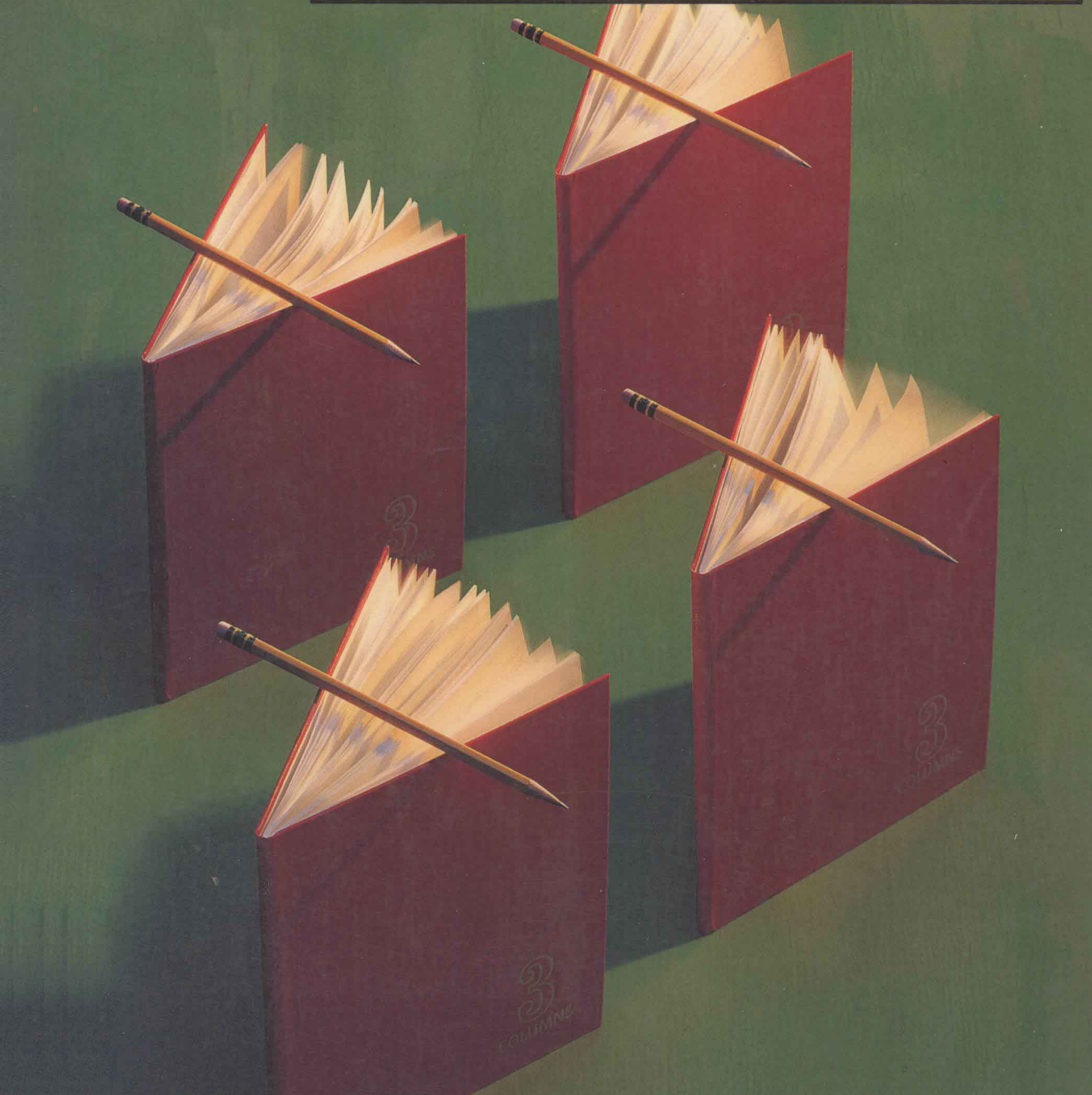
HARPERCOLLINS COLLEGE OUTLINE

BUSINESS CALCULUS

Ronald Smith

Comprehensive Outline in Easy-To-Use Narrative Format, Supplements Major Textbooks, Solved Problems, Graphs, Tables, Figures, Fully Indexed



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Business Calculus

Ron Smith, Ph.D.
Edison Community College

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Business Calculus

Preface

Business Calculus is designed to be a companion text to that which is used in any business calculus course. This book contains the topics commonly found in the courses on business calculus taught at colleges and universities.

