MANAGEMENT SCIENCE

HSIAO · CLEAVER

Management Science

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To Jean, Eugene, and Huey Hsiao and William G. Cleaver

PREFACE

Readers of this book—students, faculty, and practitioners—will find it to be a comprehensive introduction to management science. The treatment of the quantitative techniques used in management science begins at ground zero with logical development of elementary terms and concepts. Topics are developed to the point where the reader who is confronted with a practical application will have gained sufficient knowledge to tackle the situation and properly interpret the results. Since courses in operations research techniques for managerial decision making have become a standard part of the business curriculum, we have designed this text for upper-level undergraduate or beginning graduate courses offered in business or public administration and in economics.

A unique feature of our presentation is the way in which we have tried to explain their use of quantitative methods. Insofar as the development of management science has benefited from basic economic theory, we often present underlying economic concepts to fully clarify why a particular method works as it does. This means that we treat basic trade-off concepts, in particular marginal analysis and opportunity costs. This approach, we believe, leads toward a better understanding of the methods involved. It also allows us to offer reasonable explanations of difficult topics, such as integer programming and quadratic programming. Furthermore, our goal is to inspire interest in using management science's tools of analysis. We avoid rigorous mathematical proofs; rather, we stress the application and illustration of techniques. The reader, by using this approach of economic reasoning and motivation, will be able to master the techniques rather than be enslaved by them. Techniques may soon be forgotten, but the concepts behind the techniques should remain with students after they complete their tour of this book.

The text is organized in two general divisions. The first of these treats deterministic, programming models including, of course, linear programming and duality concepts, the transportation model, and the assignment problem. Also featured in this first section are comprehensive introductions to integer programming, goal programming, and quadratic programming. The second major division presents probability methods and stochastic decision-making models. Included here are Bayesian models, inventory models, queuing models, simulation techniques, game theory, and Markov processes. Refer to the organization chart for an overview of these topics in the textbook. Notice also that dynamic programming, PERT, and forecasting techniques are presented at appropriate points. Relating to all of these, a short introduction to methods of solving complex quantitative decision problems using computer routines is given.

The introduction of linear programming in Chapter 2 provides a beginning for the subsequent discussion of quantitative optimization methods. Three elementary examples in this chapter are carried through in later chapters as illustrations of linear programming and goal programming models. This approach will enable students, we believe, to apply new ideas and techniques to familiar problems, an approach that will reduce "new material shock." Underlying assumptions are carefully presented and discussed to help students in mastering the tools of analysis.

In Chapter 3, we incorporate the economic and logical explanations of procedural operations to be used in applying the simplex method. We believe this is pertinent because it enables students to acquire both mechanical orientation and theoretical understanding without being

bogged down in sophisticated mathematical details.

We consider duality to be the central underlying concept of linear programming and essential for a complete understanding of linear programming, transportation problems, quadratic programming, and game theory. Presenting the simplex tableau with the objective function in the last row—see Chapters 3 and 4—eases conversion to the study of dual problems by providing a convenient symmetry to the simplex tableaus of the primal and dual problems. Furthermore, this device not only makes the subsequent primal-dual formulation for quadratic programming more straightforward, but also simplifies the presentation of the simplex tableau for goal programming models. We consider this to be a pedagogical innovation.

Transportation problems and the closely related assignment problems are presented in Chapter 5. Heavily influenced by Koopmans' original contribution, the transportation algorithm is fully explained using duality and economic interpretation—considerations that are seldom presented in other management science or operations research textbooks.

Following a logical arrangement, with appropriate development from elementary to more complex operations, we introduce the chapters on integer programming, goal programming, quadratic programming and dynamic programming. Naturally, as in other management science or quantitative textbooks, the procedural operations and techniques of these topics require much calculation. Consequently, carefully formulated, step-by-step instructional sequences have been prepared to assist readers in grasping these basics. Here again, the underlying logic using economic reasoning is given. The broad scope of programming methods is completed by covering dynamic programming. This is followed by Chapter 10 where PERT techniques are introduced.

