World Scientific – Nobel Laureate Series: Vol 2

WILLIAM F. SHARPE

Selected Works

edited by

WILLIAM F. SHARPE

World Scientific

World Scientific – Nobel Laureate Series: Vol 2

WILLIAM F. SHARPE

Selected Works

edited by

WILLIAM F. SHARPE

Stanford University, USA

World Scientific

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

World Scientific — Nobel Laureate Series: Vol. 2
WILLIAM F SHARPE
Selected Works

ISBN-13 978-981-4329-95-8
ISBN-10 981-4329-95-9

Printed in Singapore by B & Jo Enterprise Pte Ltd

INTRODUCTION

When World Scientific Publishing first asked me if I would be willing to prepare a volume of selected works, I was reluctant to agree. Like most researchers, I am more interested in new ideas and results than old ones. But over the course of time I was persuaded that some people might find such a volume useful. Hence this book.

My first task was to select a subset of the work that I have done over many years. The second was to organize the papers in some helpful fashion. This required a number of iterations and the result may or may not be the best possible. Assuming that readers will have varied backgrounds and interests, I have tried to find a reasonable middle ground. Given limited space, I have concentrated on papers in my main lines of research, leaving out scattered work in computer economics, algorithms, programming languages, operations research and banking.

I have grouped the material in six broad subject areas. Within each of the resulting sections, the papers are in chronological order. I have also included comments about each of the papers. Some of these include information about the circumstances surrounding the preparation of the material and connections with other parts of my work and that of others. The readers indulgence is begged for the frequent references to the first person. Please do not assume that I mean to imply that I was the only or even the first person to have discovered or described a theory or empirical result. Such comments are simply intended to refer to the fact of authorship without using unnecessarily stuffy or stilted constructions.

The first section includes papers that deal with portfolio choice. These follow the seminal work of Harry Markowitz, whom I count as my mentor and who continues to be a close friend. Harry's fundamental research assumed that an investor has inputs about some aspects of the joint probability distribution of the returns on a number of individual securities (usually common stocks) and wishes to find one or more portfolios that are efficient (not dominated by other portfolios). Most of the papers included here explore ways to provide economical approximations of results using the standard Markowitz formulation. Several present algorithms and their implementation in computer programs. While some of the papers mention equilibrium aspects in passing, the focus is on the formation of efficient portfolios given estimates of security risks, returns and correlations, however they might be obtained.

The second section deals with the determination of asset prices when capital markets have reached equilibrium. Most of the papers rely on a mean/variance setting but one uses only an assumption of no arbitrage and some of the results in other papers are derived directly from this premise. The early work focuses on what is sometimes called the traditional Capital Asset Pricing Model, while a number of the others deal with extensions to include additional aspects of the capital markets. Index funds, which are to an extent motivated by equilibrium asset pricing theories, are discussed in two short pieces addressed to practitioners. The section ends with a paper that addresses equilibrium issues using the Arrow–Debreu time-state model of asset pricing.

The third section deals with the allocation of an overall fund among major asset classes. Over the course of time, practitioners found that mean/variance optimization was not especially practical when choosing a mix of individual securities such as the stocks to include in a portfolio. On the other hand, it seemed to represent a way to adopt a scientific approach to the allocation of a multi-asset fund among key asset classes such as domestic stocks, foreign stocks, government bonds and the like. For such tasks. a number of consultants as well as managers of pension and other institutional funds use some of the approaches described in the first two sections of this book, although often with constraints on formal optimizations and inputs adjusted judgmentally to reflect market conditions. This section addresses a number of issues relating to asset allocation, from early applications of mean/variance approaches to more recent approaches involving expected utility maximization and alternative ways to adapt a policy to reflect changing asset market values.