Each topic is addressed with the basic concepts in mind. The text examples are extensively annotated at each point along with step-by-step solutions. Many often-asked questions are answered during the solution process. Exercises that mirror the examples are included at the end of each chapter to check for understanding, so that the student can return to the primary text for the course.

I hope that this text provides insight by which a student can build a strong knowledge of calculus.

Ron Smith

Business Calculus

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Algebra for Calculus

1.1 FACTORING

Introduction

Many students who have difficulty with calculus find that their difficulty lies not with the calculus concepts, but with an unfamiliarity with algebraic techniques. This chapter will emphasize the algebra needed for calculus.

Prior to reading this section, please make sure that you are comfortable with (a) removing common factors, (b) factoring by pairing, and (c) trinomial factoring. These topics are covered in *Elementary Algebra* or *Intermediate Algebra*—other books in this series.

Factoring concepts covered here will be:

a) Factoring the sum of two cubes and the difference of two cubes
b) Using radicals in factoring
c) Removing common factors from complicated binomials

The Sum and Difference of Two Cubes

The general formulas for the sum of two cubes and the difference of two cubes are as shown below. The letters a and b represent any algebraic term.

1.1 Sum of Two Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

1.2 Difference of Two Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Please note that $a^3 + b^3$ does *not* equal $(a+b)^3$ and that $a^3 - b^3$ does *not* equal $(a-b)^3$. The factorization of the sum of two cubes and the difference of two cubes is demonstrated in the following example.

EXAMPLE 1.1

Use the formulas for the sum and difference of two cubes to factor the following.

a) $x^3 + 8y^3$
b) $27x^3 - 1$
c) $8a^3 + b^6$
d) $c^3d^3 - 64$

SOLUTION 1.1

a) $x^3 + 8y^3$ — Copy the original binomial.

$x^3 + 2^3y^3$ — Rewrite 8 as 2^3.

$(x)^3 + (2y)^3$ — Referring to the general formula, 1.1, let $a = x$ and $b = 2y$.

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ — Place the new terms in the format of the general formula.

$$(x)^3 + (2y)^3 = (x+2y)((x)^2 - (x)(2y) + (2y)^2)$$

$$x^3 + 8y^3 = (x+2y)(x^2 - 2xy + 4y^2)$$

Multiply within terms and simplify.

b) $27x^3 - 1$ — Copy the original binomial.

$3^3x^3 - 1^3$ — Rewrite 27 as 3^3, and 1 as 1^3.

$(3x)^3 - (1)^3$ — Referring to the general formula, 1.2, let $a = 3x$ and $b = 1$.

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ — Place the new terms in the format of the general formula.

$$(3x)^3 - (1)^3 = (3x-1)((3x)^2 + (3x)(1) + 1^2)$$

$$27x^3 - 1 = (3x-1)(9x^2 + 3x + 1)$$

Multiply within terms and simplify.

c) $8a^3 + b^6$ — Copy the original binomial.

$2^3a^3 + (b^2)^3$ — Rewrite 8 as 2^3 and b^6 as $(b^2)^3$.

Note that $b^6 = b^{2 \cdot 3} = (b^2)^3$.

$(2a)^3 + (b^2)^3$ — Use the general formula, 1.1, letting $a = 2a$ and $b = b^2$.

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ — Place the new terms in the format of the general formula.

$$(2a)^3 + (b^2)^3 = (2a + b^2)((2a)^2 - (2a)(b^2) + (b^2)^2)$$

$$8a^3 + b^6 = (2a + b^2)(4a^2 - 2ab^2 + b^4)$$

Multiply within terms and simplify.

d) $c^3d^3 - 64$ — Copy the original binomial.

