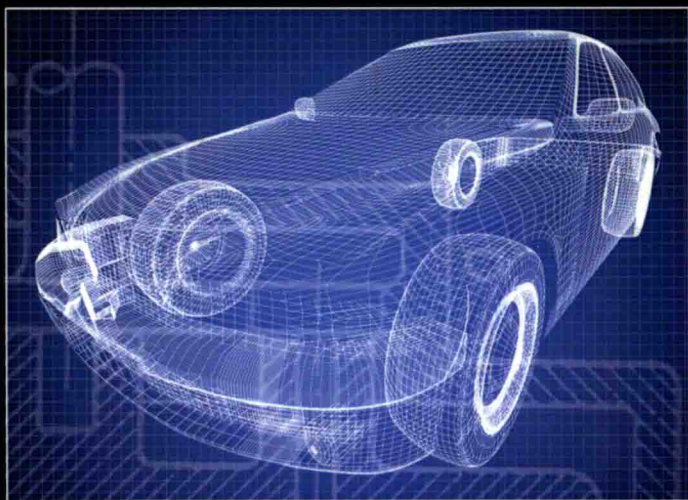


SECOND EDITION

**VEHICLE
DYNAMICS,
STABILITY,
AND CONTROL**



DEAN KARNOPP



CRC Press
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Preface

This book is the second edition of a book originally titled *Vehicle Stability*. The new title, *Vehicle Dynamics, Stability, and Control*, better describes this revised and extended version.

This book is the result of two activities that have given the author a great deal of pleasure and satisfaction over a period of more than forty years. The first was the initiation and teaching of a course for seniors and first-year graduate students in mechanical and aerospace engineering at the University of California, Davis. The course is intended to illustrate the application of techniques the students had learned in courses dealing with such topics as kinematics, rigid body dynamics, system dynamics, automatic control, stability theory, and aerodynamics to the study of the dynamic behavior of a number of vehicle types. In addition, specialized topics dealing specifically with vehicle dynamics such as the force generation by pneumatic tires, railway wheels and wings are also presented.

The second activity was a short course entitled "Vehicle Dynamics and Active Control," given by the author and his colleague, Professor Donald Margolis, numerous times in the United States and in several European countries. This short professional course was intended primarily for engineers in the automotive industry.

Although the short course for engineering professionals contained much of the material found in the academic course and in the present book, it was specialized in that it dealt only with automotive topics. The unique feature of the present book lies in its treatment of the dynamics and stability aspects of a variety of vehicle types. Anyone who has experience with vehicles knows that stability (or instability) is one of the most intriguing and mysterious aspects of vehicle dynamics. Why does a motorcycle sometimes exhibit a wobble of the front wheel when ridden "no hands" or a dangerous weaving motion at high speed? Why would a trailer suddenly begin to oscillate over several traffic lanes just because its load distribution is different from the usual? Why does a locomotive begin "hunting" back and forth on the tracks when traveling a high speed? Why is an airplane hard to fly when the passenger and luggage load is too far to the rear? Could it be that a car or truck could behave in an unstable way when driven above a critical speed? In addition, there are control questions such as "How can humans control an inherently unstable vehicle such as a bicycle?"

Many of these questions are answered in the book using the analysis of linear vehicle dynamic models. This allows the similarities and critical differences in the stability properties of different vehicle types to be particularly easily appreciated. Although analysis based on linearized mathematical models cannot answer all questions, general rigid body dynamics

and nonlinear relations relating to force generation are discussed for several cases. Furthermore, many of the nonlinear aspects of vehicle dynamics are discussed, albeit often in a more quantitative manner.

It is possible, of course, to extend the models beyond the range of small perturbation inherent in the analysis of stability. Through the use of computer simulation, for example, one can discover the behavior of unstable vehicles when the perturbation variables grow to an extent that the linearized equations are no longer valid.

An aspect of the professional course that was of particular interest to working engineers was the discussion of active means of influencing the dynamics of automobiles. It is now fairly common to find active or semi-active suspension systems, active steering systems, electronically controlled braking systems, torque vectoring drive systems, and the like that actively control the dynamics of automotive vehicles. Many of these control systems follow the lead of similar systems first applied in aircraft. Throughout the book, whenever it is appropriate, the idea that active means might be used to improve the dynamics of a vehicle is presented. In particular, Chapter 11 discusses some of the active means used to improve the dynamics of vehicles such as cars and airplanes.

