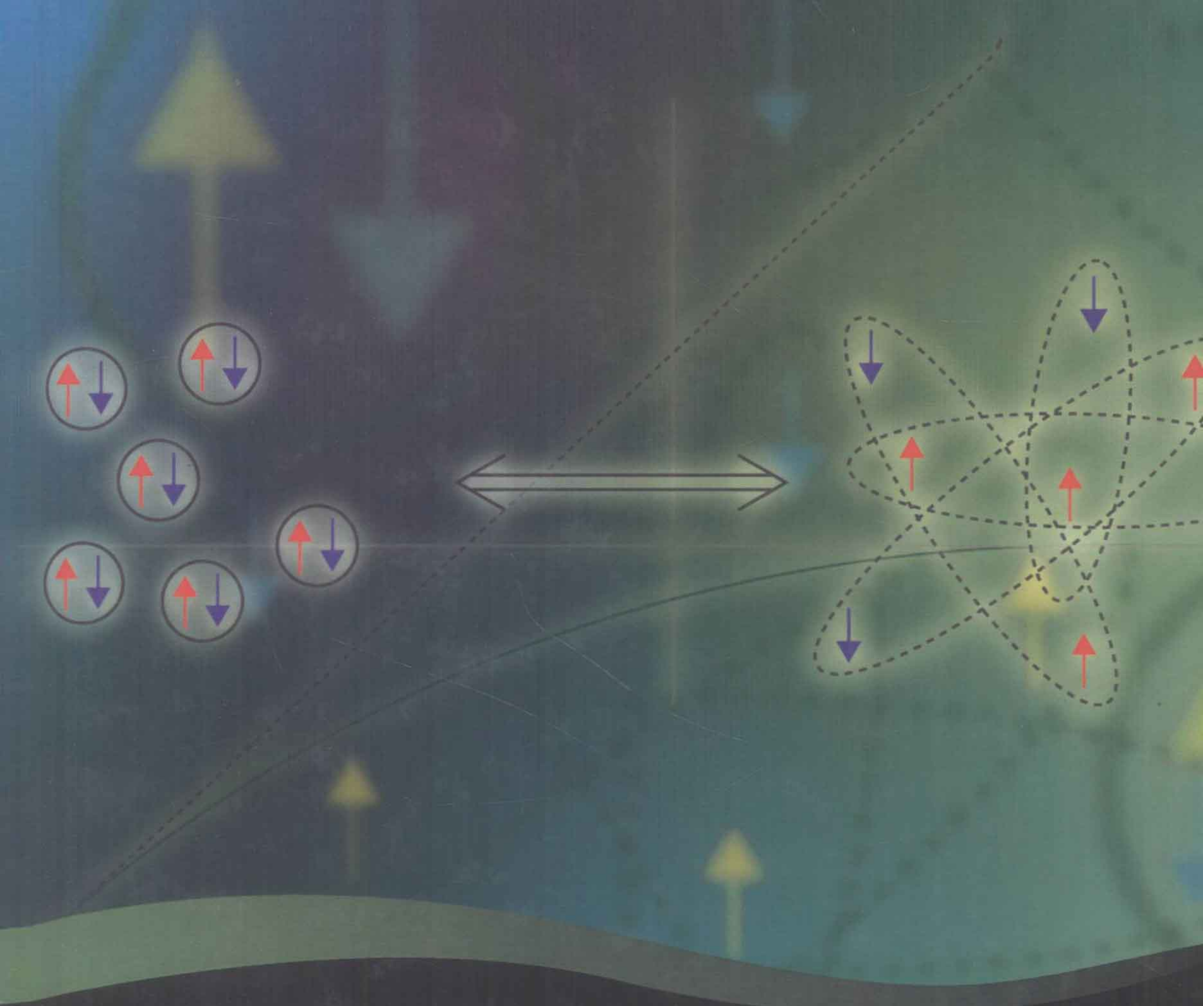

Quantum Liquids

Bose Condensation and Cooper Pairing in Condensed-Matter Systems

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Preface

When the father of low temperature physics, Heike Kamerlingh Onnes, received the Nobel prize in 1913 for his liquefaction of helium, he concluded his acceptance speech by expressing the hope that progress in cryogenics would "contribute towards lifting the veil which thermal motion at normal temperature spreads over the inner world of atoms and electrons." Speaking only months after Bohr's publication of his atomic model, and long before the advent of modern quantum mechanics, Onnes could not have guessed how prophetic his words would prove. For of all the novelties revealed by the quest for low temperatures, by far the most dramatic have been the phenomena which result from the application of quantum mechanics to systems of many particles – in a word, of quantum statistics. And of these phenomena in turn, the most spectacular by far are those associated with the generic phenomenon which is known, when it occurs in a degenerate system of bosons, as Bose–Einstein condensation, or in a system of degenerate fermions as Cooper pairing. In this phenomenon (which for brevity I shall refer to generically in this Preface as "quantum condensation"), a macroscopic number either of single particles or of pairs of particles – a fraction of order one of the whole – are constrained to behave in exactly the same way, like a well-drilled platoon of soldiers (see the cover of *Science*, December 22, 1995). While the best-known consequence of quantum condensation is superfluidity (in a neutral system like ^{4}He) or superconductivity (in a charged one such as the electrons in metals), this is actually only a special case of a much more general pattern of behavior which has many other spectacular manifestations.

There are many good books on specific quantum condensates (liquid ^{4}He, the alkali Bose gases, superconductors, liquid ^{3}He, etc.); I list a selection below. The present book is in no sense intended as a substitute for these more specialized texts; rather, by giving an overview of the whole range of terrestrial condensates and their characteristic behaviors in what I hope are relatively simple and understandable terms, I aim to put the individual systems in context and motivate the reader to study some of them further.