$c^3d^3 - 4^3$ — Rewrite 64 as 4^3.

$(cd)^3 - (4)^3$ — Referring to the general formula, 1.2, let $a = cd$ and $b = 4$.

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ — Place the new terms in the format of the general formula.

$$(cd)^3 - (4)^3 = (cd - 4)((cd)^2 + (cd)(4) + (4)^2)$$

$$c^3d^3 - 64 = (cd - 4)(c^2d^2 + 4cd + 16)$$

Multiply within terms and simplify.

Factoring Using Radicals

A calculus course is the first time radical expressions are used extensively in factoring. A radical sign is, as you may recall, the symbol $\sqrt{\ }$.

Usually, factoring with radical signs involves the use of the factoring technique for the **difference of two squares.**

By using radicals, every binomial with terms separated by a minus sign can be factored using the formula for the **difference of two squares.**

Note the following two formulas:

1.3 The Difference of Two Squares

$$a^2 - b^2 = (a+b)(a-b)$$

1.4 Using the Square Root Radical

$$a = \sqrt{a} \cdot \sqrt{a} = (\sqrt{a})^2$$

Formulas 1.3 and 1.4 are combined to form the basis for factoring using radical signs.

1.5 Factoring Using Radicals

$$a - b = (\sqrt{a})^2 - (\sqrt{b})^2 = (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$$

EXAMPLE 1.2

Factor the following binomials using Formula 1.5.

a) $x^2 - 3$

b) $y - 16$

c) $x - 2$

SOLUTION 1.2

a) $x^2 - 3$ — Copy the given binomial.

$(x)^2 - (\sqrt{3})^2$ — Rewrite x^2 as $(x)^2$ and use Formula 1.4 to convert 3 to $(\sqrt{3})^2$.

$(x + \sqrt{3})(x - \sqrt{3})$ — Using the difference of two squares process, 1.3, factor the binomial.

$x^2 - 3 = (x + \sqrt{3})(x - \sqrt{3})$ — The final answer is based on Formula 1.5.

b) $y - 16$ — Copy the given binomial.

$(\sqrt{y})^2 - 4^2$ — Rewrite 16 as 4^2 and use Formula 1.4 to convert y to $(\sqrt{y})^2$.

$(\sqrt{y} + 4)(\sqrt{y} - 4)$ — Using the difference of two squares process, 1.3, factor the binomial.

$y - 16 = (\sqrt{y} + 4)(\sqrt{y} - 4)$ — The final answer is based on Formula 1.5.

c) $x - 2$ — Copy the given binomial.

$(\sqrt{x})^2 - (\sqrt{2})^2$ — Use Formula 1.4 to convert x to $(\sqrt{x})^2$ and 2 to $(\sqrt{2})^2$.

$(\sqrt{x} + \sqrt{2})(\sqrt{x} - \sqrt{2})$ — Using the difference of two squares process, 1.3, factor the binomial.

$x - 2 = (\sqrt{x} + \sqrt{2})(\sqrt{x} - \sqrt{2})$ — The final answer is based on Formula 1.5.

We will use Formula 1.5 in the chapter on limits (chapter 2).

Factoring Complicated Binomials

In subsequent chapters we will use a process called the Product Rule for derivatives. This process results in large binomials that can often be factored. The factorization of these binomials will be completed using the procedure outlined in the following exercise. This procedure is similar to removing a common factor from a polynomial.

> **1.6 Removing a Common Factor**
>
> $$ca + cb = c(a + b)$$

EXAMPLE 1.3

Factor the following expressions by removing common factors and simplifying.

a) $8(x-3)^4(2x-1)^2 + 2(x-3)^3(2x-1)^3$
b) $3(x-1)^2(3x-2)^7 + 7(x-1)^3(3x-2)^6$
c) $12(x+6)^5 + 4(x-2)(x+6)^4$

SOLUTION 1.3

a) $\underbrace{8(x-3)^4(2x-1)^2}_{\text{1 term}} + \underbrace{2(x-3)^3(2x-1)^3}_{\text{1 term}}$

Copy the large binomial.