Since all the studies of vehicle dynamics begin with the formulation of mathematical models, there is a great deal of emphasis in this book on the use of methods for formulating equations of motion. Chapter 2 deals with the description of rigid body motion. In basic dynamics textbooks, the use of a body-fixed coordinate system that proves to be very useful to describe the dynamics of many vehicle types is typically not discussed at all. Chapter 2 describes the fundamental principles of mechanics for rigid bodies as well as the general equations when expressed in a body-fixed coordinate system.

In other chapters, the more conventional derivations of the equations of motion using Newton's laws directly or Lagrange equations with inertial coordinate systems are illustrated by a series of examples of increasing levels of complexity. This gives the reader the opportunity to compare several ways to formulate equations of motion for a vehicle.

For those with some familiarity with bond graph methods for system dynamics, Chapter 2 contains a section that shows how rigid body dynamics using a body-fixed coordinate frame can be represented in a graphical form using bond graphs. The Appendix gives complete bond graph representations for an automobile model used in Chapter 6 and for a simplified aircraft model used in Chapter 9. A number of bond graph processing programs are available. This opens up the possibility of using computer automated equation formulation and simulation for nonlinear versions of the linearized models used for stability analysis.

The academic course has proved to be very popular over the years for several reasons. First of all, many people are inherently interested in the dynamics of vehicles such as cars, bicycles, motorcycles, airplanes, and trains. Second, the idea that vehicles can exhibit unstable and dangerous

behavior for no obvious reason is in itself fascinating. These instabilities are particularly obvious in racing situations or in speed record attempts, but in everyday life it is common to see trailers swaying back and forth or to see cars slewing around on icy roads and to wonder why this happens. Third, for those with some background in applied mathematics, it is always satisfying to see that relatively simple mathematical models can often illuminate dynamic behavior that would otherwise be baffling.

The book does not attempt to be a practical guide to the design or modification of vehicles. The reader will have an appreciation of how an aircraft designer goes about designing a statically stable aircraft but will not find here a complete discussion of the practical knowledge needed to become an expert. The reader will gain an appreciation of the automotive terms understeer, oversteer, and critical speeds, for example, but there are no rules of thumb given for modifying an autocross racer. The References do, however, include books and papers that will prove helpful in building up a practical knowledge base relating to a particular vehicle dynamics problem.

For those interested in using the book as a text, it is highly recommended that experiments and demonstrations be used in parallel with the classroom lectures. The University of California, Davis is fortunate to be located near the California Highway Patrol Academy, and their driving instructors have been generous in giving demonstrations of automobile dynamics on their high-speed track and their skid pans. The students are always impressed that the instructors use the same words to describe the handling of patrol cars that the engineers use such as “understeer” and “oversteer” but without using the formal definitions that engineers prefer. Also, a trip to a local dirt track provides a demonstration of the racers’ terms “tight” and “loose” as well as some spectacular demonstrations of unstable dynamic behavior.

Furthermore, our university has its own airport. This has permitted the students to experience aircraft dynamics personally as well as analytically in the discussions of Chapter 9. The relation between stability and control is much more obvious as a passenger in a small airplane than it is in commercial aircraft. A good pilot can easily demonstrate several oscillatory modes of motion (these are all stable for production small airplanes except possibly for the low-frequency phugoid mode) and, if the passengers are willing, can show the beginnings of some divergent modes of motion for extreme attitudes.

In addition, it is possible to design laboratory experiments to illustrate many of the analyses in the book. As examples, we have designed a demonstration of trailer instability using a moving belt and a stationary model trailer as well as a small trailer attached to a three-wheeled bicycle. These models show that changes in center of mass location and moment of inertia do indeed influence stability just as the theory in Chapter 5 predicts. Model gliders have been designed to illustrate static stability and instability as discussed in Chapter 9. Even a rear-steering bicycle was fabricated to illustrate the control difficulties described in Chapter 7.

A number of exercises are included that may be assigned if the book is used as a text. (A solutions manual for instructors is available.) Some of these problems are included to help students appreciate assumptions behind derivations given in the book. Other problems extend the analyses of the corresponding chapter to new situations or relate topics in one chapter to other chapters. Still other problems are of a much more extensive nature and can form the basis of small projects. They are intended to illustrate how mathematical models of varying degrees of complexity can be used to suggest design rules for improving the dynamics, stability, and control of vehicles.

The author hopes that the readers of this book will be as fascinated with vehicle dynamics, stability, and control as he is and will be inspired to learn even more about these topics.

