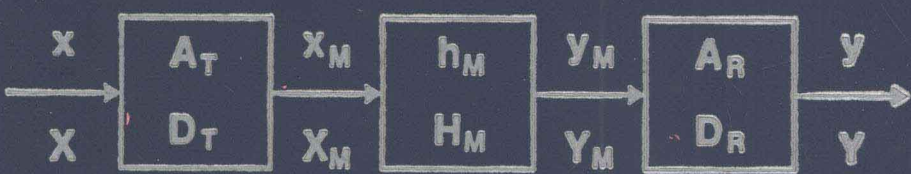
UNDERWATER ACOUSTICS

A Linear Systems Theory Approach



Lawrence J. Ziomek

Underwater Acoustics

A Linear Systems Theory Approach

LAWRENCE J. ZIOMEK

Department of Electrical and Computer Engineering
Naval Postgraduate School
Monterey, California

1985

ACADEMIC PRESS, INC.

(Harcourt Brace Jovanovich, Publishers)

Orlando San Diego New York London
Toronto Montreal Sydney Tokyo

ACADEMIC PRESS, INC.
Orlando, Florida 32887

United Kingdom Edition published by
ACADEMIC PRESS INC. (LONDON) LTD.
24–28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

Ziomek, Lawrence J.
Underwater acoustics.

Includes index.
1. Underwater acoustics. 2. Signal processing.
3. System analysis. I. Title.
QC242.2.Z56 1985 534'.24 84-18606
ISBN 0-12-781720-4 (alk. paper)

PRINTED IN THE UNITED STATES OF AMERICA

85 86 87 88 9 8 7 6 5 4 3 2 1

UNDERWATER ACOUSTICS
A Linear Systems Theory Approach

To my parents
CASIMIR and LOTTIE,
my wife
VIRGINIA,
and my daughters
NICOLE and KIRSTEN

Preface

This book is based, in part, upon lecture notes that I wrote for courses in Sonar Systems Engineering (in the Department of Electrical and Computer Engineering) and Radiation and Scattering of Waves in Fluids (in the Department of Physics) at the Naval Postgraduate School. I took an interdisciplinary approach in writing this book, involving the fields of electrical engineering (linear systems theory, statistical communication theory, and digital signal processing) and physics (underwater acoustics). However, the main approach was that of an electrical engineer with acoustic theory provided as needed.

The philosophy that I followed was to treat the ocean medium as a linear, random filter. This idea is by no means new; it has been part of the research literature since the mid-1960s. However, this book represents an attempt to write in the style of a textbook to make the material more easily approachable.

This book is rigorous primarily in the sense that its emphasis is on novel, general derivations of results from first principles whenever possible and practical, using a consistent and mainly standard notation from the first chapter to the last. However, it is not devoted to theory for theory's sake, since by the use of simplifying assumptions and examples, numerous practical results are obtained. It is hoped that by providing novel derivations whenever possible, a fresh point of view is expressed and duplication avoided.

The field of underwater acoustics is so broad and interdisciplinary that it is difficult to write a single book that treats in sufficient depth and clarity all the various topics, methods, and theories considered as part of the field. The danger is that one produces either an encyclopedic handbook, which treats many topics with elementary discussions and developments, or a research monograph, which reads like a several-hundred page journal article. While each has its merits and fills a definite need, it is my opinion that neither is totally suitable for use as a classroom textbook. It was therefore my philosophy to concentrate on developing in sufficient theoretical depth and clarity basic, fundamental results that are useful in a variety of underwater acoustic applications so that the overall level of the book lies between the two extremes of a handbook and a research monograph. The topics covered in this book indicate that it is just as important for the signal processor to understand the fundamentals of wave propagation as it is for the underwater acoustician to understand the fundamentals of array theory and signal processing. The vehicle I chose to bring together the areas of wave propagation and array theory and signal processing is *linear systems theory*. This book does not discuss the sonar equations or waveguides, not because I felt these topics were unimportant, but because there are already several books available that contain good discussions of these subjects.

This book could well be used by first-year graduate students and advanced college seniors in the fields of electrical engineering, ocean engineering, acoustics, and oceanography for a treatment of sonar systems engineering. It is quite suitable for self-study.

