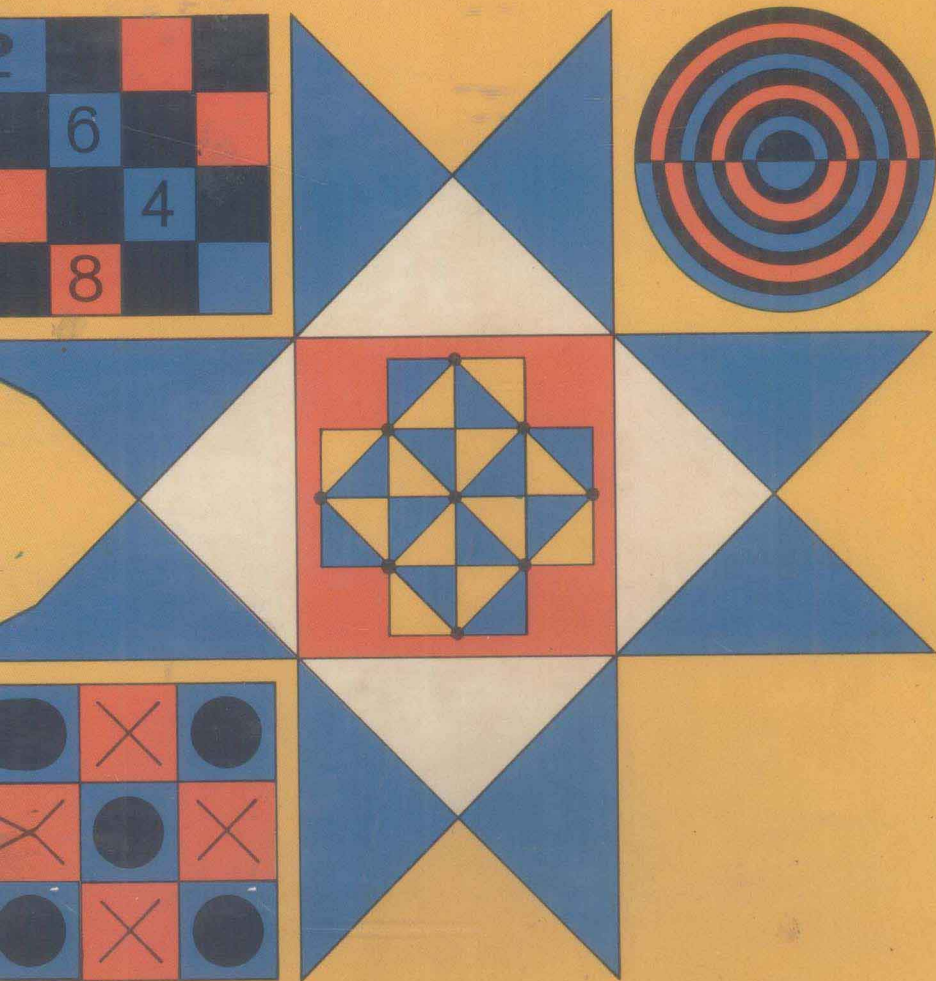


FRED. SCHUH

# THE MASTER BOOK OF MATHEMATICAL RECREATIONS



**the master book of  
MATHEMATICAL  
RECREATIONS**

BY

**FRED. SCHUH**

Translated by F. Göbel

*Translation edited by T. H. O'Beirne*

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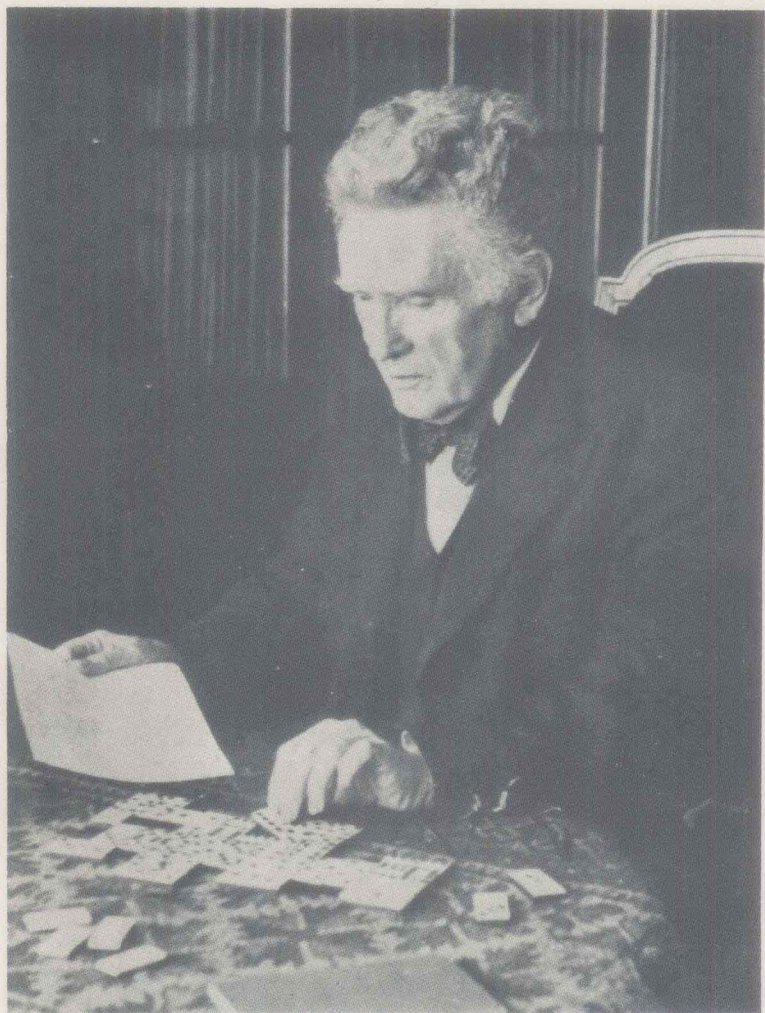
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*Fred Schuch*

