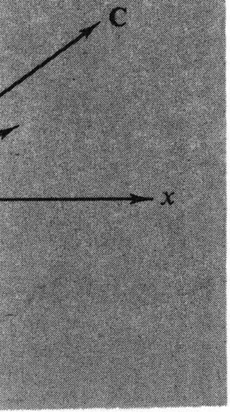

MATHEMATICAL METHODS FOR PHYSICISTS

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SECOND EDITION

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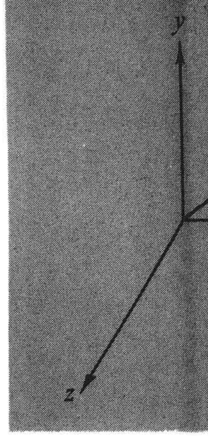
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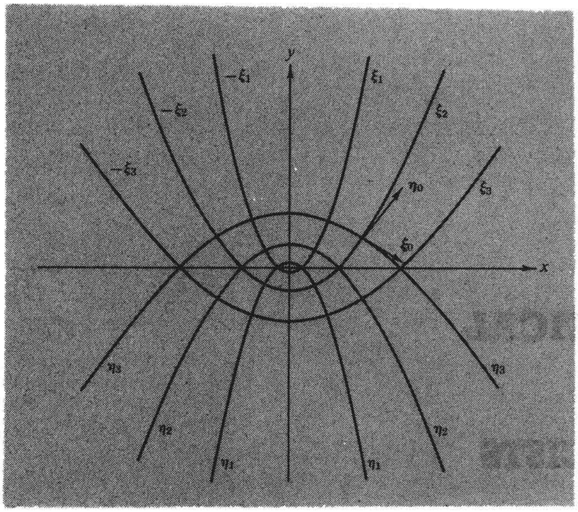
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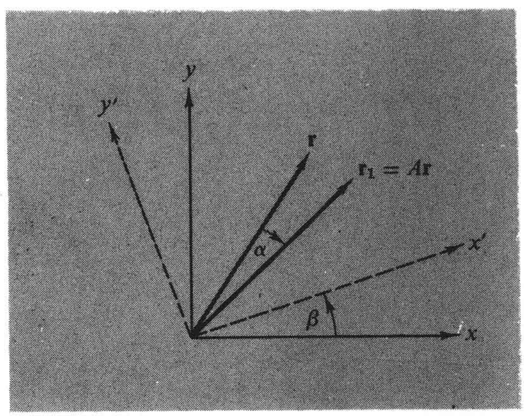
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TO CAROLYN

PREFACE TO THE SECOND EDITION

This second edition of *Mathematical Methods for Physicists* incorporates a number of changes, additions, and improvements made on the basis of experience with the first edition and the helpful suggestions of a number of people. Major revisions have been made in the sections on complex variables, Dirac delta function, and Green's functions. New sections have been included on oblique coordinates, Fourier-Bessel series, and angular momentum ladder operators. The major addition is a series of sections on group theory. While these could have been presented as a separate group theory chapter, there seemed to be several advantages to include them in Chapter 4, Matrices. Since the group theory is developed in terms of matrices the arrangement seems a reasonable one.

PREFACE TO THE FIRST EDITION

Mathematical Methods for Physicists is based upon two courses in mathematics for physicists given by the author over the past eighteen years, one at the junior level and one at the beginning graduate level. This book is intended to provide the student with the mathematics he needs for advanced undergraduate and beginning graduate study in physical science and to develop a strong background for those who will continue into the mathematics of advanced theoretical physics. A mastery of calculus and a willingness to build on this mathematical foundation are assumed.

This text has been organized with two basic principles in view. First, it has been written in a form that it is hoped will encourage independent study. There are frequent cross references but no fixed, rigid page-by-page or chapter-by-chapter sequence is demanded.