$2[4(x-3)^4(2x-1)^2 + (x-3)^3(2x-1)^3]$

Remove a common factor of 2 from each term.

$2(x-3)^3[4(x-3)^1(2x-1)^2 + (2x-1)^3]$

Remove three common factors of $(x-3)$ from each term (three factors are the largest number of factors found in each term).

$2(x-3)^3(2x-1)^2[4(x-3) + (2x-1)]$

Remove two common factors of $(2x-1)$ from each term (two factors are the largest number of factor present in both terms).

$2(x-3)^3(2x-1)^2[4x-12+2x-1]$

Distribute 4.

$2(x-3)^3(2x-1)^2[6x-13]$ Combine like terms.

$$8(x-3)^4(2x-1)^2 + 2(x-3)^3(2x-1)^3$$
$$= 2(x-3)^3(2x-1)^2(6x-13)$$

b) $\underbrace{3(x-1)^2(3x-2)^7}_{\text{1 term}} + \underbrace{7(x-1)^3(3x-2)^6}_{\text{1 term}}$

Copy the large binomial.

$(x-1)^2[3(3x-2)^7 + 7(x-1)^1(3x-2)^6]$

Remove two common factors of $(x-1)$ from each term (two factors are the largest number of factors present in each term).

$(x-1)^2(3x-2)^6[3(3x-2)^1 + 7(x-1)^1]$

Remove six common factors of $(3x-2)$ from each term (six factors are the largest number of factors present in each term).

$(x-1)^2(3x-2)^6[9x-6+7x-7]$ Distribute 3 and 7.

$(x-1)^2(3x-2)^6[16x-13]$ Combine like terms.

$$3(x-1)^2(3x-2)^7+7(x-1)^3(3x-2)^6$$
$$= (x-1)^2(3x-2)^6(16x-13)$$

c) $\underbrace{12(x+6)^5}_{\text{1 term}} + \underbrace{4(x-2)(x+6)^4}_{\text{1 term}}$ Copy the large binomial.

$4[3(x+6)^5 + (x-2)(x+6)^4]$ Remove the common factor of 4 from each term.

$4(x+6)^4[3(x+6) + (x-2)]$ Remove four common factors of $(x + 6)$ from each term (four factors are the largest number of factors present in each term).

$4(x+6)^4[3x+18+x-2]$ Distribute 3.

$4(x+6)^4[4x+16]$ Combine like terms.

$16(x+6)^4(x+4)$ Remove a factor of 4 and move it to the left of the factors.

$$12(x+6)^5+4(x-2)(x+6)^4$$
$$= 16(x+6)^4(x+4)$$

1.2 RATIONAL EXPONENTS

Converting Radical Expressions

Before some calculus operations can be completed, all algebraic expressions in radical form must be converted to rational (fractional) exponents. The rule that describes this conversion is:

1.7 $\sqrt[b]{x^a} = x^{a/b}$
where a and b are positive whole numbers $[b \geq 2]$

Note: b is called the index of the radical.

EXAMPLE 1.4

Convert the following radical expressions to terms with rational exponents.

a) $\sqrt[4]{x^7}$

b) $\sqrt[6]{x^4}$

c) $\sqrt{x}$

SOLUTION 1.4

a) $\sqrt[4]{x^7}$ — Copy the expression.

$= x^{7/4}$ — Write the 7 from within the radical sign in the numerator of the exponent and the index, 4, in the denominator.

b) $\sqrt[6]{x^4}$ — Copy the expression.

$= x^{4/6}$ — The number 4 from within the radical sign is the numerator of the exponent. The index, 6, is the denominator.

$= x^{2/3}$ — Reduce the fraction, if possible.

c) $\sqrt{x}$ — Copy the expression.

$\sqrt[2]{x^1}$ — Express the understood 1 exponent of the x within the radical sign. Express the understood index of 2.

