

# LOGIC

AND LOGICAL THINKING

A MODULAR APPROACH

Peter A. Facione/Donald Scherer

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Peter A. Facione/Donald Scherer

Bowling Green State University

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# PREFACE

As the main weapons against the ever-present threats to existence and to the quality of that existence, humans were given the abilities to reason, to acquire and transmit knowledge, to anticipate and plan for the future, and, in general, to think, reflect upon experience, and communicate with others. If we develop these abilities, we can use them to secure food, shelter, cures for disease, harmonious social structures, recreation, indeed almost anything. But if we fail to develop them, we guarantee our own more or less swift end.

This book has two main purposes, which go hand in hand: It aims to help you improve your powers of logic and it introduces logic as a field of study. These purposes, considered jointly, motivate the selection of material, the choice of examples, and the tone of the presentation. In this way the book develops practical skills in logical thinking and also provides the theoretical framework that explains why the skills can be successfully applied.

Almost every aspect of our collective and individual lives involves thinking or reasoning. All things being equal, the more logically you think, the better you can use the opportunities open to you. This book is an aid. It will alert you to the deceptiveness of fallacious arguments and help you to see the logical correctness of other arguments and the reasonableness of often-used proof strategies. Through examples and exercises it reinforces and sharpens your natural logical abilities. Use it to improve your critical thinking skills and, thereby, your chances of making good use of your opportunities.

As you study logic, you will learn what an argument is, what validity amounts to, what a fallacy is, and what indirect proofs and conditional proofs are. Most importantly, you will learn when an argument is worthy of acceptance. One of the most efficient and powerful tools that has been developed by twentieth-century logicians is the systematic use of symbolization. By uncovering the logical structure of arguments and representing these structures symbolically, logicians have devised reliable ways to test arguments for validity. Logic, like physics or medicine, is an ongoing study. Since Aristotle's time an ever larger class of arguments has become amenable to analysis through application of logic. We will look at both traditional logic and modern symbolic logic. We will study techniques and learn to apply them.

We wish to offer a special thanks to the thousands of students at Bowling Green State University who had a part in motivating this book, selecting its contents, and working through its exercises and its earlier editions. We also are most grateful to the many colleagues and graduate teaching assistants who did so much to help with

the selecting and testing of examples, checking of exercises, and organizing of materials over the years. We are especially grateful to Michael Bradie, James Stuart, and Paul Lastas for their helpful ideas and comments. We also thank all those many people who spent so many hours in the careful typing, reviewing, and proofreading of our manuscript, most especially S. Jack Odell and Pat Bressler. We dedicate this book to Charlotte and Noreen with love.

Peter A. Facione  
Donald Scherer

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# INTRODUCTION

## How to Use This Book

We designed this book for anyone who wishes to learn about logic and logical thinking. It aims at being self-contained so that it can be used in any type of classroom setting, traditional or innovative. You can study it totally on your own or with the guidance of an instructor. We try to present material in an intuitive and lucid way, using a generous number of examples, exercises, and explicitly stated decision procedures to aid you in both understanding and applying what you learn.

There are four major parts in this text. Part One explores the relationship between logic and language. We will explain what logic is all about, how logic relates to our everyday language and use of words, why it makes a difference how we define our terms, and what the strengths and weaknesses of various types of definitions are. We will also develop careful, systematic definitions for many of the central concepts of logic. Part Two develops skills normally associated with the study of formal logic. We will learn how to render English arguments into symbolic notation in order to show their logical structure more clearly. These skills pay off as we develop the ability to use mechanical decision procedures to examine arguments of propositional logic for validity. We will also learn techniques for testing syllogistic arguments and other arguments of predicate logic for validity.

Part Three explains illogical thinking in terms of mistakes in structure or content of arguments. These mistaken arguments, called fallacies, result from common human errors like those cataloged and explained in Part Three. Part Three tells us what makes illogical arguments illogical, and Part Four explains what makes logical arguments logical. In Part Four we discuss various kinds of direct and indirect proof strategies which can be used to build reasonable proofs. We will apply our discussion of proof strategies to expand both the deductive techniques of symbolic logic and the inductive techniques of modern research design. All four major parts end with comprehensive self-quizzes which you can use to measure your understanding of each.