## PREFACE

In this book I have endeavoured to show how pure puzzles (that is, puzzles which differ from crosswords, word-play riddles and the like, in that they are not limited to one or more specific languages) can be solved by systematic trial, with the maximum possible assistance from reasoning. This reasoning may often shorten the solving time considerably. Sometimes the reduction is due to the puzzle being put in a simpler form; we may invert it, for example, and solve it backwards. In many cases the solution of the puzzle is found by breaking it up into simpler puzzles; this is a very important strategy, which plays a significant role in mathematics, too. Much attention has been given not only to puzzles, but also to puzzle games (for two players) and their complete analysis. This topic is developed by various worked examples, which include the game of noughts and crosses (also known by the name of ticktacktoe).

Also, those parts of mathematics that are of importance for puzzles have been treated as simply as possible, avoiding algebraic formulae. In the front rank of these is the enumeration of possible cases, the so-called theory of permutations and combinations (§§125–130); I have tried to make this accessible even to non-mathematicians through a treatment based on examples. This theory is indispensable for the determination of the number of solutions for various puzzles—and frequently for the solution itself, too—while it also plays an important role in the preeminently interesting theory of probability. The basic theorems for calculating probabilities are discussed in detail and exhaustively, again without formulae, while numerous applications have been made, especially to games (§§131–164). I have also considered it a good idea to develop in a simple way (as with probability theory, giving historical details) the concept of a number system (§§78–88), because this, too, is applied in various puzzles and games, e.g., in the weight puzzle (§§89–91) and in the particularly enjoyable game of nim (§§113–123), which is played with piles of matches.

In Chapter XV, I give a short discussion of the basic ideas of mechanics. I have done this to be able to mention some simple-sounding, but really not so simple questions on the motion of objects,

among which are some questions that even a professional is likely to answer incorrectly. In addition, a few interesting phenomena of motion have been mentioned. Here I hope to have contributed toward dispelling the idea that mechanics has to be a dry subject.

Only here and there has use been made of algebraic formulae, and even then to a very modest extent. The sections in which this occurs have been marked with an asterisk; they can be omitted. Some sections in which the puzzles or the arguments are somewhat difficult have also been marked with an asterisk. So the reader should judge for himself whether he wishes to skip these sections or, perhaps, even give them special attention.

Many of the puzzles in this book and various of the puzzle games are original. However, all sorts of well-known puzzles and games have been treated as well, but even these have been elucidated in as original a way as possible.

The original puzzles that seem to me to be most successful include: numbers which are written with the same digits as one of their multiples (§§23–28);

the domino puzzles with the smallest and with the largest numbers of corners (§§37–39);

the multiplication sum with two digits 0, two digits 1, two digits 2, and so on (§§235–237);

the puzzle of sixteen numbers in a square with 24 prime sums (§§244–247);

the puzzle of the multiples of 7 with the maximum product (§§250–254);

the multiplication puzzle which involves the first line of the quotation on page vi (§255);

the repeating division puzzle (§258);

the puzzle with the eight dice (§§263 and 264);

the broken lines across sixteen dots in a square (§§274 and 275);

the road puzzle with concentric circles (§§289 and 292);

the counting-out puzzle 1–2–3 (§§301 and 304).

Among the original puzzle games I would like to mention the 5-subtraction game (§222); the modified subtraction game (§§225–228); the Game of the Dwarfs or “Catch the Giant!” (§§170–180), a simplified version of the soldiers’ game which, notwithstanding its extreme simplicity, is not easy at all; as likewise another game that is played on the same board and with the same pieces as the soldiers’ game (§191). I can also instance various games with piles of matches

(§§101–112) and an extension of the game of nim (§§122–123). I shall confine myself to these examples, although I could mention many other original puzzles.

I take this opportunity to express my thanks to my assistant, Mr. W. T. Bousché, who did most of the drawings, and to my pupil Mr. M. L. van Limborgh, who did some of the others; their work leaves nothing to be desired. To my former pupil Mr. J. Ploeg, M.E., thanks are due for some suggestions in the field of mechanics. I also thank my friends Mr. J. C. N. Graafland, M.E., Mr. J. Spanjersberg, M.A., and Mr. L. A. de Vries for proofreading; their suggestions have led to many an improvement. I am very grateful to the Publisher, who has taken pains to provide an attractive format.

FRED. SCHUH

*The Hague, Autumn 1943*

Est modus in rebus; sunt certi denique fines,  
Quos ultra citraque nequit consistere rectum.

*(There is a measure in things; and there are fixed limits  
beyond or short of which Right is unable to exist.)*

HORACE



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[Asterisks indicate sections that involve algebraic formulae.]

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