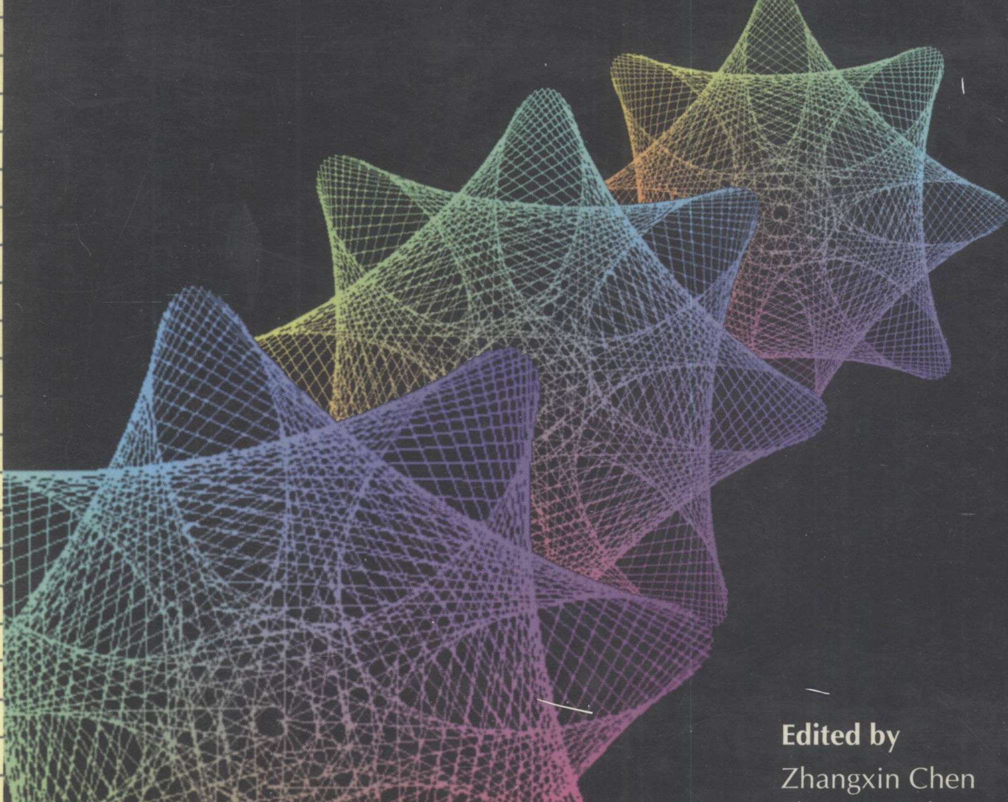

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Bifurcation Theory & Its Numerical Analysis

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PREFACE

Static bifurcation theory deals with the changes that occur in the structure of the set of zeros of a mapping as parameters in the mapping are varied, while dynamic bifurcation theory is concerned with the changes that occur in the structure of the limit sets of solutions of differential equations as parameters in a vector field are varied. Extensive research on the theoretical characterization of their behavior has been conducted. In parallel, the numerical analysis and related numerical algorithms for computing the associated solutions have been developed.

The Second International Conference on Bifurcation Theory and its Numerical Analysis was successfully held in Xi'an, China, June 29-July 3, 1998. The first international conference of this series was held at the same place ten years ago. Their aim was to bring together active researchers with different backgrounds to discuss recent and prospective advances in bifurcation theory and its numerical analysis. Over seventy people from Canada, China, France, Germany, Italy, Japan, Singapore, and the United States of America attended the second conference and more than forty papers were presented on a variety of subjects in Bifurcation Theory, Differential Equations, Dynamical Systems, Nonlinear Analysis, Numerical Analysis, and their applications.

This book contains eighteen selected papers presented at the second conference. They cover recent development of a wide range of theoretical and numerical issues of the subjects mentioned above. They also involve applications to such important areas as fluid flows, elasticity, elastic-plastic solids, neutron transport, robotics, activator-inhibitor modeling, and biology.