Chapter 1 provides a general overview of the book and background discussion concerning the treatment of the ocean medium as an underwater acoustic communication channel. Chapters 3 and 4 cover the fundamentals of complex apertures and arrays, respectively, while Chapter 5 covers basic topics in sonar signal processing. Complex aperture theory is discussed before array theory, analogous to the general practice of discussing the theory of continuous-time signals before discussing discrete-time signals. Once the general principles of aperture theory are developed, the array theory results can be derived quickly and easily. Chapters 3–5 contain many examples and problems with answers provided at the end of the book. Several of the problems at the end of Chapter 4 are well suited for computer projects, and I have used them for such purposes. Recommended prerequisites for this course include introductory-level courses in communication theory, probability, and random processes. An introductory-level course in acoustics would be desirable but is not absolutely necessary.

Another type of course that could be taught from this book is an advanced special topics course on mathematical models of the ocean medium. Chapters 1, 2 (Fundamentals of Linear, Time-Variant, Space-Variant Filters), 6 (Wave Propagation in Inhomogeneous Media), and 7 (Random Ocean Medium Transfer Functions) could form the foundation for such a course and could be supplemented by additional material of the instructor's choice. Chapters 2, 6, and 7 do contain examples and are written in a tutorial fashion suitable for classroom use. Chapter 7 is the essence of the book for it is here that most of the ideas contained in Chapters 1 through 6 are brought together.

I would like to thank my colleagues Profs. Donald E. Kirk and Robert D. Strum of the Department of Electrical and Computer Engineering at the Naval Postgraduate School for their early encouragement and support. My students also deserve special credit for their many thought-provoking questions and comments, which caused me to make many revisions for the better. I would also like to acknowledge the Naval Postgraduate School Foundation Research Program and the Defense Advanced Research Projects Agency (DARPA). Their support of my research efforts is responsible for the material appearing in Chapters 2 and 7 of this book. And finally, I would like to acknowledge Lily T. Nimri and Elaine R. Christian for their typing assistance and Alvin W. Lau who drew all of the artwork.

Lawrence J. Ziomek
Carmel, California

Contents

Preface ix

1 INTRODUCTION AND BACKGROUND DISCUSSION

1.1 The Propagation of Acoustic Signals in the Ocean 1
1.2 The Ocean Medium as an Underwater Acoustic Communication Channel 3
Bibliography 6

2 FUNDAMENTALS OF LINEAR, TIME-VARIANT, SPACE-VARIANT FILTERS

2.1 Deterministic Filters 7
2.2 Random Filters 16
Bibliography 28

3 COMPLEX APERTURES

3.1 Coupling the Transmitted and Received Signals to the Medium 29
3.2 Directivity Functions 32
3.3 Linear Apertures and Far-Field Directivity Functions 41
3.4 Linear Apertures and Near-Field Directivity Functions 67
3.5 Planar Apertures and Far-Field Directivity Functions 69
3.6 Planar Apertures and Near-Field Directivity Functions 80
3.7 Directivity Index 83
Problems 88
Bibliography 92

4 ARRAYS

4.1 Linear Arrays and Far-Field Directivity Functions 94

4.2 Linear Arrays and Near-Field Directivity Functions 116
4.3 Grating Lobes 118
4.4 Array Gain 120
4.5 Planar Arrays and Far-Field Directivity Functions 124
4.6 Planar Arrays and Near-Field Directivity Functions 136
4.7 Volume Arrays and Far-Field Directivity Functions 138
Problems 143
Bibliography 151

5 SIGNAL PROCESSING

5.1 FFT Beamforming for Planar Arrays 153
5.2 Complex Envelopes 176
5.3 The Auto-Ambiguity Function 188
5.4 Time-Compression–Stretch Factor, Time Delay, and Doppler Shift Expressions 200
Problems 204
Bibliography 206

6 WAVE PROPAGATION IN INHOMOGENEOUS MEDIA

6.1 The WKB Approximation 208
6.2 Ray Acoustics 220
6.3 The Parabolic Equation Approximation 241
Bibliography 245

7 RANDOM OCEAN MEDIUM TRANSFER FUNCTIONS

7.1 Coupling Equations and the Generalized Coherence Function 246
7.2 The WKB Approximation Revisited 250
7.3 The Parabolic Equation Approximation Revisited 263
Bibliography 275