In Chapter 11, Bayesian decision theory and the basic concepts of probability theory lead into the treatment of decision-making models in Chapters 12 through 16. Chapter 11 offers careful treatment of Bayes' Theorem and of differences between the Bayesian and classical approaches. In Chapters 12 and 13, the inventory and queuing models demonstrate fundamental trade-off concepts. By placing these chapters in sequence, the textbook emphasizes the similarity of underlying economic concepts even in seemingly different operational problems. Basic simulation techniques are begun in Chapter 13 on queuing, and are then expanded in Chapter 14, which shifts away from queuing examples and provides a unique business decision-making model involving competing firms and sequential periods of decision making. The model can either be operated by hand calculations or be computerized. It is suitable for teams of students to use in competition with one another. This broad section of the textbook dealing with stochastic models is completed with chapters on game theory and Markov chains.

Among the indispensable tools of management science for business decision making, we feel that one must include forecasting techniques and an appropriate orientation to computer methodology. There is no doubt that realistic business situations require knowledge of these methods as part of the decision-making process, and treatment of these topics therefore has been presented in Chapters 17 and 18, respectively. These chapters, together with the chapters on goal programming and quadratic programming, offer a most comprehensive coverage of quantitative decision-making techniques.

As a final perspective, insights are presented in Chapter 19 into the

future opportunities and potentials in the field of management science for business decision making. Above all, an attempt has been made throughout all chapters to conform to a fundamental goal of this text—that it should be readable and teachable.

With regard to mathematical prerequisites, no calculus or matrix algebra is needed for using this text. Extensive exercises are presented at the end of each chapter. The overall purpose of these exercises is to reinforce the reader's understanding of the basic concepts of quantitative methods and to provide the reader with a *working* knowledge of the application of quantitative techniques. To achieve this purpose, all exercises were selected so that the need to do computations by hand should not intimidate students. Each chapter is also followed by a list of suggested readings.

Many persons have given us help in the preparation of this book. Our chief acknowledgment must go to our colleagues who have reviewed the entire manuscript and made helpful suggestions and comments that have led to substantial improvement. They are: Professor Nesa L. J. Wu of Eastern Michigan University, Professor Donald S. Miller of Emporia State University, Professor Stanley R. Schultz of Cleveland State University, and Professor Whewon Cho of Tennessee Technological University. We also extend our thanks to both Professors Howard A. Plotkin and Andrew J. Thacker, Jr., of the University of Houston, who evaluated several preliminary chapters of our manuscript in its early stages. Others who have provided valuable assistance are Dr. Andrew Hook of the New York Federal Reserve Bank, Professor T. C. Lee of the University of Connecticut, and Professors Yu-Chu Hsu and Frank E. Whelan of Southern Connecticut State College. To them we are indebted.

The unedited version of the manuscript has been used several times in both undergraduate and graduate classes at Southern Connecticut State College. Needless to say, we are extremely grateful to all students who have used these materials and helped improve them by their reactions.

J. C. H. D. S. C.

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19.2

Introduction

Everyone must make decisions. How well these decisions are made affects the success of individuals, the performance of organizations, and ultimately the well-being of society. Many decision problems are deeply rooted in the economic problem of allocating scarce resources.

In recent years, the industrialized world has witnessed the rapid development of quantitative tools for solving increasingly complex management problems. The primary purpose of this book is to present and illustrate these quantitative analyses.

1.1 Economizing Resources and Decision Making

The fundamental problem of economics and business is the need to choose among alternative uses of scarce resources in order to achieve the greatest return from them. The key words are *scarce*, *alternative*, and *greatest*.

Scarcity means that resources are not free; any scarce resource has a price attached to it. If resources were free, there would be no economic problem—everyone would be able to obtain enough of everything to satisfy all wants. Although air may still be a free resource in such places as Alaska and Wyoming, it can no longer be considered free (at least not to society, which must pay for keeping it fit to breathe) in such cities as Pittsburgh and New York. The supply of all resources (including land, labor, capital, and managerial ability) is limited; we must use them efficiently to receive the optimal return from them.