In each section of the book you will find lists of instructional objectives. An instructional objective is, as you may know, a complex statement of a particular educational goal. These statements usually have three parts: (1) a specification of some goal activity, (2) a specification of the conditions under which the goal activity is to be achieved, and (3) a specification of the minimum level of achievement that will be accepted as a demonstration of mastery. The four parts of this text are divided into chapters on specific topics. Chapters, in turn, are broken into brief learning units called modules. Each section of the text (module, chapter, or part) is organized around its own specific instructional objectives. The larger the section, the broader

and more abstract are its educational goals; the smaller the section, the more specific are its objectives. The goals and objectives of each portion are explicitly stated. Use these statements as guides to what you should learn in that part, chapter, or module. Each module of the text begins with a precise statement of its objectives. Within each module the text, examples, and exercises are organized to develop and test precisely those objectives. You can use the exercises at the end of a module either to better learn the skills of that module through practice or to measure how well you have acquired those skills.

Exercises are the key to learning logic and logical thinking. Used texts that have exercises and answer spaces marked up are no bargain. What you may save in initial cost is not worth what you lose by not doing the exercises from scratch yourself. You should strive to meet the objectives of any given module after having read the module, studied its examples, worked its exercises, checked your answers against the ones provided, and done any review work suggested by the comments that appear by the exercise answers. Complete one module at a time, and meet its objectives before going on to later modules in the same chapter.

Although within a chapter the modules are to be studied in order, there are a large number of different ways to sequence the chapters. They can be organized to suit many different goals, abilities, interests, and time-frames. For example, if your interest is formal logic, you might wish to study Chapters 1, 3, 4, 11, 12, 13, 5, and 6 in that order. If you are more interested in developing your critical thinking skills in the area of informal logic, study Chapters 1, 2, 3, 7, 9, 10, 11, 12, and 14 in that order. If you prefer a more integrated study of both logic and logical thinking, work through the chapters in the order they are presented.\*

Just as each section of the text has its own instructional goal, there is an overall objective for the text itself. Let's assume you already have some initial intuitive ability to distinguish between acceptable arguments and unacceptable arguments. All of us make judgments of this kind every day. Some people are more *accurate* in doing so, which means they make the right evaluation more often than others do. Some people are more *efficient*, which means they can tell acceptable arguments from unacceptable arguments relatively quickly, that is, with less hesitation, doubt, and guesswork. After having worked through this text, you should be able to discriminate between acceptable and unacceptable arguments with at least 50 percent greater efficiency and at least 30 percent greater accuracy.† To help you achieve this goal the text provides discussions of all the relevant evaluative concepts of logic, schema for classifying acceptable arguments and unacceptable arguments, and explicit decision procedures that allow you to apply efficiently and accurately the evaluative concepts and theoretical explanations. The text is supplemented by examples all along the way, as well as a large number of exercises.

Please share with us any suggestions you might have for improving this text. We wish you well in your study.

\* Several alternative sequences are suggested in the Instructor's Manual.

† Pretests and posttests on logical thinking bear out these predictions. See Donald Scherer and Peter A. Facione, "A Pre/Post Test for Introductory Logic," *Metaphilosophy*, Summer 1977.

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# ONE

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## LOGIC AND LANGUAGE

Part One lays the foundation for the integrated study of logic and logical thinking. It clarifies what logic is and defines its main concern as the study of arguments. All the basic ideas necessary to develop your critical thinking skills through the study of logic are carefully presented here. Part One starts with common and familiar ideas about the issues: what is logic, what are arguments, how do you recognize an argument, how do you begin to evaluate arguments, and what is logical correctness? It then refines these ideas, using examples first and then presenting careful definitions, adding the technical precision and sophistication needed so that in later parts of the book practical skills can be both learned and understood. Not only does Part One present systematic and useful definitions for basic terms like 'statement,' 'argument,' 'induction,' 'deduction,' 'valid,' 'justified,' 'logically correct argument,' and 'sound argument,' but it also explains what definitions are and how they are used, and presents some virtues and vices of a variety of types of definitions.



# 1

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## WHAT LOGIC IS ALL ABOUT

Bertrand Russell is supposed to have said “People would rather die than think—and most do.” We are not sure whether his generalizations are true, but we agree with the cautionary note implicit in his remark. Each of us uses logic daily, sometimes in trivial and sometimes in momentous matters. We ask ourselves whether or not a particular set of views or beliefs warrants our concluding that certain other positions or ideas ought to be accepted as true. We might be concerned about nothing more important than the chances of our winning a hand of poker with two pairs. But we might be concerned about whether or not we can afford to buy a car or a house given our present financial condition.

Logicians concern themselves with much the same kind of thing but in more theoretical ways. They ask what *would be true if* we were to assume a certain set of facts or beliefs. They ask what *follows* from a given set of views or beliefs. Whereas we ask these things about the particular facts and beliefs that are the circumstances of our own lives, they ask these questions more abstractly. Their aim is to determine the procedures to use to tell what follows from any given set of premises, to tell what the *logical consequences of premises are*. Once they develop these procedures, they can be passed on to others. These procedures are tools we can use in judging the logical correctness of the particular inferences we deal with daily.