Answers to Problems 277
Index 283

1

Introduction and Background Discussion

1.1 THE PROPAGATION OF ACOUSTIC SIGNALS IN THE OCEAN

The propagation of *small-amplitude* acoustic signals in the ocean can be described by the *linear*, inhomogeneous, scalar wave equation

$$\nabla^2 \varphi(t, \mathbf{r}) - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} \varphi(t, \mathbf{r}) = x_M(t, \mathbf{r}), \tag{1.1-1}$$

where $\varphi(t, \mathbf{r})$ is the *velocity potential* at time t and position $\mathbf{r}$, $x_M(t, \mathbf{r})$ is the input acoustic signal to the medium or the *source distribution*, which represents the rate at which fluid volume is added (volume flow rate) at time t and position $\mathbf{r}$ per unit volume of fluid, and $c(\mathbf{r})$ is the speed of sound in the ocean in meters per second. If we assume that $c(\mathbf{r})$ is equal to a *constant* c, then the solution of Eq. (1.1-1) is given by

$$\varphi(t, \mathbf{r}) = -\frac{1}{4\pi} \int_{V_0} \frac{x_M(t - (|\mathbf{r} - \mathbf{r}_0|/c), \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|} dV_0, \tag{1.1-2}$$

where the quantity

$$t - (|\mathbf{r} - \mathbf{r}_0|/c) \tag{1.1-3}$$

is known as the *retarded time* and the integration is performed (in general) over the volume occupied by the source (see Fig. 1.1-1). The *acoustic pressure* $p(t, \mathbf{r})$ in pascals (newtons per square meter) and the *acoustic fluid (particle) velocity*

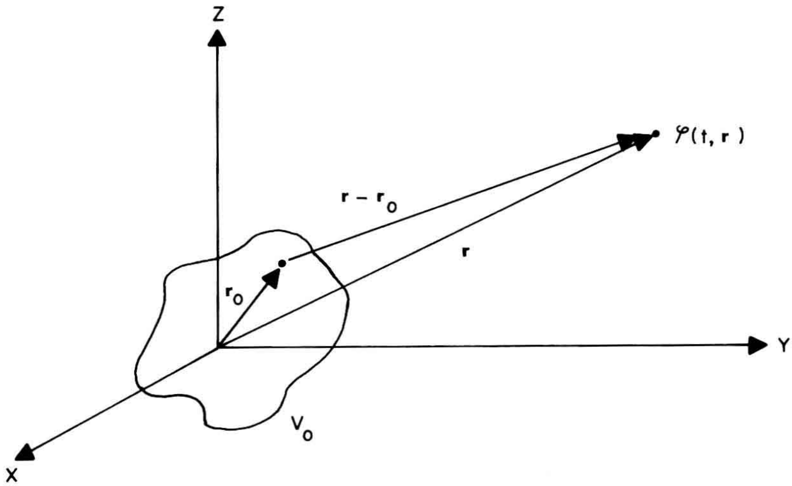


FIG. 1.1-1 Source distribution shown occupying a volume V_0.

vector $\mathbf{u}(t, \mathbf{r})$ in meters per second can be obtained from $\varphi(t, \mathbf{r})$ as follows:

$$p(t, \mathbf{r}) = -\rho_0 \frac{\partial}{\partial t} \varphi(t, \mathbf{r}), \tag{1.1-4}$$

where ρ_0 is the *equilibrium density* (assumed to be constant) of the ocean in kilograms per cubic meter and

$$\mathbf{u}(t, \mathbf{r}) = \nabla \varphi(t, \mathbf{r}). \tag{1.1-5}$$

Note that if we take the curl of both sides of Eq. (1.1-5), then

$$\nabla \times \mathbf{u}(t, \mathbf{r}) = \nabla \times \nabla \varphi(t, \mathbf{r}) = 0, \tag{1.1-6}$$

since the curl of the gradient of a scalar function [in this case, $\varphi(t, \mathbf{r})$] is equal to zero. Equation (1.1-6) indicates that the *vorticity* of the fluid, which is defined as $\nabla \times \mathbf{u}(t, \mathbf{r})$ and is proportional to the local angular velocity of the fluid, is equal to zero, and thus $\mathbf{u}(t, \mathbf{r})$ is said to be *irrotational*.

