

A. Koutsoyiannis

**Theory of
Econometrics**

Second edition

THEORY OF ECONOMETRICS

An Introductory Exposition of Econometric Methods

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SECOND EDITION

Foreword by C. F. CARTER

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Foreword

Additional textbooks require some excuse: but the reason for this one is easily stated. Most econometric textbooks are either too advanced for all but a minority of students, or achieve simplicity by leaving too many things out. Dr Koutsoyiannis set herself the difficult task of being simple and reasonably comprehensive at the same time; and, because she had a clear aim, and knows the subject intimately in every detail, she has succeeded. Furthermore, she has produced a book which is written by an economist, about the real things which ought to be the province of economics — ‘a study of mankind in the ordinary business of life’. This book is not pure mathematics dressed up with a few economic names for the variables, but an effort to introduce, to a large number of economists, tools which will be of value to them in the practical business of their subject. It deserves a wide welcome.

The University of Lancaster
November 1972

CHARLES F. CARTER

Preface to the Second Edition

I have attempted in this second edition to stress the importance of the application of econometric methods to economic relationships. The structure of this edition is basically the same as that adopted in the first edition. However, the numerous comments and suggestions which I received from colleagues in several universities, reviewers and students, gave me the opportunity to rewrite many sections of the book, bearing always in mind my basic goal: the emphasis of the economic implications of the various measurement techniques. I hope, as a result, that economists will become increasingly aware of the ease with which the powerful tools of econometrics can be applied meaningfully to specific economic problems, without the heavy mathematical requirements of the other main textbooks in this field.

Complying to the many requests of teachers, instructors and students, I have added an Appendix with numerous examples and questions relating to economic applications. It is my belief that a thorough understanding of the econometric methods cannot be attained unless the student is exposed to as many examples as possible which have some degree of economic realism. Hence, I have minimised the questions which are purely theoretical, and I have concentrated on examples from economic theory which can be worked out with observations (data) of the relevant economic variables. This is a unique feature to the second edition, which, I believe, will prove most useful and will contribute to the wider use of econometric techniques.

Some of the examples have been taken from various sources and adapted to economic situations. The majority of problems, however, have been prepared by the author with the help of teaching assistants, to whom I am greatly indebted. I would like to express my thanks in this respect to A. Taher, M. Nosko, and J. Collins of the University of Waterloo. I am also grateful to D. Pallas and D. Wagg for the speed and efficiency with which they typed a difficult typescript.

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Preface to the First Edition

For a long time econometrics has been considered a highly specialised tool of research. Yet its rapid growth and increasing use in economic planning and research require its simplification and wider diffusion among students of economics and professional economists. It is the contention of the author that econometrics is much simpler than is commonly believed. It is hoped that the present textbook will substantiate this contention and, by simplifying the econometric methods, will provide a good guide to the more advanced textbooks on econometrics such as J. Johnston's *Econometric Methods*, A. Goldberger's *Econometric Theory*, or E. Malinvaud's *Statistical Methods in Econometrics*.

The thought of writing an introductory textbook on econometrics originated from the author's experience both as a student and as a teacher of econometrics. As a student, having gone through the formidable task of acquiring the background in mathematics and statistics needed for a formal course on econometrics, the author became increasingly aware that most of the ideas pertaining to the application of econometric methods may be presented clearly with the use of fairly simple mathematical and statistical notions. This conviction was strengthened by the author's teaching experience. Of course, by adopting a simple mathematical exposition, rigorous treatment is sacrificed to simplicity; but it has been the author's experience that unless students are gradually introduced to the sophisticated mathematical and statistical exposition of the standard econometric textbooks, they are often discouraged no matter how enthusiastic they are about the subject.

Recently several introductory books on econometrics have been published. However, most of these books omit many essential topics on the grounds that they are 'beyond the scope' of an introductory textbook. As a consequence these textbooks provide an incomplete treatment of the subject.

It is true that there is no substitute for the classical textbooks (such as the ones mentioned above) for the thorough understanding of econometric theory. But frequently in the advanced textbooks the economic implications of the econometric methods, which are of paramount importance for the practising economist, are shadowed by the elaborate mathematical exposition of the subject-matter. Thus even those students with an advanced background knowledge in mathematics and statistics sometimes fail to grasp the economic implications and relevance of the mathematical assumptions needed to build up the econometric methods.

The present book is an attempt to provide students of economics and professional economists, not formally trained in econometrics, with the necessary

tools for econometric research in a mathematically simple way. It is intended to be a textbook of *econometrics for economists* and to serve as a stepping-stone to the more advanced books and treatises on the subject.

The book covers a range of topics similar to the classical textbooks of econometrics. The author has attempted to simplify the mathematical presentation of the econometric methods. The exposition is based on the rules of simple algebra. Matrix algebra, while allowing a neat and concise presentation, often has the effect of bewildering and discouraging the mathematically unsophisticated student for whom this book is intended. Thus the book does not assume knowledge of mathematics beyond elementary college algebra, and of statistics beyond an introductory course. The essential definitions and tools of statistics and mathematics required for the understanding of the book are summarised in two brief appendices. Mathematical proofs are mostly presented in small print so as not to interrupt the main course of exposition of the econometric techniques.

The greatest attention is given to the economic aspects of econometrics. The assumptions made in the various methods and their economic implications are treated in detail. For each method the assumptions are first explicitly stated and their economic meaning examined. Particular attention is paid to the analysis of the consequences of the violation of the assumptions on the measurements of the coefficients of economic relationships. Finally, the tools available for testing the assumptions of the econometric methods are explained and ways for improving the measurements, when the assumptions are violated, are discussed.