This book is born of the conviction that it ought to be possible to present the essentials of Bose–Einstein condensation and Cooper pairing, and their principal consequences, without invoking advanced formal techniques, but at the same time without asking the reader to take anything on trust. Thus, the most advanced technique I have introduced is the language of second quantization, which for those not already practised in it is reviewed in a self-contained fashion in Appendix 2A. (However, most of Chapters 1–4 can actually be read even without fluency in the second-quantization language.) While this policy has the drawback of precluding me from introducing the Bogoliubov–de Gennes equations for superfluid Fermi systems (a technique which certainly needs to be learned, eventually, by anyone intending to do serious theoretical research on such systems), I hope it will mean that the book is relatively easily readable by, for example, beginning graduate students in theory or by experimentalists who do not wish to invest the time and effort to cope with more advanced formalism.

It will become clear to the reader from an early stage that I have at least two rather strong convictions about the theory of quantum condensation which are not necessarily shared by a majority of the relevant theoretical community. The first is that it is neither necessary nor desirable to introduce the idea of "spontaneously broken $U(1)$ symmetry," that is to consider (alleged) quantum superpositions of states containing different total numbers of particles; rather, I take from the start the viewpoint first enunciated explicitly by C.N. Yang, namely that one should simply think, in nontechnical terms, about the behavior of single particles, or pairs of particles, averaged over the behavior of all the others, or more technically about the one- or two-particle density matrix. At the risk of possibly seeming a bit obsessive about this, I have tried to derive all the standard results not only for Bose but for Fermi systems using this picture; the idea of spontaneous $U(1)$ symmetry breaking is mentioned only to make contact with the bulk of the literature.[1] My second strong conviction is that many existing texts on superconductivity and/or superfluidity do not adequately emphasize the distinction, which to my mind is absolutely crucial, between the equilibrium phenomenon which in the context of a neutral superfluid is known as nonclassical rotational inertia (or the Hess–Fairbank effect) and in a charged system underlies the Meissner effect, and the metastable phenomenon of persistent currents in a neutral or charged system; again, I have tried to place some emphasis on this, see in particular Chapter 1, Section 1.5.