In addition to acoustic (sound) waves, there is water motion resulting from *internal waves*. Thus, in general, we must deal with the superposition of two wave motions: (1) acoustic (sound) waves, whose motions are *irrotational* and *compressional*; and (2) internal waves, whose motions are *rotational* and *incompressible*. The total fluid-velocity vector is therefore given by

$$\nabla \varphi(t, \mathbf{r}) + \nabla \times \mathbf{\Phi}(t, \mathbf{r}), \tag{1.1-7}$$

where $\nabla\varphi(t, \mathbf{r})$ is the fluid-velocity vector due to sound waves [see Eq. (1.1-5)] and $\nabla \times \mathbf{\Phi}(t, \mathbf{r})$ is the fluid-velocity vector due to internal waves. The expression $\mathbf{\Phi}(t, \mathbf{r})$ is called the *vector velocity potential.* Note that $\nabla \cdot \nabla \times \mathbf{\Phi}(t, \mathbf{r}) = 0$ (vector identity), and thus the wave motion $\nabla \times \mathbf{\Phi}(t, \mathbf{r})$ due to internal waves is said to be incompressible, as mentioned earlier. We will not be considering internal wave motion in this book, so Eq. (1.1-5) will be our defining relationship for the fluid-velocity vector.

Equation (1.1-2) is a representation of the acoustic field at distances close to the source, where the speed of sound can be considered constant. The analysis and results presented in Chapters 3 and 4 on the directivity functions of complex apertures and arrays, respectively, are based on Eq. (1.1-2). In addition, the various signal-processing topics considered in Chapter 5 also assume that c is constant. Of course, as is well known, the speed of sound in the ocean is not constant in general but is instead a function of temperature, depth, and salinity, that is, it is a function of position in the medium. Thus, different approximate solutions of Eq. (1.1-1) for variable speed-of-sound profiles $c(\mathbf{r})$ are discussed in Chapter 6. These solutions are then incorporated into the derivation of transfer functions of the random ocean medium, which is the subject of Chapter 7. Once a transfer function has been derived, it can then be *coupled* to the far-field directivity functions of the transmit and receive apertures (arrays) and to the frequency spectrum of the transmitted electrical signal. This systems approach is introduced next in more detail.

1.2 THE OCEAN MEDIUM AS AN UNDERWATER ACOUSTIC COMMUNICATION CHANNEL

Since the wave equation for small-amplitude acoustic signals is linear, we can represent the ocean medium (in general) as a *linear, time-variant, space-variant, random filter* (*system* or *communication channel*). With this interpretation in mind, refer to Fig. 1.2-1, which illustrates the geometry of a basic bistatic communication channel, and Fig. 1.2-2, which is a mathematical block-diagram representation of Fig. 1.2-1. With respect to Fig. 1.2-1, both the transmit and receive apertures (arrays) are, in general, volume apertures (arrays) and are in motion.

Figure 1.2-2 serves as a good introduction to the following concepts and associated notation:

(1) complex transmit and receive aperture functions, $A_T(f, \mathbf{r})$ and $A_R(\eta, \mathbf{r})$;

(2) transmit and receive far-field directivity functions, $D_T(f, \boldsymbol{\alpha})$ and $D_R(\eta, \boldsymbol{\beta})$;

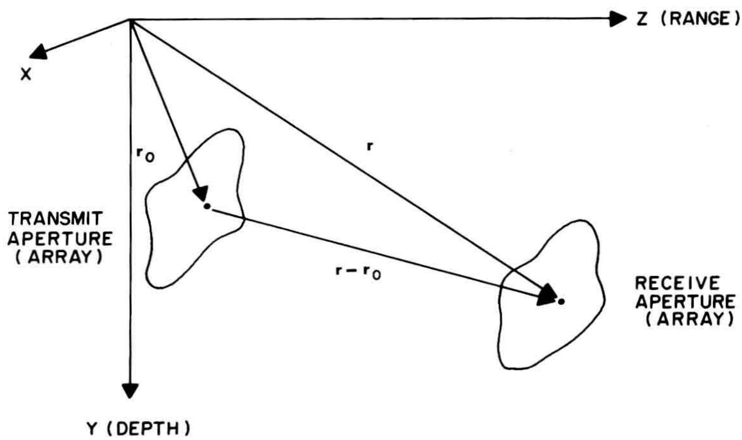


FIG. 1.2-1 Basic geometry of a bistatic communication channel.

(3) vectors whose components are spatial frequencies, $\boldsymbol{\alpha}$, $\mathbf{v}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$;

(4) frequency and angular spectra, $X(f,\boldsymbol{\alpha})$, $X_M(f,\mathbf{v})$, $Y_M(\eta,\boldsymbol{\beta})$, and $Y(\eta,\boldsymbol{\gamma})$;

(5) linear, time-variant, space-variant, random ocean medium impulse-response and transfer functions, $h_M(\tau,\mathbf{r}_0;t,\mathbf{r})$ and $H_M(f,\mathbf{v};t,\mathbf{r})$.