The book is divided in three parts.

Part One begins with a brief discussion of the procedure which must be followed in any applied econometric research. It proceeds with the examination of the theory of correlation and the simple linear regression model. In a separate chapter the method of the Analysis of Variance and its use in connection with regression analysis is developed.

Part Two is devoted to the examination of the assumptions of the linear regression model, their economic meaning, their implications for the values of the parameters of the economic relations, their tests, and the corrective action which must be adopted whenever these assumptions are violated.

Part Three includes the examination of the problems arising from the simultaneous dependence of the economic variables. The systems of simultaneous relations require the application of more elaborate econometric techniques for their measurement. An attempt is made to present the most important of these methods, starting from the simpler ones and gradually proceeding to the development of the more sophisticated maximum likelihood methods. In the final chapter the ranking of the various econometric techniques is attempted, based on the evidence provided by extensive studies generally known as Monte Carlo studies.

The author is heavily indebted to Professor C. F. Carter, Vice-Chancellor of the University of Lancaster and former Editor of the *Economic Journal*, without whose guidance, encouragement and support the project would not have been possible. I am also greatly indebted to Professor A. Goldberger of the University

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I would like to thank Katherine Kossentos for her assistance in the design of the cover of this book. I believe that the impressions of the reader of any book start from its cover. I think that Miss Kossentos's simplicity of design conveys the simplicity of approach attempted in this book.

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PART ONE

Correlation Theory

The Simple Linear Regression Model

1. Definition, Scope and Division of Econometrics

1.1. DEFINITION AND SCOPE OF ECONOMETRICS

Econometrics deals with the measurement of economic relationships. The term 'econometrics' is formed from two words of Greek origin, *οικονομία* (economy), and *μέτρον* (measure).

Econometrics is a combination of economic theory, mathematical economics and statistics, but it is completely distinct from each one of these three branches of science.

The following quotation from the opening editorial of *Econometrica* written by R. Frish in 1933 may give a clear idea of the scope and method of econometrics:

But there are several aspects of the quantitative approach to economics, and no single one of these aspects, taken by itself, should be confounded with econometrics. Thus, econometrics is by no means the same as economic statistics. Nor is it identical with what we call general economic theory, although a considerable portion of this theory has a definite quantitative character. Nor should econometrics be taken as synonymous with the application of mathematics to economics. Experience has shown that each of these three viewpoints, that of statistics, economic theory, and mathematics, is a necessary, but not by itself sufficient, condition for a real understanding of the quantitative relations in modern economic life. It is the *unification* of all three that is powerful. And it is this unification that constitutes econometrics.

Thus econometrics may be considered as the integration of economics, mathematics and statistics for the purpose of providing numerical values for the parameters of economic relationships (for example, elasticities, propensities, marginal values) and verifying economic theories. It is a special type of economic analysis and research in which the general economic theory, formulated in mathematical terms, is combined with empirical measurement of economic phenomena. Starting from the relationships of economic theory, we express them in mathematical terms (i.e. we build a model) so that they can be measured. We then use specific methods, called *econometric methods*, in order to obtain numerical estimates of the coefficients of the economic relationships. Econometric methods are statistical methods specifically adapted to the peculiarities of economic phenomena. The most important characteristic of economic relationships is that they contain a random element, which, however, is ignored by

economic theory and mathematical economics which postulate exact relationships between the various economic magnitudes. Econometrics has developed methods for dealing with the random component of economic relationships.

An example will make the above clear. Economic theory postulates that the demand for a commodity depends on its price, on the prices of other commodities, on consumers' income and on tastes. This is an exact relationship, because it implies that demand is completely determined by the above four factors. No other factor, except those explicitly mentioned, influences the demand. In mathematical economics we express the above abstract economic relationship of demand in mathematical terms. Thus we may write the following demand equation

$$Q = b_0 + b_1P + b_2P_0 + b_3Y + b_4t$$

where Q = quantity demanded of a particular commodity

P = price of the commodity

P_0 = prices of other commodities

Y = consumers' income

t = tastes

b_0, b_1, b_2, b_3, b_4 = coefficients of the demand equation.

The above demand equation is exact, because it implies that the only determinants of the quantity demanded are the four factors which appear in the right-hand side of the equation. Quantity will change only if some of these factors change. No other factor may have any effect on demand. Yet it is common knowledge that in economic life many more factors may affect demand. The invention of a new product, a war, professional changes, institutional changes, changes in law, changes in income distribution, massive population movements (migration), etc., are examples of such factors. Furthermore, human behaviour is inherently erratic. We are influenced by rumours, dreams, prejudices, traditions and other psychological and sociological factors, which make us behave differently even though the conditions in the market (prices) and our incomes remain the same. In econometrics the influence of these 'other' factors is taken into account by the introduction into the economic relationships of a random variable, with specific characteristics, which will be discussed in later chapters. In our example the demand function studied with the tools of econometrics would be of the (stochastic) form

$$Q = b_0 + b_1P + b_2P_0 + b_3Y + b_4t + u$$

where u stands for the random factors which affect the quantity demanded.

It is essential to stress that econometrics presupposes the existence of a body of economic theory. Economic theory should come first, because it sets the hypotheses about economic behaviour which should be tested with the application of econometric techniques. In testing a theory we start from its mathematical formulation, which constitutes *the model* or the *maintained hypothesis*. In our