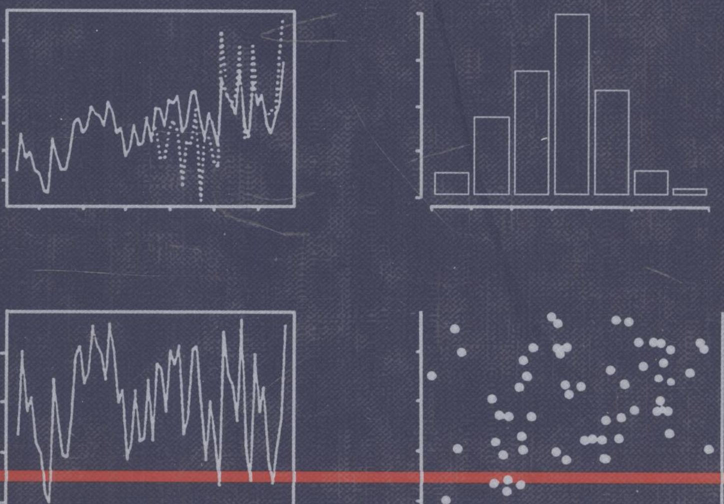
Foundations of Time Series Analysis and Prediction Theory

Mohsen Pourahmadi



WILEY SERIES IN PROBABILITY AND STATISTICS

Foundations of Time Series Analysis and Prediction Theory

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A Wiley-Interscience Publication
JOHN WILEY & SONS, INC.
New York • Chichester • Weinheim • Brisbane • Singapore • Toronto

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Library of Congress Cataloging-in-Publication Data:

Pourahmadi, Mohsen.
Foundations of time series analysis and prediction theory / Mohsen Pourahmadi.
p. cm. — (Wiley series in probability and statistics)
Includes bibliographical references and index.
ISBN 0-471-39434-3 (cloth : alk. paper)
1. Time-series analysis. 2. Prediction theory. I. Title. II. Series.

QA280 .P68 2001
519.5'5—dc21 2001017918

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

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To
Terri, Nina and Adam

Preface

This book provides a solid *foundation* for time series analysis and prediction theory. The need for a foundation is manifested by the maturity, widespread use and truly interdisciplinary nature of time series analysis lying at the intersection of the mathematical, statistical, computational, physical, engineering and system sciences. The goal is to join the two lines of developments in *time series analysis* centered around the work of Schuster, Yule, Slutsky, Wold, ..., and *prediction theory* based on the work of Szegö, Wold, Cramèr, Kolmogorov, Wiener, ... into a logically sound and pedagogically reasonable theory of modern statistical time series analysis.

We use the language of regression (projection) and the Hilbert space which is powerful, natural, intuitive, spoken widely, and hence capable of facilitating interactions, interdisciplinary efforts to problem solving and communicating results among researchers from diverse and growing fields where time series problems may arise. There is great need for a book emphasizing the fundamental results and structural underpinnings of stationary processes to explain, extend and unify in a mathematically coherent manner the diverse and useful developments of the last few decades. Special effort is made to motivate, present and prove the results on the structure and prediction of stationary processes in the time-domain using the celebrated Wold decomposition. Statistical methods and concepts, however, are mostly motivated using autoregressive (AR) models. In addition due emphasis is given to the spectral-domain results, data analysis and computational issues so as to entice the reader to pursue these areas further.

Our approach does not compete with but rather complements those pursued by Hannan (1970), Anderson (1971), Hannan and Deistler (1988), Brockwell and Davis

(1991) and Fuller (1996), in that we do not present the proofs of consistency and asymptotic normality of the sample covariances, the maximum likelihood estimators of the ARMA parameters and the spectral density estimators. Such classical results, though extremely important, are well established. Instead, we emphasize the mathematical/probabilistic, statistical/data analytic and computational concepts along with some novel and useful results from the research papers that have not appeared in books yet. Proofs of some of these mathematical/probabilistic results are lengthy and tedious or may require advanced background in probability, harmonic and functional analysis. In such cases, we merely motivate, state the results, sketch the proofs and highlight their applications. The books by Robinson (1954), Lambert and Poskitt (1983) and Pollock (1999) are relevant to the topic of developing foundations for time series analysis and prediction theory.

This book is suitable for researchers and advanced students in probability and statistics, mathematics, engineering and system sciences, physical and natural sciences, economics and social sciences who are interested in the deeper aspects of time series analysis. Familiarity with probability and statistics at the level of Casella and Berger's (1990) *Statistical Inference* and knowledge of linear algebra and Hilbert spaces are helpful. Advanced topics including those on Hilbert spaces are used and developed gradually in later chapters as needed. With a careful selection of topics and appropriate supplementation if necessary, the book can be used as a graduate text for either a one- or a two-semester course on time series analysis (Chapters 2–5) and (second-order) stochastic processes (Chapters 5–10). The problems at the end of each chapter and the indicated projects in Chapters 2–4 are useful for gaining skill, deeper appreciation of the covered topics and indicating possible directions of extending the methodology and the theory. However, the book does not attempt to be a traditional manual for time series data analysis and forecasting.