$= x^{1/2}$ — The 1 from within the radical sign is the numerator of the exponent and the index 2 is the denominator.

Adding Rational Exponents

The addition of rational exponents occurs when expressions containing the same base—but varying exponents—are multiplied. The addition of rational exponents can be stated as follows in Rule 1.8.

1.8 $x^{a/b} \cdot x^{c/d} = x^{ad/bd} \cdot x^{cb/bd} = x^{(ad+cb)/bd}$ where a, b, c, and d are integers and $b, d \neq 0$.

EXAMPLE 1.5

Compute the following products involving the multiplication of expression with rational exponents.

a) $(x^{2/3})(x^{1/5})$
b) $(x^{1/2})(x^{3})$
c) $(x^{1/3})(x^{1/2} + x^{3/2})$
d) $(x^{3/4} + x^{1/4})(x - x^{2})$

SOLUTION 1.5

a) $(x^{2/3})(x^{1/5})$ — Copy original factors.

$= x^{10/15} \cdot x^{3/15}$ — Using Rule 1.8, convert the two rational exponents so that they have the same denominator.

$= x^{(10+3)/15} = x^{13/15}$

b) $(x^{1/2})(x^{3})$ — Copy original factors.

$= x^{1/2} \cdot x^{3/1}$ — Rewrite the exponent as $\frac{3}{1}$ (a fraction).

$x^{1/2} \cdot x^{3/1} = x^{1/2} \cdot x^{6/2}$ — Using Rule 1.8, the two rational exponents are converted so that they have the same denominator.

$x^{1/2} \cdot x^{6/2} = x^{(1+6)/2} = x^{7/2}$ — Add the exponent numerators.

c) $(x^{1/3})(x^{1/2} + x^{3/2})$ — Copy the two expressions.

$x^{1/3} \cdot x^{1/2} + x^{1/3} \cdot x^{3/2}$ — Distribute $x^{1/3}$.

$x^{2/6} \cdot x^{3/6} + x^{2/6} \cdot x^{9/6}$ — Use Rule 1.8 to convert the exponent denominators to the same numbers.

$x^{(2+3)/6} + x^{(2+9)/6} = x^{5/6} + x^{11/6}$ — Add the numerators of the exponents in each term.

d) $(x^{3/4}+x^{1/4})\,(x-x^2)$ — Copy the two binomials.

$(x^{3/4}+x^{1/4})\,(x-x^2) = (x^{3/4}+x^{1/4})\,(x^{1/1}-x^{2/1})$

Express the whole numbers as fractional exponents.

$= x^{3/4}\cdot x^{1/1} - x^{3/4}\cdot x^{2/1} + x^{1/4}\cdot x^{1/1} - x^{1/4}\cdot x^{2/1}$

Multiply each part of one algebraic expression by every part of the other expression.

$= x^{3/4}x^{4/4} - x^{3/4}x^{8/4} + x^{1/4}x^{4/4} - x^{1/4}x^{8/4}$

Use Rule 1.8 to convert the denominators of each rational exponent to the same number.

$= x^{(3+4)/4} - x^{(3+8)/4} + x^{(1+4)/4} - x^{(1+8)/4}$

Add the numerators of the exponents.

$= x^{7/4} - x^{11/4} + x^{5/4} - x^{9/4}$

Subtracting Rational Exponents

The subtraction of rational exponents often occurs when one algebraic expression is divided by another. We will limit our discussion to the case in which the divisor is one term.

Three rules that assist in the subtraction of rational exponents are shown here:

1.9 $\dfrac{1}{x^{a/b}} = x^{-a/b}$

1.10 $x^{c/d}\cdot x^{-a/b} = x^{bc/bd}\cdot x^{-ad/bd} = x^{(bc-ad)/bd}$

1.11 $\dfrac{a+b+c+\ldots}{d} = \dfrac{a}{d}+\dfrac{b}{d}+\dfrac{c}{d}+\ldots$

Here, a, b, c, and d are real numbers and $b, d \neq 0$.