After two introductory chapters on the general phenomenon of quantum condensation, in the remaining chapters I treat in order liquid ^{4}He (Chapter 3), the alkali Bose gases (Chapter 4), "classic" (BCS) superconductivity (Chapter 5), superfluid ^{3}He (Chapter 6), the cuprate superconductors (Chapter 7), and finally, in Chapter 8, a miscellany of mostly recently realized quantum-condensed systems; in particular, Section 8.4 deals with very recent experiments which have made the long-conjectured "crossover" from Bose condensation to Cooper pairing a reality. The flavor of the various chapters is rather diverse, reflecting differences in the history and current status of our understanding of these systems; in particular, while in Chapters 3–6 I try to provide a reasonable theoretical basis for understanding the phenomena described, in Chapters 7 and 8 the treatment is much more descriptive and cautious. It will be noted that in Chapter 5 I have said essentially nothing about a topic which is a major part of most textbooks, namely the way in which properties associated with the normal phase (ultrasound absorption, tunnelling, spin susceptibility, etc.) are modified in the superconducting phase; this is a deliberate policy, so as not to distract attention from the central topic of condensation. Whenever possible, I have tried to provide derivations of standard results which differ somewhat from the conventional ones. While some of these alternative arguments (e.g. that given in Section 5.7 for the Ginzburg–Landau

[1] And in Appendix 5C as a purely formal device to streamline an otherwise cumbrous algebraic step.

formulation of superconductivity theory) may be less rigorous than the standard derivations, I hope they may complement the latter by giving a more physical picture of what is going on.

Two further points of general policy are that in order to keep the focus in the text on the main line of the argument, I have tried wherever possible to relegate cumbrous mathematical derivations to appendices; and that I have for the most part not attempted to trace the detailed history of the various theoretical ideas I discuss. (Any colleagues who feel thereby slighted might want to note that even the original BCS paper on superconductivity does not appear in the list of references!) Generally, I have given specific theoretical references only at points where the discussion in the text needs to be supplemented.

The original delivery date for the manuscript of this book was in the summer of 2004, but unforeseen events forced a postponement of 18 months. This was serendipitous, in the sense that during that period the topics covered in Sections 8.3 and 8.4 of Chapter 8 have undergone explosive experimental expansion, but the down side is that in some other chapters which were written earlier, in particular Chapter 4, the coverage of the most recent developments is not always complete.

It would be pointless to list all the good books which exist on individual quantum-condensed systems. For what it is worth, here are some which I feel readers of this book might find natural further reading:

- On helium-4: Pines and Nozières (1966). Wilks (1967) is a very useful general compendium.
- On the alkali Bose gases: Pethick and Smith (2002) and Pitaevskii and Stringari (2003).
- On "classical" superconductivity: De Gennes (1966) and Tinkham (1975, revised 1996). These two are as useful today as when they were first published.
- On superfluid ^{3}He: Vollhardt and Wölfle (1990) is a very good compendium, but hardly bedtime reading. The more general texts by Tsuneto (1998) and Annett (2004) have useful chapters.
- On cuprate superconductivity: see comments in Chapter 7.

I would like to thank the many colleagues, at UIUC and elsewhere, who have helped my understanding of the various systems treated in this book. Particular thanks are due to Lance Cooper, Russ Giannetta, Laura Greene, Myron Salamon and Charlie Slichter for their comments on a first draft of Chapter 7, Sections 7.3–7.7 and to Man-Hong Yung for proof-reading parts of the manuscript; needless to say, the responsibility for any remaining deficiencies is entirely mine. I am also very grateful to Linda Thorman and Adam D. Smith for their sterling efforts in typing the manuscript against a tight deadline. The writing of this book was supported by the National Science Foundation under grants nos. DMR-99-86199, DMR-03-50842 and PHY-99-07949.

Finally, I am very conscious that there are a number of points in this book where, owing in part to publishers' deadlines, I have not been able to spend all the time and thought that I would ideally have liked.[2] There is no doubt also the usual

[2] This is particularly true of the last few paragraphs of Chapter 5, Section 5.7 and Chapter 7, Section 7.5 and Appendix 7B.

crop of hopefully minor mistakes, typographical errors, etc. In the hope that the book may at some time in the future merit a revised edition, I shall be very grateful to any readers who bring such deficiencies to my attention.