According to the mathematics/statistics level, the book can be divided loosely into three parts comprising of Chapters 1–4, 5–8 and 9–10. At the expense of some duplication, the topics are arranged so that each part can be studied virtually independently of the others. The first part is statistical and covers most ingredients needed for time series data analysis. The second part is probabilistic and develops structural results on stationary processes. Finally, the third part is more mathematical reviewing some background material on Hilbert spaces and function theory and providing proofs of deeper results such as the spectral representation and the Szegö-Kolmogorov prediction theorems. Some of these topics are of independent interest to statisticians due to their widespread use in modern statistics; see Wahba (1990), Small and McLeish (1994) and Vidakovic (1999).

The scope of the subject is wide and the topics covered reflect my interests and the need to control the length of the book. Topics not covered here have received book-length coverage in other sources. Some examples are nonlinear models (Tong, 1990), nonstationary models (Priestley, 1988), long-memory models (Beran, 1994), random fields (Rosenblatt, 1985; Yaglom, 1986), Bayesian models (West and Harrison, 1997), continuous-time processes (Dym and McKean, 1976) and multivariate processes (Rozanov, 1967; Hannan and Deistler, 1988). A broad theory of statistical inference for these diverse cases is provided by Taniguchi and Kakizawa (2000). My hope is that the present book will prepare and motivate the reader for further study and research in these diverse areas centered around the concept of stationarity.

Some of the datasets used in this book are available via my website `http://www.math.niu.edu/~pourahm/book/` at Northern Illinois University. The computations and graphics are done in Splus (MathSoft Inc.). The datasets and the sample programs can also be obtained from the author at the address below. Please feel free to let me know of your questions, comments and constructive criticism.

March 20, 2001

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Acknowledgments

Special thanks are due to many people and organizations for contributing directly or indirectly to this work. The project began in 1988 while on sabbatical in the Division of Statistics at University of California, Davis; George Roussas and Bob Shumway were helpful with facilitating the visit and explaining the roles of AIC and BIC, respectively. My understanding of the subject has been shaped and reshaped by many formal and informal discussions with Manny Parzen, P. Brockwell, R. Davis, D. Findley, D. Tjøstheim, A.G. Miamee, H. Salehi, P.R. Masani, S. Cambanis, G. Kallianpur, V. Mandrekar, A.R. Soltani, S. Axler, ..., and my colleagues in the Division of Statistics at Northern Illinois University. Many students have suggested changes and corrections over time, however, the help of B. Tarami and D. Olive in proof-reading various drafts deserves to be singled out. Susan Catron did a superb job of preparing the document in LaTeX and finishing the work of several of her predecessors. Steve Quigley and the editorial staff at John Wiley & Sons were generous with their assistance, timely suggestions and reminders. Research support from the NSF, AFOSR and NSA has contributed greatly to this work. Finally, I would like to thank my family for their patience, understanding, support and good humor while I was working on the project. Leaving a bunch of red pens in my Christmas, 1999, stocking was probably their gentle way of reminding me to bring it to a timely closure.

M. P.

Acronyms

ACF	Autocorrelation function
AO	Additive outlier
AR	Autoregression
ARIMA	Autoregressive integrated moving average
ARMA	Autoregressive moving average
ARR	Autoregressive representation
BFS	Bayesian Forecasting System
CAPM	Capital asset pricing model
CS	Compound symmetry
DE	Damped Exponential
EWMA	Exponentially weighted moving average
GARCH	Generalized autoregressive conditionally heteroschedastic
GARP	Generalized autoregressive parameters
GLS	Generalized least squares
IACF	Inverse autocorrelation function
IO	Innovation outlier

IV	Innovation variances
MA	Moving average
MLE	Maximum likelihood estimator
MMSE	Minimum mean square error
OSM	Ordinary Scatterplot Matrix
PACF	Partial autocorrelation function
PRISM	Partial Regression-On-Intervenors Scatterplot Matrix
RKHS	Reproducing kernel Hilbert space
SFS	Statistical forecasting system
SUR	Seemingly unrelated regressions
VMO	Vanishing mean oscillation

Contents

1

Introduction

Unlike classical statistics, time series analysis is concerned with statistical inference from data that are not necessarily independent and identically distributed (i.i.d.). The ideal framework for time series analysis is stationarity whereas the data encountered in practice are often nonstationary for a variety of reasons. Thus, the challenges facing a time series analyst are to transform the data to stationarity first, and then transform stationary data into an i.i.d. sequence. Some unique features of time series analysis and this book in dealing with these two challenges are described briefly in this chapter.

1.1 NONSTATIONARY DATA AND ORTHOGONALIZATION

A discrete-time stochastic process $\{X_t\}$ representing the evolution of a system over time could be nonstationary because either its *mean* (level) or its *dependence* as gauged by the covariance between two measurements is a function of time. For a given time series data, separating or distinguishing these two aspects of nonstationarity known as mean-nonstationarity and covariance-nonstationarity, respectively, is a difficult task. Traditionally, however, much attention is paid to correcting the mean nonstationarity by relying on simple transformations such as logarithm, differencing and smoothing. These transformations and others from the familiar and popular area of regression analysis are reviewed and illustrated in the early parts of Chapters 2 and 3.

Unfortunately, no simple statistical methodology like regression is known for dealing with covariance-nonstationarity. In addition, the presence of dependence in the data, whether of stationary or nonstationary kind, is problematic for someone who is