathematical problems of the life sciences and to enhance cooperation and exchanges between mathematical scientists, biologists and medical researchers. This volume has both a synthetic and

analytic effect. The different chapters of the volume have been mostly concerned with model building and verification in different areas of biology and the medical sciences.

I believe people in the entire spectrum of those with interest in ecology, from field biologists seeking a conceptual framework for their observations to mathematicians seeking fruitful areas of application, will find stimulation here. It may so happen that some readers may find some parts of this volume trivial and some of the parts incomprehensible. Keeping this in view the extensive bibliographies given at the end of each chapter do attempt to provide an entry to the corresponding areas of study.

For over 35 years I have been engaged in teaching and research at several well-known institutions of India, Germany and North America. Publication of the series of books has been the fruit of a long period of collaboration together with relentless perseverance. My labour will be deemed amply rewarded if at least some of those for whom the book is meant derive benefit from it.

I feel highly indebted to the contributors of this volume who have so kindly accepted my invitation to contribute chapters. The enormous pleasure and enthusiasm with which they have accepted my invitation have touched me deeply, boosting my interest in the publication of the book.

I constantly remember the extent of care my parents have taken to impart proper education to me. I am highly indebted to Srimat Swami Shankaranandaji Maharaj, seventh President of the Ramakrishna Math and the Ramakrishna Mission, Belur Math, Swami Tejasanandaji and Swami Gokulanandaji, the then Principal and Vice-Principal of the Ramakrishna Mission Vidyamandira, Belur Math and to the monastic members of the Ramakrishna Mission Calcutta Students' Home, Belgharia for their kind guidance and suggestions and for instilling in me, while I was still a college and university student, a deep sense of total involvement in pursuing academic goals and a strong commitment to human values.

It is a pleasure to acknowledge the moral support, help and encouragement that I have been receiving constantly in all my academic activities from my wife Shorasi and my children Subhas, Sumita and Sudip.

I.I.T. Kharagpur

J. C. Misra

January, 2005

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