Financial support for the conference was generously provided by the State Educational Ministry of China, the National Natural Science Foundation of China, the State Key Basic Research Project, the U.S. Army Research Office-Far-East, the U.S. Office of Naval Research-Asia, the U.S. Air Force Office for Scientific Research/Asian Office of Aerospace Research and Development, and the Numerical Algorithms Group (NAG) ltd., U.K. We would also like to thank the local organizers at Xi'an Jiaotong University, China for their hospitality. Especially, we would like to express our gratitude to the U.S. Army Research Office-Far-East, the U.S. Office of Naval Research-Asia, and the U.S. Air Force Office for Scientific Research/Asian Office of Aerospace Research and Development for supporting publication of the present book.

Zhangxin Chen, Shui-Nee Chow and Kaitai Li

January 29, 1999

CONTENTS

Invariant Foliations of Overflowing Manifolds for Semiflows in Banach Space

PETER BATES, KENING LU, AND CHONGCHUN ZENG

Abstract

In this paper, we prove that each normally hyperbolic overflowing invariant manifold has a unique unstable manifold in a neighborhood of the overflowing manifold and that this unstable manifold uniquely persists under small perturbations of the semiflow. We prove that the unstable manifold can be uniquely decomposed into a disjoint union of C^k submanifolds which are along the unstable direction. This family of submanifolds forms an invariant foliation of the unstable manifold.

KEYWORDS: Invariant foliations, overflowing manifolds, unstable manifolds, semiflows.

1 Introduction

We consider a C^1 semiflow defined on a Banach space X; that is, it is continuous on $[0, \infty) \times X$, and for each $t \geq 0, T^t : X \to X$ is C^1, and

$$T^t \circ T^s(x) = T^{t+s}(x)$$

for all $t, s \geq 0$ and $x \in X$. A typical example is the solution operator for a differential equation.

In [6], we studied the persistence of an overflowing manifold M ("negatively invariant and the semiflow crosses the boundary transversally") for the semiflow T^s under perturbations. We do not assume that M is compact or finite dimensional. Also, M is not necessarily an imbedded manifold, but an immersed manifold. In brief, our main results on the overflowing manifolds may be summarized as follows. We assume that the immersed manifold M

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does not twist very much locally, M is covered by the image under T^t of a subset a positive distance away from the boundary, DT^t contracts along the normal direction and does so more strongly than it does along the tangential direction, and DT^t has a certain uniform continuity in a neighborhood of M. If the C^1 perturbation $\tilde{T}^t$ of T^t is sufficiently close to T^t, then $\tilde{T}^t$ has a unique C^1 immersed overflowing manifold $\tilde{M}$ near M. Furthermore, if $\tilde{T}^t$ is C^k and a spectral gap condition holds, then $\tilde{M}$ is C^k.

In the present paper, we consider the case that the normal bundle of the overflowing manifold is split into a stable bundle and an unstable bundle. We assume that the linearized semiflow DT^t contracts along the stable direction and more strongly than it does along the tangential direction and expands along the unstable direction more strongly than it does along the tangential direction. We prove that each such overflowing manifold has a unique unstable manifold in a neighborhood of the overflowing manifold and that this unstable manifold uniquely persists under any small perturbations of the semiflow. We prove that the unstable manifold can be uniquely decomposed into a disjoint union of C^k submanifolds which forms an invariant foliation of the unstable manifold. Each submanifold is a leaf or a fiber of the foliation. Here, the invariance means that the preimage of each leaf under the semiflow is contained in a leaf. The existence of invariant foliations near compact normally hyperbolic invariant manifolds for semiflows was established in [5].

Invariant foliations with invariant manifolds have become a fundamental tool to study the qualitative properties of a flow or semiflow near invariant sets. They are extremely useful in that they can be used to track the asymptotic behavior of solutions and to provide coordinates in which systems of differential equations may be decoupled and normal forms derived. This theory has a long history. In the 1950's, Peixoto [28] used this technique to study planar systems. Starting in the 1960's, applying the invariant foliation as a major tool, Anosov [1] proved the structural stability of hyperbolic sets and Smale [30] and Palis and Smale [27] also used invariant foliations in the proof of Ω-stability. Later, Hirsch, Pugh, and Shub [17] extended this from hyperbolic sets to normally hyperbolic invariant manifolds, but mainly for finite dimensional cases. About the same time, Fenichel [12], [13] and [14] independently proved similar theorems and also pioneered the use of invariant foliations to handle some singular perturbation problems. For several recent applications, see, for example, [11], [15], [16], [18], [19], [20], [21], [23], [31], and [32]. Kirchgraber and Palmer [22] applied the invariant foliation technique to the linearization of finite dimensional systems. In [8], invariant foliations are used to produce smooth conjugacy of flows on different center manifolds.