The fourth section includes papers that deal with the measurement of a fund's performance and risk. In this area, mean/variance analyses drawn from both portfolio choice and asset pricing theories have had substantial impact on the world of practical asset management. The section starts with my first attempt to provide a useful measure that combines both portfolio risk and return in a single number and employs it to analyze the performances of a set of mutual funds and possible factors contributing to differences among them. Subsequent papers further examine the properties of this measure (now generally termed the Sharpe Ratio) and other measures designed to take into account aspects of both risk and return. In later papers, the role of factor models is discussed, and the technique often described as "returns-based style analysis" introduced and applied. The section ends with analyses of methods for using measures of risk and correlation in budgeting and monitoring a pension fund's allocations of funds among investment managers.

The fifth section includes papers which emphasize empirical analyses. Many focus on factors causing differences in the performance of groups of stocks with different attributes. In many of these, the emphasis is on risks and correlations, where the importance of a number of factors is well established. However, some papers deal with average returns and the possibility that securities exposed to some factors may have higher or lower expected returns than those predicted by variants of the Capital Asset Pricing Model (which provides a reference point for many of the analyses). One paper estimates the likely costs of pursuing a socially desirable goal by divesting stocks doing business in a country (South Aftrica) until it changes an undesirable policy (apartheid). Not all the papers deal solely with stock returns; one analyzes the effects of differences in societal wealth on the premium for a market portfolio that includes stocks and bonds. The common aspect across the papers, however, is an emphasis on data analysis and tests of theories with less attention paid to their derivation.

The sixth and final section focuses on issues relating to the manner in which individuals finance their lives. Two papers, based on academic talks, deal with key decisions made both before and after retirement. The remaining papers focus on decisions made at or after retirement, including strategies for spending and investing savings and immediate or planned purchases of annuities. Although much of my work over the years has been devoted to issues faced by those saving for retirement, I am now focusing most of my research and writing on the issues that arise after retirement. These are increasingly important both for society and individuals; moreover, they are challenging both practically and theoretically. The papers included in this section provide more questions than answers, but reflect the unsettled state of the work in this difficult, yet important and fascinating area.

Contents

Section 1: Portfolio Choice

1-1 "The Diagonal Security Model," Chapter II in William F. Sharpe, *Portfolio Analysis based on a Simplified Model of the Relationships Among Securities*, A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics, University of California, Los Angeles, June 1961, pp. 9–23.

This paper is reprinted from Chapter II of my dissertation, completed in June of 1961. It was based on research done in the fall of 1960 and the spring of 1961. During this period, I worked with Harry Markowitz, who was at the RAND Corporation, as was I. Although Harry was not on the faculty at U.C.L.A., my Chairman, Armen Alchian, agreed to let him serve as a *de facto* adviser. The dissertation explored a number of aspects of a simple model of security returns suggested in Harry's *Portfolio Selection* book. In the literature, this was subsequently termed the single index or one-factor model. The chapter describes the model and shows how efficient portfolios can be obtained using an algorithm that requires little computer memory and processing time (a crucial aspect at the time when both were in limited supply and very expensive). The approach relies on a transformation of the security covariance matrix to one with elements along the principal diagonal, leading me to use the term "diagonal security model". While I propose here that the index represent the market in general, no assumption is made that this is related to any sort of equilibrium in the capital market — a subject that I took up later in the dissertation in a chapter reprinted in Section 2.

1-2 "A Simplified Model for Portfolio Analysis," *Management Science*, Vol. 9, No. 2, January 1963, pp. 277–293.

This paper was submitted to *Management Science* in December 1961 (as indicated in a footnote) although it was not published until January 1963. It presents the diagonal model, shows how a portfolio can be characterized as investment in a common factor and in security residual returns, and provides estimates of the efficacy of simplifying the relationships among securities in this manner. The first part of the paper is based on my dissertation but there is additional discussion of the effects of the ability to borrow and/or lend at fixed rates of interest on the solution to the portfolio optimization problem and ways to take such opportunities into account when finding efficient portfolios. Since the time required to solve a portfolio optimization problem will be affected by both the number of securities considered and the number of "corner portfolios" obtained, the paper uses Monte Carlo simulations to estimate the relationships between these attributes and their effects on computer processing time.