Urbana, IL
30 January 2006

List of Symbols (not defined)

$\|\uparrow>$	state $\sigma = +1$
$\|\downarrow>$	state $\sigma = -1$
c_V	specific heat
e	electron charge
g	gravitational acceleration [also interchannel coupling constant, pp. 159–64]
H	Hamiltonian
h	Planck's constant
$\hbar$	Dirac's constant ($\equiv h/2\pi$)
$\boldsymbol{J}(\boldsymbol{r})$	current density
$\boldsymbol{k}$	wave vector
k_B	Boltzmann's constant
m	particle mass
n	particle density
P	pressure
t	time
T_1	nuclear spin relaxation time
$\delta n(p\sigma)$	deviation of (quasi)particle occupation number from normal-state value
λ	optical wavelength [also dimensionless ratio Na_s/a_{2p}, pp. 127–28: relative channel weight, pp. 163–64]
μ	micron (10^{-6}m)
μ_B	Bohr magneton
μ_o	permeability of free space
σ	conductivity
χ	magnetic susceptibility
ω	angular frequency

List of Symbols*

Symbol	Meaning	Page where defined/ introduced
$[A, B]$	commutator $AB - BA$	64
$\{A, B\}$	anti-commutator $AB + BA$	66
$\vert 00>$	doubly-unoccupied state of $(\mathbf{k}\uparrow, -\mathbf{k}\downarrow)$	244
$\vert 11>$	doubly-occupied state of $(\mathbf{k}\uparrow, -\mathbf{k}\downarrow)$	244
$^4He^*$	excited state of ^4He atom	20
A	hyperfine interaction constant	114
$\mathbf{A}(\mathbf{r})$	electromagnetic vector potential	25
$\hat{a}$	Bose/Fermi annihilation operator	64
$\hat{a}^\dagger$	Bose/Fermi creation operator	64
a_b	bound-state radius	158
a_{bg}	background scattering length	160
a_s	s-wave scattering length	118
a_{zp}	width of single-particle groundstate in harmonic trap	127
B_{1g}	group-theoretic notation for $d_{x^2-y^2}$ symmetry	321
B_{c1}	lower critical field (of superconductor)	197
B_{c2}	upper critical field (of superconductor)	198
B_{hf}	characteristic hyperfine field	7
c_k	coefficient of pair wave function	105, 175, 186
c_s	speed of (hydrodynamic) sound	98
$\boldsymbol{d}$	constant value of $\mathbf{d}(\hat{\mathbf{n}})$ in ^3He-A	260
$\hat{\mathbf{d}}(\hat{\mathbf{k}})$ (or $\hat{\mathbf{d}}(\hat{\mathbf{n}})$)	d-vector notation for spin triplet OP	260
$dn/d\varepsilon$	density of states of both spins at Fermi surface of metal	166
$d_{x^2-y^2}$	most popular symmetry of cuprate order parameter	321
E_{BP}	broken-pair energy	186
E_{EP}	excited-pair energy	186

*Symbols that are used only close to their definition are not included in the list below; note that some of these duplicate symbols listed here.

Symbol	Meaning	Page where defined/ introduced
E_{GP}	ground-pair energy	186
E_J	Josephson coupling energy	227
E_k	BCS excitation energy, $\equiv (\varepsilon_k^2 + \|\Delta_k\|^2)^{1/2}$	181
E_R	recoil energy	146
f	filling fraction	149
F	Helmholtz free energy	104
F	Helmholtz free energy density	104
F	total atomic spin	7
$f(\mathbf{p}\mathbf{p}'\tau\sigma')$	Landau interaction function	230
$F(\mathbf{r}\,\sigma\,\mathbf{r}'\sigma' : t)$	order parameter in Fermi systems	51
$f(r_{ij})$	two-particle ingredient of Jastrow function	109
F_k	Fourier transform of Cooper-pair wave function $F(\boldsymbol{\rho}, \mathbf{R})$ with respect to relative coordinate $\boldsymbol{\rho}$	179
F_n	coefficient in expansion of Cooper-pair wave function	217
$f_n(T)$	normal fraction	24
$f_s(T)$	superfluid franction	23
$F^{s,a}$	dimensionless Landau parameters	231
$F_{\alpha\beta}(\mathbf{k})$	matrix form of Cooper pair wave function	258
g_D	nuclear dipolar coupling constant	266
H_D	nuclear dipole energy	265
I_c	critical current of Josephson junction	227
I_{cl}	classical moment of inertia	22
K	total dimer/molecule intrinsic angular momentum	7
k_F	Fermi wave vector	365
k_{FT}	$\equiv q_{TF}$	240
K_{mn}	matrix elements of time-reversal operator	220
ℓ	mean free path	215
$\hat{\boldsymbol{\ell}}$	direction of apparent relative angular momentum of pairs in ^{3}He-A	273
L	orbital angular momentum	22
$m*$	effective mass	231
m_F	projection of total atomic spin on z-axis	115
$\bar{n}$	time reverse of state n	213
N(0)	density of states of one spin at Fermi surface ($\equiv \frac{1}{2}(dn/d\varepsilon)$)	176
N_o	condensate number	12, 34
p	doping (of cuprates) (= no. of holes per CuO_2 unit)	290
p_F	Fermi momentum	229
q_{TF}	Thomas-Fermi wave vector	171
R	center-of-mass coordinate	49