In the infinite-dimensional setting, Ruelle [29] proved a result giving invariant stable and unstable fibers (leaves) almost everywhere on a compact invariant set for a semiflow in Hilbert space. It was assumed that the linearized time-t map is compact and injective with dense range. Mañé [26] extended Ruelle's results to semiflows in Banach space, under the same conditions on the time-t map.

Lu [25] used the foliation to prove the Hartman-Grobman theorem for scalar parabolic equations and later in an abstract setting applicable to the Cahn-Hillard equation with Bates [3]. Also, Chow, Lin, and Lu [10] constructed an invariant foliation near an invariant manifold.

Very recently, Li, McLaughlin, Shatah, and Wiggins [24] and Zeng [33] used invariant foliations in the study of the existence of homoclinic orbits for nonlinear Schrödinger equations.

Recently, Aulbach and Garay [2] used invariant foliations to study partial linearization for noninvertible mappings near fixed points. Chen, Hale and Tan [9] obtained invariant foliations near fixed points for C^1 semigroups in Banach spaces.

We should mention that our results in this paper are not a trivial extension of the results of Fenichel. Here, we had to overcome the difficulties caused by the irreversibility of the semiflow and the noncompactness of the manifold, which means we do not have a global tubular neighborhood of the manifold. Our result, even in the case of finite dimension is new, giving the results for an immersed overflowing manifold.

The proof for the existence and persistence of an unstable manifold of an overflowing manifold is based on Hadamard's graph transform, while the proof for the existence of the unstable foliation of the unstable manifold is based on Liapunov-Perron's method. We first obtain results for maps, and then apply them to semiflows.

2 Main Results

In this section, we introduce basic notation and hypotheses, and then state our main results for maps and semiflows.

Let X be a Banach space and $T \in C^1(X, X)$. Suppose M is a C^1 Banach manifold (with boundary removed) and $\psi : M \to X$ is an immersion from M into X.

For a subset $A \subset X$, and $a > 0$, let

$$B(A, a) = \{x \in X \ : \ d(x, A) < a\}.$$

For $m_0 \in M$, let $B_c(m_0, a)$ denote the connected component of

$$\psi^{-1}(B(\psi(m_0), a))$$

containing m_0.

Definition 2.1 *M is said to be overflowing if the following conditions hold*

(1) There exist an open $M_1 \subset M$ and a homeomorphism $u : M \to M_1$ such that $\psi(m) = T(\psi(u(m)))$ for all $m \in M$;

(2) There exists an $r > 0$ such that for any $m_0 \in M_1$, $\psi(\overline{B_c(m_0, r)})$ is closed in X.

Condition (1) means that the image of $\psi(M_1)$ under T covers $\psi(M)$. Condition (2) essentially says that the 'distance' from $\psi(M_1)$ to the boundary of $\psi(M)$ is at least r. Traditionally, a manifold is called overflowing for a vector field if the vectors on the boundary point outside. We modified this definition for maps, which ensures that when M is overflowing for a vector field, it is so for the time-maps.

(H1) For each $m \in M$ there is a decomposition

$$X = X^c_m \oplus X^u_m \oplus X^s_m$$

of closed subspaces with X^u_m, X^s_m being transversal to $(D\psi(m))(T_m M)$, where $T_m M$ is the tangent space of M at m. Furthermore, for any $m_1 \in M$,

$$\Pi^\alpha_{m_1} DT(\psi(m_0)) : X^\alpha_{m_0} \to X^\alpha_{m_1}$$

are isomorphisms for $\alpha = c, u$, where $m_0 = u(m_1) \in M_1$ and Π^α_m is the projection onto X^α_m with kernel X^β_m for $\beta \neq \alpha$, and there exists $\lambda \in (0,1)$ such that

$$\begin{aligned}&||\Pi^s_{m_1} DT(\psi(m_0))|_{X^s_{m_0}}|| \\ &< \lambda \min\{1, \inf\{|\Pi^c_{m_1} DT(\psi(m_0))x^c| \ : \ x^c \in X^c_{m_0}, |x^c| = 1\}\},\end{aligned}$$

and

$$\lambda \inf \left\{ \left| \Pi^u_{m_1} DT(\psi(m_0)) x^u \right| : x^u \in X^u_m, |x^u| = 1 \right\}$$
$$> \max \left\{ 1, ||\Pi^u_{m_1} DT(m) \left|_{X^c_m} \right|| \right\}.$$

Here $\Pi^s_m = I - \Pi^c_m - \Pi^u_m$.