The final part of the paper describes empirical tests included in the dissertation that suggest relatively small differences between the compositions of efficient portfolios based on a full covariance matrix and those obtained using a single index approximation. However, the samples included relatively few securities and relatively few periods of historic data. Appropriately, the final paragraph cautions that they are "far too fragmentary to be considered conclusive". Not discussed was the fact that the optimal portfolios included only a minority of the securities in the sample, calling into question the inputs, whether taken directly or simplified using a single index model – a subject covered in later papers in this volume. Here, as in my dissertation, I chose to plot portfolio risk/return combinations with expected return on the horizontal axis and standard deviation on the vertical axis. In this, I followed the graph using expected return and variance in Markowitz' 1959 book. I also used "B" for the concept later termed "beta" due primarily to the lack of greek letters on my typewriter.

1-3 "Mathematical Investment Portfolio Selection — Some Early Results," *University of Washington Business Review*, April 1963.

This paper is based on a presentation that I gave at a joint meeting of the Econometric Society and the Institute of Management Sciences in September 1962. Shortly before, IBM had produced a program designed to solve the portfolio optimization problem. It was designed for formulations with full covariance matrices although pre-processing algorithms could be invoked for those who wished to use inputs from a single-index or multi-index model. In this paper, I argued that a single index model, if employed with a special-purpose algorithm of the sort that I had designed, could not only reduce computer costs but also was more amenable to use with subjective inputs obtained from security analysts. The latter part of the paper is devoted to the results of an experiment that I conducted in 1961, described in one of the chapters of my dissertation. In it, a practicing security analyst provided estimates of the sensitivities of the prices of a set of securities to changes in the security market (measured by the Dow-Jones average) as well as the probability distributions of returns if the overall market remained at its present level. From these, I computed the parameters needed for estimates based on the diagonal model and derived efficient portfolios. The analyst also provided three portfolios that he would recommend for investors with different degrees of conservatism. The results were encouraging in two respects. The recommended portfolios had increasing expected returns and standard deviations, given the security estimates. Moreover, they dominated a majority of portfolios of comparable diversification that had been selected randomly. However, none of the three recommended portfolios were on the (purportedly) efficient frontier. The final sections of the paper ruminate on the implications of these results, suggesting that Markowitz's approach with or without index model assumptions, may be too powerful for use with inputs derived from security analysts but that it provides a powerful way to think about and teach some of the key principles of investment management.

1-4 "Portfolio Analysis," (excerpt) *The Journal of Financial and Quantitative Analysis*, Vol. II, No. 1, June 1967, pp. 76–81.

Paper 1–4 is the general portion of my introduction to a special issue of the *Journal of Financial and Quantitative Analysis* devoted to Portfolio Analysis. It shows some of the elements of the impact of the subject on academic and, to a considerably lesser extent, practical finance. Concern with returns that are not normally distributed was substantial at the time and many were considering other probability distributions, including some with infinite variance. By 1967, normative (prescriptive) applications had been joined by positive (descriptive) theories and a few researchers were considering the impact of equilibrium on investment practice. The Arrow–Debreu state-preference model of equilibrium had at the time been little noticed in the area of financial economics but I indicated that it might have greater influence in the future. Finally, I suggested (rather prophetically, as it turned out) that the broader area of business finance would adopt many of the ideas that started with and were derived from those of portfolio analysis.

1-5 "Mean-Absolute Deviation Characteristic Lines for Securities and Portfolios," *Management Science*, October 1971, pp. B-1–B-13.