Symbol	Meaning	Page where defined/ introduced
R_N	normal state resistance (of Josephson junction)	228
r_o	range of interatomic potential (= van der Waals length)	118
$S(k)$	static structure factor	98
$S(\boldsymbol{r}, t)$	entropy density	83
t	tunneling matrix element [also time, throughout]	147
$T^*(p)$	crossover line (in phase diagram of cuprates)	292
T_c	critical temperature	12
T_F	Fermi temperature	11, 166
T_λ	lambda-temperature of liquid ^4He	72
$U(\mathbf{r})$	external potential	25
$u_\mathbf{k}$	parameter in BCS wave function	244
U_o	coefficient of δ-function in interparticle potential	41
V	visibility (of interference fringes)	137
$V(r)$	external potential	18
$V(r_i - r_j)$	interparticle potential	47
v_c	Landau critical velocity	101
v_F	Fermi velocity	167
$v_\mathbf{k}$	parameter in BCS wave function	244
$V_{\mathbf{kk}'}$	pairing interaction (in BCS problem)	180
V_o	coefficient of BCS contact interaction	174
V_o	height of optical-lattice potential	146
$\mathbf{v}_s(\mathbf{r}t)$	superfluid velocity	35, 191
$\mathbf{v}_s(\boldsymbol{R}t)$	superfluid velocity (in Fermi systems)	52
$Y(T)$	Yosida function	206
Z	atomic number	170
β	$1/k_BT$	9
γ	coefficient of linear term in specific heat (of normal Fermi system)	296
Γ_K	time-reversal-breaking parameter	219
δ	control parameter for interatomic potential (relative to position of resonance)	120
δ_c	characteristic value of δ	123, 162
Δ	constant value of Δ_k	182
Δ	detuning of laser	116
$\Delta_\mathbf{k}$	BCS gap parameter	181
$\Delta_{\mathbf{k},\alpha\beta}$	matrix form of gap in spin space	259
ΔN	number imbalance (relative number)	69, 225
$\Delta\phi$	relative phase	44
$\varepsilon(\omega)$	dielectric constant	301
ε_1	real part of dielectric constant	299
ε_2	imaginary part of dielectric constant	299
ε_c	cutoff energy in BCS model	175

Symbol	Meaning	Page where defined/ introduced
ε_F	Fermi energy	10
$\varepsilon_{\mathbf{k}}$	kinetic energy relative to Fermi energy [also used earlier for absolute value of kinetic energy]	175
ζ	$-(k_F a_S(\delta))^{-1}$ [also diluteness parameter $(na^3{}_S)^{1/2}$ in dilute Bose gas, pp. 132–4]	367
ζ	depletion parameter	106
θ_D	Debye temperature	2
κ	circulation of vortex line/ring [also GL parameter, 203: bulk modulus, 238]	91
κ_o	quantum of vorticity h/m [also bulk modulus, 171–5]	95
$\lambda_{ab}(T)$	ab-plane penetration depth (of cuprates)	305
$\lambda_c(T)$	c-axis penetration depth (of cuprates)	307
λ_L	London penetration depth	25
λ_T	thermal de Broglie wavelength	118
μ	chemical potential	9
μ_n	nuclear magnetic moment	15
ξ'	order of magnitude of Cooper pair radius	184
$\xi(T)$	healing length	127, 201
$\xi_{ab}(T)$	in-plane Ginzburg-Landau healing length	324
$\xi_c(T)$	c-axis Ginzburg-Landau healing length	316
ξ_k	absolute value of kinetic energy	372
ξ_o	Cooper pair radius	190
$\boldsymbol{\rho}$	relative coordinate	49
$\rho(E)$	density of single-particle states per unit energy	10
ρ (r)	single-particle density	44
$\rho(\boldsymbol{R}, \boldsymbol{R}' : \beta)$	many-body density matrix (in coordinate representation)	110
$\rho_1(\mathbf{rr}'t)$	single-particle density matrix	32
$\rho_2(\boldsymbol{r}_1\sigma_1\boldsymbol{r}_2\sigma_2 : \boldsymbol{r}'_1\sigma'_1 \boldsymbol{r}'_2\sigma'_2 : t)$	2-particle density matrix	47
$\rho_n(T)$	normal density	78
$\rho_s(T)$	superfluid density	78
σ	spin projection (in units of $\hbar/2$)	47
τ	relaxation time	167, 234, 297
$\phi(r_1r_2\sigma_1\sigma_2)$	"pseudo-molecular" wave function in BCS problem	178
$\phi(\mathbf{r}t)$	phase of condensate wave function (or order parameter)	35