This hypothesis essentially says that the linearized map DT contracts along the stable direction and more strongly than it does along the tangential direction and expand along the unstable direction more strongly than it does along the tangential direction. We do not require the families $\{X^c_m\}$, $\{X^u_m\}$, and $\{X^s_m\}$ to be invariant. Note that $\{X^c_m\}$ is an approximation of the tangent space of $\psi(M)$, which is invariant.

In order to establish local tubular neighborhoods with a uniform size, in the following we shall assume that the projections Π^α_m for $\alpha = c, u, s$ are Lipschitz in m and the manifold M does not 'twist' too much.

(H2) For any $m_0 \in M$, $m_1, m_2 \in B_c(m_0, r)$, $m_1 \neq m_2$, *and* $\alpha = c, u, s$,

$$||\Pi^\alpha_{m_1} - \Pi^\alpha_{m_2}|| \le L|\psi(m_1) - \psi(m_2)|$$

and

$$\frac{|\psi(m_1) - \psi(m_2) - \Pi^c_{m_0}(\psi(m_1) - \psi(m_2))|}{|\psi(m_1) - \psi(m_2)|} \le \epsilon_1 < 1,$$

where $1 \le L < \frac{\sqrt{2}-1}{r}$ *and* ϵ_1 *are constants.*

As an example, when M is a C^2 compact manifold imbedded in a Hilbert space, the hypothesis (H2) holds if Π^c_m is the orthogonal projection onto the tangent space of $\psi(M)$ and r is chosen to be small enough. When M is a C^1 manifold, this particular projection is only C^0 in m. However, if M is finite dimensional and has countable basis, one may construct a C^1 approximation of any C^0 family of projections. This issue is addressed in [7]. We do not know if such approximations exists for general infinite dimensional manifolds.

Since we do not assume that M is compact or finite dimensional, for technical reasons, we need to assume that T, Π^c_m, Π^u_m, and Π^s_m have some uniform properties:

(H3) (1) There is a constant $B > 0$ *such that* $||\Pi^\alpha_m|| \le B$ *for all* $m \in M$, *and* $\alpha = c, u, s$;

(2) There exists $\mu_0 > 0$ *such that for any* $m \in M$ *and* $\alpha = u, s$,

$$||\Lambda^\alpha_m|| \le \mu_0,$$

where $\Lambda^u_m \in L(X^c_m, X^u_m)$ *and* $\Lambda^s_m \in L(X^c_m, X^s_m)$ *are determined by*

$$(D\psi(m))(T_m M) = (I + \Lambda^u_m + \Lambda^s_m)X^c_m;$$

(3) For any $\eta > 0$*, there exists* $\epsilon > 0$*, such that for any* $x_1, x_2 \in B(\psi(M_1), \epsilon)$*,* $|x_1 - x_2| < \epsilon$*,*

$$||DT(x_1) - DT(x_2)|| \leq \eta;$$

(4) There are constants $a > 0$ *and* $B_1 > 0$ *such that*

$$\inf\{|\Pi^c_{m_1} DT(\psi(m_0))x^c| \;:\; x^c \in X^c_{m_0}, |x^c| = 1\} \geq a$$

and

$$||DT|_{B(\psi(M_1),r)}|| \leq B_1.$$

$$||\Pi^\alpha_m DT(\psi(m_1))|_{X^u_{m_1}}|| \leq B_1.$$

for $\alpha = c, s$*.*

Condition (2) implies that the space X^c_m is an approximation of the tangent space of $\psi(M)$ at m with an error bounded by μ_0. Condition (3) automatically holds when $\psi(M)$ is contained in a compact set. Roughly, it is an assumption on the uniform continuity of DT, but weaker than that. The reason for this assumption is that the graph transform is a global transform and some uniform estimates are needed.