This paper was written after I completed my 1970 book, *Portfolio Theory* and *Capital Markets*, published by McGraw-Hill. The paper was motivated by considerable interest at the time in the possibility that *ex ante* return distributions of security returns might be "fat-tailed". Following ideas put forth by Mandelbrot, Fama and others argued that the best representation of such distributions might be taken from members of the stable Paretian class which have infinite variances. This had led some to argue that the mean absolute deviation of returns should be used as a measure of risk, rather than the standard deviation or variance. At the time, the importance of a security's characteristic line had been recognized in both normative and positive applications. Such relationships had typically been estimated from historic data using traditional least-squares analysis in which a line is chosen to minimize the sum of the squared deviations of the observations from the fitted relationship. The statistical literature left open the question of whether linear relationships might better be estimated by minimizing the mean absolute deviation of the differences between actual and fitted values. In this paper, I provided a simple algorithm for doing this with a two-variable relationship such as that utilized for security and portfolio characteristic lines. I then compared results using this approach with traditional least-squares for both a sample of individual securities and another of mutual funds. The results showed that the slope (beta) estimates obtained using the two methods were very similar. The intercepts differed substantially for individual securities, but much less so for diversified mutual fund portfolios. My conclusion was that traditional least-squares methods were probably sufficient for the majority of applications in this area.

1-6 "Imputing Expected Returns From Portfolio Composition," *Journal of Financial and Quantitative Analysis*, June 1974, pp. 463–472.

This paper introduced the idea of inferring an investment organization's predictions from its portfolio choice. The idea was to start with an estimate of the covariance matrix, then make inferences about the set of expected returns that would make the current holdings optimal. The key relationships are those from the first-order conditions for maximizing a quadratic function (including the mean and variance of portfolio return) subject to linear constraints (the full-investment constraint and possibly upper and lower bounds). Without bounds, this leads to the derivatives of a Lagrangean function; with bounds the associated Kuhn–Tucker conditions are relevant. If a riskless security is available, these conditions allow one to infer the relative expected excess returns of the portfolio components. This approach is, in effect, the basis for procedures widely used today for risk measurement and management, as I showed in a later paper on budgeting and measuring portfolio risk (included in Section 4). This paper also shows that if security markets are assumed to be consistent with the equilibrium conditions of the Capital Asset Pricing Model, one can infer expected measures of abnormal performance (here, denoted "A" but now commonly called "alpha") for the positions. An empirical analysis of the holdings of a large institutional investor showed that actual holdings were consistent with relatively small expected departures from equilibrium values — a result that has been found over and over again as this procedure has been applied in the investment industry.

1-7 "An Algorithm for Portfolio Improvement," in K. D. Lawrence, J. B. Guerard, Jr., and G. D. Reeves, Editors, *Advances in Mathematical Programming and Financial Planning*, JAI Press, Inc., 1987, pp. 155–170.

In 1978, when I published the first edition of my textbook, *Investments*, programs had been developed to solve general quadratic programming problems (most notably the one that I had used at the RAND Corporation). There were also programs designed to efficiently solve portfolio choice problems using the single index model, including one produced by IBM for use on its 1401 business computer (since the program was based on my diagonal model algorithm, I was asked to meet with the developers for a morning but my input was minimal, at best). Another program had been developed at Dartmouth College and my own program was available at the University of Washington and elsewhere. But a simple algorithm for a problem with a general covariance matrix, a full-investment constraint and upper and lower bounds on security holdings was not widely available. To fill the gap, and to provide an approach that could be used to help others better understand the economics of mean/variance portfolio optimization, I developed the gradient method described in this paper.

In the later 1970's, I was also serving as a consultant for the Investment Systems Group of Wells Fargo Investment Advisors, members of which provided valuable input and an opportunity to test the procedures in a real-world setting. I provided the results in this paper which was submitted for publication in *Management Science.* I then included the essence of the algorithm in the second edition of my textbook, published in 1981, indicating that the source was the paper, forthcoming in *Management Science.* Unfortunately, confusion among the journal's editorial board members led to years of delay. At some point I was asked by John Guerard to contribute a paper to the book that he and others were editing. I asked the journal for a release of my submission and the paper finally was published in the cited volume (in excellent company).