The educational goal of Chapter 1 is to learn what it means to say that logic is concerned with procedures for evaluating arguments. In Modules 1 and 2 we examine a variety of ideas that are more or less closely related to logic in order to clarify by examples the differences between these ideas and logic. Then we refine our ideas of logic by focusing on its chief concern, the process of evaluating arguments. In Modules 3 and 4 we begin again with common examples, trying to distinguish the logician’s technical meaning of ‘argument.’ We then learn how to identify the premises and conclusions of arguments, a skill which is a prerequisite for the evaluation of an argument as worthy or unworthy of acceptance.

As we learn what logic is all about, the relevance of logic to the concerns of our own lives will grow clearer. It will become more and more obvious that the ability to think logically and to analyze arguments logically is of tremendous practical importance.

# MODULE 1

## BEING LOGICAL

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If you were to teach someone to play golf, you would begin by giving them an easy golf club to use, teaching them the proper grip and swing, and allowing them to master the skill of hitting a golf ball with reasonable ease, distance, accuracy, and loft. As the person acquired that skill and others, like putting and hitting woods off tees and fairways, you would begin to concentrate more of your teaching on the rules of the game, the strategies of club selection and ball placement, and definitions of key terms. Of course you would also teach more advanced skills like how to punch a ball low under branches, chip out of a sand trap, or deliberately hit a slice or hook to avoid natural hazards like large trees or roughs. Learning logic and logical thinking is a lot like learning golf. There are basic skills to master; there are strategies of proof, rules for the proper use of techniques, and definitions of key terms. There are also more advanced skills, which should be easy to learn later if the fundamental skills are mastered properly. At the outset of our study you are like the beginning golfer, concerned with what golf clubs and a golf ball look like and how they are held and used. We are concerned with what being logical is. Our initial strategy for finding out will be to distinguish instances of logical thinking from examples of related but different things like being sharp, persuasive, emotional, or reasonable. If we can manage even a rough understanding of what being logical is from the examples in Module 1, we can then go on in Module 2 to formulate an idea about what logic is.

● After reading Module 1 you should be able to:

1. Distinguish a concern for trying to be logical from similar yet different human concerns like trying to be reasonable, sharp, emotional, or persuasive, given a brief passage expressing one of these concerns

**1:1\* Key words** What is it like to be a really logical person? Is that different from being a reasonable person? How about being sharp? How do all these differ from being an emotional person or, again, from being persuasive?

There are answers to these questions. They may not be hard and fast, definite answers, but they are good answers. **There are answers because each of the key terms 'logical,' 'reasonable,' 'sharp,' 'smart,' etc., has a slightly different meaning in our language.** If you know how to speak English, you know that saying "Star Trek's Mr. Spock and Agatha Christie's fictional detective Hercule Poirot are logical" is different from saying "they are reasonable" or "they are persuasive."

Often when trying to get a clear idea about an abstract term, it is useful to begin by contrasting that term's conventional use with the ordinary uses of terms with similar meanings. If we wanted to clarify our idea of what 'knowledge' means, we would begin by reflecting on and contrasting the meanings of statements like "I know that the bread should be baked at 194°C," "It's my opinion that we should bake the bread at 194°C," "I think that bread is to be baked at 194°," and "I've heard

\* Throughout the text we number each section so that you can find specific discussions on various topics easily. Our coding is to list the module number first and then the section number, separating the two by a colon. So 8:4 would mean Module 8, section 4; 8:5.1 would mean Module 8, section 5, sub-section 1.

that . . .” We may not end up with a precise definition of knowledge this way, but we will come to understand better what it does mean because we will have a better idea of how the concept of knowledge contrasts with similar ideas—like ‘belief,’ for instance. Of course, much more can be said about ‘knowing’ and ‘believing,’ but more advanced philosophical treatments usually are not undertaken unless our original purposes call for highly technical and refined definitions. Since our purposes involve learning logic and how to think logically, we do need to be able to contrast the ability of being logical from similar but different abilities. We do not, at least for now, require a highly refined definition.

**1:2 Two puzzles** To help clarify the differences between being reasonable, being sharp, being logical, being emotional, and being persuasive we will look at two puzzles. After stating the puzzle we will shift our attention first to the skills (being logical, reasonable, or sharp) required for solving the puzzles and second to the qualities (being logical, persuasive, emotional) exhibited by some of the characters in the puzzles. As we describe these skills and characteristics, you should reflect on and contrast the meanings of words like ‘logical,’ ‘reasonable,’ ‘sharp,’ ‘persuasive’ in our descriptive statements. If you want to enjoy the puzzles for their own sakes, you can cover up our discussions of the solutions and have some fun solving them for yourself.