The reader will see that mathematics as a language is beautiful and elegant. Unfortunately, elegance all too often means elegance for the expert and obscurity for the beginner. While still attempting to point out the intrinsic beauty of mathematics, elegance has occasionally been reluctantly but deliberately sacrificed in the hope of achieving greater flexibility and greater clarity *for the student.*

Mathematical rigor has been treated in a similar spirit. It is not stressed to the point of becoming a mental block to the use of mathematics. Limitations are explained, however, and warnings given against blind, uncomprehending application of mathematical relations.

The second basic principle has been to emphasize and re-emphasize physical examples in the text and in the exercises to help motivate the student, to illustrate the relevance of mathematics to his science and engineering.

This principle has also played a decisive role in the selection and development of material. The subject of differential equations, for example, is no longer a series of trick solutions of abstract, relatively meaningless puzzles but the solutions and general properties of the differential equations the student will most frequently encounter in a description of our real physical world.

ACKNOWLEDGMENTS

Any work of this sort necessarily represents the influence and help of many people. I acknowledge gratefully my debt to the professors who taught me physics and mathematics and who instilled in me a love of these disciplines. Among these men, Professors G. Breit, H. Margenau, and E. J. Miles deserve special mention. I thank my colleagues, particularly Professors D. C. Kelly and P. A. Macklin, for their assistance and helpful criticism. A special note of thanks is due to my students over the past eighteen years for their criticisms and reactions which have played a major part in shaping this book, Mr. J. Clow and Mr. S. Orfanides were particularly helpful. A special acknowledgment is owed Mrs. Juanita Killough for so patiently and conscientiously typing this manuscript.

INTRODUCTION

Many of the physical examples used to illustrate the applications of mathematics are taken from the fields of electromagnetic theory and quantum mechanics. For convenience the main equations are listed below and the symbols identified. References in these fields are also given.

Electromagnetic theory

MAXWELL'S EQUATIONS (MKS UNITS—VACUUM)

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

Here **E** is the electric field defined in terms of force on a static charge and **B** the magnetic induction defined in terms of force on a moving charge. The related fields **D** and **H** are given (in vacuum) by

$$\mathbf{D} = \varepsilon_0 \mathbf{E} \qquad \text{and} \qquad \mathbf{B} = \mu_0 \mathbf{H}$$

The quantity ρ represents free charge density while **J** is the corresponding current. For additional details see: J. M. Marion, *Classical Electromagnetic Radiation*, New York: Academic Press (1965); W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, Reading, Mass.: Addison-Wesley (1955); J. D. Jackson, *Classical Electrodynamics*, New York: Wiley (1962).

Note that Marion and Jackson prefer Gaussian units. A glance at the last two texts and the great demands they make upon the student's mathematical competence should provide considerable motivation for the study of this book.

Quantum Mechanics

SCHRÖDINGER WAVE EQUATION (TIME INDEPENDENT)

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

ψ is the (unknown) wave function. The potential energy, often a function of position, is denoted by V while E is the total energy of the system. The mass of the particle being described by ψ is m. $\hbar$ is Planck's constant h divided by 2π. Among the extremely large number of beginning or intermediate texts we might note: A. Messiah, *Quantum Mechanics* (2 vols), New York; Wiley (1961): R. H. Dicke and J. P. Wittke, *Introduction to Quantum Mechanics*, Reading Mass.: Addison-Wesley (1960); E. Merzbacher, *Quantum Mechanics* (Second Edition), New York: Wiley (1970).

CONTENTS

Chapter 2. COORDINATE SYSTEMS

Chapter 3. TENSOR ANALYSIS

Chapter 4. DETERMINANTS, MATRICES, AND GROUP THEORY

Chapter 5. INFINITE SERIES

Chapter 6. FUNCTIONS OF A COMPLEX VARIABLE I. ANALYTIC PROPERTIES, CONFORMAL MAPPING

Chapter 7. FUNCTIONS OF A COMPLEX VARIABLE II. CALCULUS OF RESIDUES

Chapter 8. SECOND-ORDER DIFFERENTIAL EQUATIONS

Chapter 17. CALCULUS OF VARIATIONS