The algorithm takes advantage of the fact that with a single linear equality constraint and upper and lower bounds on holdings, a two-security swap is locally optimal for improving the objective function in a traditional mean/variance optimization. Moreover, the optimal magnitude for such a swap can be computed very quickly. By determining a series of such swaps, one can efficiently come extremely close to the formally optimal portfolio composition. The result is intuitive, extremely simple to program, and economizes on both computer memory and processing time. The algorithm was subsequently employed by a number of academic and investment organizations. In my own classes, I found it effective to present the algorithm as soon as I had introduced the concept of mean/variance portfolio efficiency, then let the students reach their own conclusions about optimality. With the first-order conditions firmly established, it was then easy to move to equilibrium in capital markets characterized by mean/variance investors, methods for risk budgeting and monitoring, as well as other aspects of then-traditional financial economics. Now that most students and practitioners have access to spreadsheet programs such as Microsoft's Excel that can solve general quadratic programming problems this algorithm's primary advantage would seem to be more pedagogical than practical.

II. THE DIAGONAL SECURITY MODEL

A. Important Attributes of Securities

Discussions with security analysts indicate that three aspects of the future performance of a security are usually considered explicitly. First, a security is classified as either high-yield or low-yield on the basis of the analyst's "best guess" concerning its future performance. Such an estimate corresponds to the expected-yield quantity (E) required for the portfolio-analysis problem.

The second aspect which receives attention in most security analyses is the risk that the most likely return may not be realized. Securities are considered more speculative, the greater is this risk. The variance parameter required by portfolio analysis makes explicit this notion of risk.

The third aspect of the performance of a security which is often considered explicitly is its relationship to the security market (and/or the economy) in general. Securities which rise and fall with the market are considered more sensitive or cyclical than those which are little affected by such broad movements. Thus the risk of a security is often considered to be due to two factors: the risk associated with the firm itself, and the risk of a market (or general business) decline with an associated effect on the firm in question.

In the diagonal model these three attributes -- expected yield, risk, and dependence on the market -- are made explicit. The simplicity of the model derives from the fact that these attributes are the only ones taken into account. Evidence presented in the next chapter suggests that any loss in precision due to this simplicity is likely to be small. It is shown below that the cost of portfolio analysis can be greatly reduced when the diagonal model is used. For these reasons the model appears worthy of the detailed examination it receives in this dissertation.

B. The Diagonal Model[1]

The basic equation of the diagonal model relates the yield of a security to its own attributes and to the performance of some index of market activity:

$$Y_i = A_i + B_i \cdot I + w_i \tag{1}$$

where A_i and B_i are parameters, and w_i is a random variable with an expected value of zero and a variance of Q_i. The parameter (I) represents the level of an index of some activity considered to be of major importance in determining the yields of most securities. In this study we use the level of the security market for this index. A number of alternative attributes are of interest and may be incorporated into future work.[2] Such measures can be substituted for the market level without altering the formulation of the

---

[1]This model is one of a number suggested by Markowitz; see his Portfolio Selection, pp. 96-101.

[2]A particularly interesting candidate is the general price

diagonal model presented here; on the other hand, if they are to be incorporated in addition to the market index, the model will have to be expanded. Models which relate the yield of a security to more than one such factor will undoubtedly prove valuable; however, in this study we will restrict our attention to the simple model in which but one element of this type influences the yields of most securities.

For a number of problems it is convenient to measure (I) in terms of deviations (either relative or absolute) from its expected value. However, we will formulate the model for the general case in which the expected value of (I) is non-zero, so that any desired measurement can be used.

Equation (1) constitutes the basic assumption of the diagonal model. Security analysis, in this model, involves the specification of three parameters for each security: A_i, B_i, and Q_i. These

---

level of the economy. The wealth of a firm can be shown to be affected by the price level and the relative importance of real and monetary assets and liabilities in its financial structure. In particular, if (M) is the firm's net monetary creditor position (monetary assets less monetary liabilities) and (R) its net real asset position (real assets less real liabilities), then the effect of changes in the price level on the firm's wealth can be shown to be proportional to R/(R+M). The diagonal model can be utilized to reflect this relationship by using R/(R+M) as an estimate of B_i for each security, with (I) representing an index of the general price level in the economy. For the general model of the effects of inflation on the wealth position of a firm, see R. Kessel, "Inflation-caused Wealth Redistribution," American Economic Review, XLVI (March, 1956), 128-141.