We shall see that if X^c_m is a good approximation of the tangent space of $\psi(M)$, then we have the existence and the persistence of unstable manifolds of the overflowing manifolds.

In some cases, a persisting imbedded unstable manifold, instead of an immersed manifold, is desired. To obtain a persisting imbedded unstable manifold, we need not only the original overflowing manifold to be imbedded, but also the following condition:

(H2') For $m_1, m_2 \in \psi^{-1}(B(\psi(m_0), r))$*, all the assumptions in (H2) hold.*

Theorem 2.1 *Suppose* M *is an overflowing manifold for the map* T*. Assume that (H1)-(H3)are satisfied. Let* $\tilde{T} \in C^1(X, X)$*. When* $||\tilde{T} - T||_{C^1}$ *is sufficiently small on* $B(\psi(M_1), r)$*, there exists an unique overflowing* C^1 *manifold* $\tilde{W}^u$ *for* $\tilde{T}$ *in a neighborhood of* $\psi(M)$*, such that* $\tilde{W}^u$ *is along the tangent and unstable subspaces.*

$$\tilde{W}^u = h(M \times X^u(\epsilon))$$

where $X^u(\epsilon) = \{x^u \in X^u_m : |x^u| < \epsilon, m \in M\}$ *and* $h : M \times X^u \to X$ *a* C^1 *immersion. Under the* C^0 *norm of* h *and* $\tilde{T}$, h *is Lipschitz with respect to* $\tilde{T}$.

To get higher smoothness of W^u, we assume that the following "spectral gap" condition holds

$$\begin{aligned} &||\Pi^s_{m_1} DT(\psi(m_0))|_{X^s_{m_0}}|| \\ &< \lambda(\inf\{|\Pi^c_{m_1} DT(\psi(m_0))x^c| \ : \ x^c \in X^c_{m_0}, |x^c| = 1\})^i, \end{aligned}$$

for all $m_1 \in M$, $m_0 = u(m_1) \in M_1$, $i = 1, 2, \cdots, k$, and some $0 < \lambda < 1$.

The next result follows from [6]

Theorem 2.2 *When* $T \in C^k(X, X)$ *has uniformly bounded the i-th order derivatives* $i = 1, 2, \ldots k$, *the unstable manifold* W^u *is* C^k. *When* $T \in C^{k,1}(X, X)$, W^u *is also* $C^{k,1}$.

Our next theorem is on the unstable foliation of the unstable manifold W^u.

Theorem 2.3 *For small* $\epsilon > 0$ *there exists a unique family of* C^k *submanifolds* $\{W^{uu}_m(\epsilon) \ : \ m \in M\}$ *of* $W^u(\epsilon)$ *satisfying:*

(1) For each $m \in M, M \cap W^{uu}_m = \{m\}$ *and* W^{uu}_m *changes continuously with respect to* m.

(2) If m_1, $m_2 \in M$, $m_1 \neq m_2$, $W^{uu}_{m_1} \cap W^{uu}_{m_2} = \emptyset$ *and* $W^u = \cup_{m \in M} W^{uu}_m(\epsilon)$.

(3) For $m \in M$, $T : W^{uu}_m \cap T^{-1}(W^{uu}_{T(m)}) \to W^{uu}_{T(m)}$ *is a diffeomorphism.*

(4) For $x, y \in W^{uu}_m(\epsilon)$, *we have* $|T^{-n}(x) - T^{-n}(y)| \to 0$ *exponentially as* $n \to +\infty$.

(5) For $x \in W^{uu}_m, m \neq m_1$, *we have* $\frac{|T^{-n}(x) - T^{-n}(m)|}{|T^{-n}(x) - T^{-n}(m_1)|} \to 0$ *exponentially as* $n \to +\infty$.

We now consider the manifolds for semiflows. Let a C^1 map T and $\psi(M)$ satisfy (H1)-(H3). Let $\tilde{T} \in C([0, +\infty) \times X, X)$ be a semiflow, i.e.,

$$\tilde{T}^0 = I, \qquad \tilde{T}^{t+s} = \tilde{T}^t \circ \tilde{T}^s, \quad \text{for } t, s \geq 0.$$

We assume that for all $t \geq 0$, $\tilde{T}^t \in C^1(X, X)$. Suppose there exists $t_0 > 0$ such that $||\tilde{T}^{t_0} - T||_{C^1(B(\psi(M_1), r))} < \sigma$. From Theorem 2.1, when σ is sufficiently small, there exists a C^1 unstable manifold ^{u} for $\tilde{T}^{t_0}$.