Before proceeding further, a word of caution concerning Fig. 1.2-2 is in order. Note, for example, that $X_M \neq XD_T$, $Y_M \neq X_M H_M$, and $Y \neq Y_M D_R$, in general. The equations required for describing the linear, time-variant, space-variant, random filter's input–output relationships and for coupling the transmitted and received electrical signals to the medium via the transmit and receive apertures are developed in Chapters 2 and 3, respectively.

Let us now describe in more detail the notation used in Fig. 1.2-2. The position vectors $\mathbf{r}_0$ and $\mathbf{r}$ refer to the spatial coordinates (x_0, y_0, z_0) and (x, y, z), respectively, in meters, and t refers to time in seconds. The parameters f and η are frequencies in hertz where f represents input or transmitted frequencies and η represents output or received frequencies. Note that if $\eta \neq f$, then Doppler spread is implied.

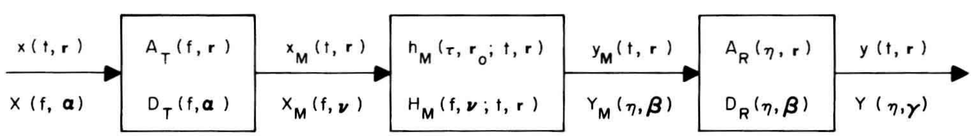


FIG. 1.2-2 Mathematical block-diagram representation of the bistatic communication channel depicted in Fig. 1.2-1.

The quantities $\boldsymbol{\alpha}$, $\boldsymbol{\nu}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ are vectors whose components are *spatial frequencies* in cycles per meter. Since spatial frequencies are related to both direction cosines and wavelength, and hence to wave-number components, they represent directions of wave propagation. The vector $\boldsymbol{\nu}$ represents input or transmitted spatial frequencies into the medium, as in $X_M(f, \boldsymbol{\nu})$, while $\boldsymbol{\beta}$ represents output or received spatial frequencies from the medium, as in $Y_M(\eta, \boldsymbol{\beta})$. Note that if $\boldsymbol{\beta} \neq \boldsymbol{\nu}$, then angular spread (scatter) is implied.

The remaining expressions found in Fig. 1.2-2 are further described in the following list:

$x(t, \mathbf{r})$	Input electrical signal to transmit electroacoustic transducer applied at time t and spatial location $\mathbf{r}$ of transducer
$X(f, \boldsymbol{\alpha})$	Frequency f and angular spectrum $\boldsymbol{\alpha}$ of input electrical signal
$A_T(f, \mathbf{r})$	Complex frequency response at spatial location $\mathbf{r}$ of transmit transducer. Also referred to as the complex transmit aperture function
$D_T(f, \boldsymbol{\alpha})$	Transmit far-field directivity function or beam pattern
$x_M(t, \mathbf{r})$	Input acoustic signal to the medium applied at time t and spatial location $\mathbf{r}$. Also, output acoustic signal from transmit electroacoustic transducer. Recall that $x_M(t, \mathbf{r})$ appeared as the source distribution in Eq. (1.1-1)
$X_M(f, \boldsymbol{\nu})$	Frequency f and angular spectrum $\boldsymbol{\nu}$ of input acoustic signal
$h_M(\tau, \mathbf{r}_0; t, \mathbf{r})$	Time-variant, space-variant, random impulse response or *Green's function* of the ocean medium. It describes the response of the medium at time t and spatial location $\mathbf{r}$ due to the application of a unit impulse at time $(t - \tau)$, or τ seconds ago, at a distance $\lvert\mathbf{r} - \mathbf{r}_0\rvert$ meters away (see Fig. 1.2-1)
$H_M(f, \boldsymbol{\nu}; t, \mathbf{r})$	Time-variant, space-variant, random transfer function of the ocean medium
$y_M(t, \mathbf{r})$	Output acoustic signal from the medium at time t and spatial location $\mathbf{r}$. Also, input acoustic signal to receive electroacoustic transducer
$Y_M(\eta, \boldsymbol{\beta})$	Frequency η and angular spectrum $\boldsymbol{\beta}$ of output acoustic signal
$A_R(\eta, \mathbf{r})$	Complex frequency response at spatial location $\mathbf{r}$ of receive transducer. Also referred to as the complex receive aperture function
$D_R(\eta, \boldsymbol{\beta})$	Receive far-field directivity function or beam pattern