Symbol	**Meaning**	**Page where defined/ introduced**
Φ_o	(superconducting) flux quantum, $\equiv h/2e$	196
$\chi_0(\mathbf{r}t)$	condensate eigenfunction/wave function	34, 125
$\chi_o(q\omega)$	bare response function	171
$\hat{\psi}(\mathbf{r}t)$	Bose/Fermi field operator	67
$\psi_\sigma^\dagger(\mathbf{r}t)$	Fermi creation operator	67
$\Psi(\mathbf{R}, t)$ [or $\Psi(\mathbf{r}, t)$]	Ginzburg-Landau order parameter	52
$\Psi(\mathbf{r}_1\mathbf{r}_2\ldots\mathbf{r}_N : t)$	many-body wave function	31
$\Psi(\mathbf{r}t)$	order parameter	34
ω	angular velocity	22
$\boldsymbol{\omega}(\mathbf{r}t)$	vorticity	91
ω_c	quantum unit of angular velocity	22
ω_L	Larmor frequency	266
ω_o	frequency of harmonic trap	19
ω_p	(electron) plasma frequency	167
$\hat{\boldsymbol{\omega}}$	spin-orbit rotation axis (in superfluid ^{3}He)	268
Ω_p	ionic plasma frequency	238

Contents

1 Quantum liquids

A "quantum liquid" is, by definition, a many-particle system in whose behavior not only the effects of quantum *mechanics*, but also those of quantum *statistics*, are important. Let us examine the conditions for this to be the case.

Needless to say, if we wish to describe the actual structure of atoms or molecules, then under just about any conditions known on Earth it is essential to use quantum mechanics; a classical description fails to account for even the qualitative properties. However, if we consider the atoms or molecules as themselves simple entities and ask about their dynamics or thermodynamics, we find that classical mechanics is often quite a good approximation. A qualitative explanation of why this should be so goes as follows: The fundamental novelty introduced by the quantum-mechanical description in the motion of particles is that it is necessary to ascribe to the particle wave-like attributes, resulting in phenomena such as interference and diffraction; the quantitative relation between the "wave" and "particle" aspects is given by the de Broglie relation

$$\lambda = h/p \tag{1.0.1}$$

where p is the momentum of the particle and λ the wavelength of the associated wave. However, we know from classical optics that a wave will behave very much like a stream of particles ("physical optics" becomes "geometrical optics") if the wavelength λ is small compared to the characteristic dimension d of whatever is obstructing it ("one cannot see around doors"); the condition to see wave-like effects is, crudely speaking

$$\lambda \gtrsim d \tag{1.0.2}$$

In the case of a many-particle system it is perhaps not immediately clear what we should identify as the length d, but for reasonably closely packed systems, at least, it seems reasonable to take it to be of the order of the interparticle distance, i.e. as $n^{-1/3}$ where n is the density (though see below). On the other hand, the typical value of λ is determined, according to Eqn. (1.0.1), by that of the momentum p, which in thermal equilibrium is determined by the mean thermal energy $k_{\mathrm{B}}T$:

$$p \sim (mk_{\mathrm{B}}T)^{1/2} \tag{1.0.3}$$

Combining Eqns. (1.0.1)–(1.0.3), we find that the conditions for quantum mechanical effects to be important in the (center-of-mass) motion of a set of atoms or molecules is roughly

$$k_{\mathrm{B}}T \lesssim n^{2/3}\hbar^2/m \tag{1.0.4}$$

where m is the mass of the atom or molecule in question.