Furthermore, we assume

(H4) For any $\eta > 0$, there exists $\zeta > 0$, such that for any $x \in B(\psi(M), r)$, $t \in [0, \zeta]$, we have

$$|\tilde{T}^t(x) - x| < \eta.$$

Thus, we have

Theorem 2.4 *The unstable manifold $\tilde{W}^u$ for $\tilde{T}^{t_0}$ is the unstable manifold for the semiflow $\tilde{T}^{t_0}$.*

From Theorem 2.3, there is an unstable foliation $W^u = \bigcup_{m \in M} W_m^{uu}$ for T^{t_0}. Furthermore, we have

Theorem 2.5 *$W^u(\epsilon) = \cup_{m \in M} W_m^{uu}(\epsilon)$ is an invariant foliation for the semi-flow T^t.*

The proof of Theorem 2.1 consists of four main steps, which are based on the Hadamard's graph transformation:

(a) Coordinate Systems. We first introduce three coordinate systems based on the splitting of the tangent bundle of the phase space X restricted to ψM, the normal bundle $X^u \oplus X^s$ and the local trivialization of the bundle, then establish the fundamental estimates relating these coordinate systems. Since ψ is not an imbedding, we can not construct the tubular neighborhoods as we did in [4]. However, since ψ does not locally twist the manifold M very much, we are able to establish local tubular neighborhoods based on $B_c(m, r)$ as we did in [6] and to obtain basic estimates. The result on persistence is obtained in the union all these local tubular neighborhoods. It may happen that some points in this set do not have globally unique representations, but this difficulty can be overcome.

(b) Invariant Cones. In this step, we establish the invariance of two types of moving cones as we did in [6]. The arguments there can be carried over here with little modification.

(c) Existence of Lipschitz Unstable Manifold. The method we use here is the same as the one we used in [4] and [6]. We regard $X^u \oplus X^s$ as a bundle over X^u and consider sections of this bundle in the tubular neighborhood. Define the complete metric space Γ^u of Lipschitz sections. Then we construct a "graph transform" F^u based on $\tilde{T}$, the perturbation of T. Using the invariance of the cones, one can prove that F^u is a contraction and the fixed point is the desired unstable manifold. One can also prove the continuous dependence of the unstable manifold with respect to the perturbation $\tilde{T}$. The proof requires many estimates.

(d) The Smoothness of the unstable manifold. The basic idea to show the smoothness is to find a candidate for the tangent bundle of the unstable manifold, which is invariant under the linearization $D\tilde{T}$, then to prove it indeed is tangent to the unstable manifold. The arguments are based on the idea of the Lipschitz jets which is borrowed from [17]. Since the trivialization of the normal bundle of $\mathcal{M}$ is not available in a Banach space, the proof is more complicated than for finite dimensional systems. We first define a space of sections of the Lipschitz jet bundle. Then we construct a graph transform based on the linearization $D\tilde{T}$ and show that it has a unique fixed point which gives the tangent space of the unstable manifold. A major difficulty for finding the fixed point is that the space of sections of the Lipschitz jet bundle is not complete. Finally, one can prove that the tangent bundle is C^0.

The proof of Theorem 2.3 is based on the Liapunov-Perron approach. This method was used in [10] to prove the existence of invariant foliations for semiflows when a global flat coordinate system is available. In our case here, there is not a global flat coordinate system on W^u or in a neighborhood of $\psi(M)$. However, each time we only construct a single fiber W_x^{uu}, $x \in \tilde{W}^u$. Since the backword orbits of the points on W_x^{uu} converge to each other, for $k > 0$, $T^{-k}(W_x^{uu})$ is in a small neighborhood so that the local Cartiasian coordinate system works. This is not the case when constructing the unstable manifold. The existence of W^u follows from the discrete version of the variation of constant formula for differential equations.

The complete proof of these results will appear in [7].

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