$y(t, \mathbf{r})$	Output electrical signal from receive electroacoustic transducer at time t and spatial location $\mathbf{r}$ of transducer
$Y(\eta, \gamma)$	Frequency η and angular spectrum γ of output electrical signal

As mentioned previously, we can represent the ocean medium as a linear, time-variant, space-variant, random filter. The term "time-variant" implies motion among targets, the ocean surface, discrete point scatterers, and the transmit and receive apertures (arrays). Discrete point scatterers in the ocean may include, for example, gas bubbles, fish, and particulate matter. The time-variant property results in both Doppler spread and spread in time-delay values. If the filter is time-invariant, then no motion is implied. As a result, there will be no Doppler spread and no spread in time delay.

The term "space-variant" implies that the sound-speed profile (index of refraction) of the ocean is a function of position. This space-variant property results in scatter or angular spread due to refraction. If the filter is space-invariant, then an isospeed medium is implied. As a result, there will be no refraction, and hence no scatter or angular spread, since the sound rays will be traveling in straight lines.

In addition, since any motion and/or the index of refraction can be decomposed into a sum of deterministic (average) and random (fluctuating) components, these random components can be accounted for via a random filter representation as opposed to a deterministic filter representation. For example, by using a systems theory approach, surface, volume, and/or bottom reverberation returns can be modeled as the outputs from linear filters. In addition, target returns can also be modeled as filter outputs. Furthermore, different transmit signals and transmit and receive far-field directivity functions can easily be coupled to various models (i.e., transfer functions) of the random, inhomogeneous ocean medium in a straightforward and logical fashion in order to study problems in pulse propagation in random media, underwater acoustic communication, target detection, and parameter estimation using various space–time signal-processing algorithms.

BIBLIOGRAPHY

S. M. Flatté (ed.), *Sound Transmission Through a Fluctuating Ocean.* Cambridge Univ. Press, London and New York, 1979.

D. Middleton, A statistical theory of reverberation and similar first-order scattered fields. Part I: waveforms and the general process. *IEEE Trans. Inf. Theory* **IT-13**, 372–392 (1967).

S. Temkin, *Elements of Acoustics.* Wiley, New York, 1981.

L. J. Ziomek, *Linear Time-Variant Space-Variant Filters and the WKB Approximation.* Naval Postgraduate School Technical Report 62-83-058, October, 1983, 93 pages. Also available from DTIC, AD Number A 134886.

2

Fundamentals of Linear, Time-Variant, Space-Variant Filters

2.1 DETERMINISTIC FILTERS

2.1.1 Impulse Response and Transfer Functions

A linear, time-variant, space-variant filter, as depicted in Fig. 2.1-1, is characterized by its corresponding time-varying, space-varying impulse response $h(\tau, \mathbf{r}_0; t, \mathbf{r})$. The function $h(\tau, \mathbf{r}_0; t, \mathbf{r})$ describes the response of the filter at time t and spatial location $\mathbf{r} = (x, y, z)$ due to the application of a unit impulse at time $(t - \tau)$, or τ seconds ago, at a distance $|\mathbf{r} - \mathbf{r}_0|$ meters away, where $\mathbf{r}_0 = (x_0, y_0, z_0)$. Note that

$$h(\tau, \mathbf{r}_0; t, \mathbf{r}) \equiv h(t, \mathbf{r}; t - \tau, \mathbf{r} - \mathbf{r}_0). \tag{2.1-1}$$

The impulse response function is also called the *Green's function.*

The relationship between the input signal $x(t, \mathbf{r})$ and the output signal $y(t, \mathbf{r})$ is given by

$$y(t, \mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t - \tau, \mathbf{r} - \mathbf{r}_0) h(\tau, \mathbf{r}_0; t, \mathbf{r}) \, d\tau \, d\mathbf{r}_0, \tag{2.1-2}$$

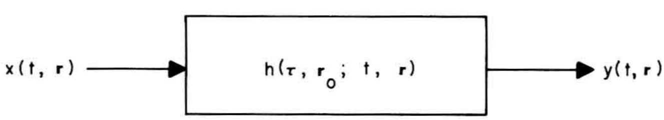


FIG. 2.1-1 Representation of a linear, time-variant, space-variant filter.