**Puzzle 1:**

The football coach at Whatsamatta U. has a problem. He has a twelve-member coed football squad. When they are dressed for a game in pads and uniforms, they all look exactly alike in both size and weight. The only way he can tell one from another is by the numbers on the back of each player’s uniform. The coach knows that eleven of the players really are identical in weight but one happens to be different. The coach wants to know two things: which one is different and whether this odd one is lighter or heavier than the others. Not only would it be viewed as rude if he should just come out and ask their respective weights, but also he is afraid they might lie. After all, they know that he plans to bench the odd person if the odd person turns out to be lighter and use the odd person if he or she turns out to be heavier. This coach is reasonable and knows that there is more than one way to weigh a football player. He is also rather resourceful and has borrowed the bronze seesaw that the Psychology Department owns. He takes his team over to the seesaw on the pretense of doing a new exercise to build up their ability to balance while in motion. To further hide his scheme he tells them that none of them is allowed to ride the seesaw more than three times altogether. Now that he has the team ready and his real intentions hidden, can you figure out how he should place his players on the seesaw to solve his problems: Who is the odd one, and is he or she lighter or heavier?

The reasonable thing to do is to first realize that there are two ways to find a solution: (1) puzzle it out or (2) read the next few paragraphs. If you decide to puzzle it out, the next reasonable thing is to note that you need a strategy. Recall that in a puzzle there is often more information given than at first seems evident, so the first step in your strategy is to reread the puzzle. If you are not sharp, though, you may have to give up trying to figure it out. To be able to figure it out means being able to hit on the right strategy and also to execute the strategy once it is discovered. But finding the winning strategy for puzzle solving (as for any game or sport) is usually more a matter of insight, coaching, practice, and experience with puzzles than simply a matter of natural mental ability.



**Solution 1:**

A sharp person might have hypothesized that the key problem could be solved if the coach could find a standard-weight player. Using a standard-weight player on one side of the seesaw, the coach could put each of the other players on the other side one at a time. In this way the coach could compare each player to the original standard weight player.

A logical person could take it from here and draw out the consequences of this hypothesis, testing it to see whether it solves the coach's problems without violating any of the restrictions imposed by the puzzle.

**Solution 1 continued:**

The logical person would begin to test the hypothesis by figuring out what could happen with a standard-weight person on one side and a person of unknown weight on the other. If the seesaw remains level, the new person is also a standard-weight person. If it tilts, then the new person is the odd one. If the odd one's side went up, then the person is lighter than the others; if it went down, heavier. The logical consequence, then, is that the proposed solution does solve the coach's problem. Unfortunately there are two further difficulties. The first remaining difficulty is how to get the process started, i.e., how to find the original standard-weight person.

The second difficulty arises if we recall that each player is restricted to no more than three weighings on the seesaw. The logical consequence of this is that if our odd player does not show up early in the process, we will have to find a substitute for our original standard-weight player.

A reasonable person would see a number of ways of handling the first problem. Perhaps the most obvious one is simply to start by picking any two players at random and sitting them one on each side of the seesaw. Logically, if it remains level they are both standard-weight people and so either could serve as the original standard-weight person to get the process of comparison going. The other person, as well as anyone who turns out to be equal in weight to the original, can serve as a substitute if more than three turns seem to be needed. So if the seesaw remains level, logic has solved both remaining problems. If the seesaw moves, on the other hand, both problems are still solved. If it moves, one of the people on it is odd. This means that the other ten are all standard weight. Any one of them could be selected to start the comparison procedure, the remaining nine standing in reserve as substitutes if needed.

Can we infer anything else from the fact that the seesaw moved? Yes. We can infer that the following two statements are true: (1) If the odd person is the one on the side that went up, the odd person is lighter than the others; (2) if the odd person is the one on the side that went down, the odd person is heavier than the others. From these facts a sharp person might hypothesize that in this case only one more use of the seesaw is really needed. Test that idea out using your powers of logic. Put one of the other ten players on the seesaw in place of the person on the down side. If the seesaw returns to level, then the person who just came off the down side is heavier than the other eleven players. If the seesaw does not move but remains tilted, the person still up in the air is lighter.

We can generalize what we have observed so far. Reasonableness shows us our choices or options and perhaps leads us to rule out obviously wrong or silly possibilities. Sharpness discovers or suggests the hypotheses, the ideas, we should test as possible solutions to our problem. Logicalness tests them: it draws out their consequences, good or bad.