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Introduction: Elementary Vehicles

Vehicles such as cars, trains, ships, and airplanes are intended to move people and goods from place to place in an efficient and safe manner. This book deals with certain aspects of the motion of vehicles usually described using the terms “dynamics, stability, and control.” Although most people have a good intuitive idea what these terms mean, this book will deal with mathematical models of vehicles and with more precise and technical meaning of the terms. As long as the mathematical models reasonably represent the real vehicles, the results of analysis of the models can yield insight into the actual problems that vehicles sometimes exhibit, and in the best case, can suggest ways to cure vehicle problems by modification of the physical aspects of the vehicle or the introduction of automatic control techniques.

All vehicles represent interesting and often complex dynamic systems that require careful analysis and design to make sure that they behave properly. In particular, the stability aspect of vehicle motion has to do with assuring that the vehicle does not depart spontaneously from a desired path. It is possible for an automobile to start to spin out at high speed or a trailer to begin to oscillate back and forth in ever-wider swings seemingly without provocation. Most of this book deals with how the physical parameters of a vehicle influence its dynamic characteristics in general and its stability properties in particular.

The control aspect of vehicle motion has to do with the ability of a human operator or an automatic control system to guide the vehicle along a desired trajectory. In the case of a human operator, this means that the dynamic properties of such vehicles should be tailored to allow humans to control them with reasonable ease and precision. A car or an airplane that requires a great deal of attention to keep it from deviating from a desired path would probably not be considered satisfactory. Modern studies of the Wright brother's 1903 Flyer indicate that, while the brothers learned to control the airplane, it apparently was inherently so unstable that modern pilots are reluctant to fly an exact replica. The Wright brothers, of course, did not have the benefit of the understanding of aircraft stability that aeronautical engineers now have. Modern light planes can now be designed to be stable enough that flying them is not the daunting task that it was for the Wrights.

For vehicles using electronic control systems, the dynamic properties of the vehicle must be considered in the design of the controller to assure that the controlled vehicles are stable and have desirable dynamics. Increasingly, human operators exercise supervisory control of vehicles with automatic

control systems. In some cases, the control system must stabilize an inherently unstable vehicle so that it is not difficult for the human operator to control its trajectory. This is the case for some modern fighter aircraft. In other cases, the active control system simply aids the human operator in controlling the vehicle. Aircraft autopilots, fly-by-wire systems, antilock braking systems, and electronic stability enhancement systems for automobiles are all examples of systems that modify the stability properties of vehicles with active means to increase the ease with which they can be controlled. Active stability enhancement techniques will be discussed in Chapter 11 after the dynamic properties of several types of vehicles have been analyzed.

In some cases, vehicle motion is neither actively controlled by a human operator nor by an automatic control system but yet the vehicle may exhibit very undesirable dynamic behavior under certain conditions. A trailer being pulled by a car, for example, should obviously follow the path of the car in a stable fashion. As we will see in Chapter 5, a trailer that is not properly designed or loaded may however exhibit growing oscillations at high speeds, which could lead to a serious accident. Trains that use tapered wheelsets are intended to self-center on the tracks without any active control, but above a critical speed, an increasing oscillatory motion called hunt can develop that may lead to derailment in extreme cases. For many vehicles, this type of unstable behavior may suddenly appear as a critical speed is exceeded. This type of unstable behavior is particularly insidious since the vehicle will appear to act in a perfectly normal manner until the first time the critical speed is exceeded, at which time the unstable motion can have serious consequences.

This book will concentrate on the mathematical description of the dynamics of vehicles. An important topic that can be treated with some generality involves the stability of the motion of vehicles. Other aspects of vehicle dynamics to be discussed fall into the more general category of “vehicle handling” or “vehicle controllability” and are of particular importance under extreme conditions associated with emergency maneuvers.

Obviously, vehicle stability is of interest to anyone involved in the design or use of vehicles, but the topic of stability is of a more general interest. Since the dynamic behavior of vehicles such as cars, trailers, and airplanes is to some extent familiar to almost everybody, these systems can be used to introduce a number of concepts in system dynamics, stability, and control. To many people, these concepts seem abstract and difficult to understand when presented as topics in applied mathematics without some familiar physical examples. Everyday experience with a variety of vehicles can provide examples of these otherwise abstract mathematical concepts.

Engineers involved in the design and construction of vehicles typically use mathematical models of vehicles in order to understand the fundamental dynamic problems of real vehicles and to devise means for controlling vehicle motion. Unfortunately, it can be a formidable task to find accurate mathematical descriptions of the dynamics of a wide variety of vehicles. Not

only do the descriptions involve nonlinear differential equations that seem to have little similarity from vehicle type to vehicle type, but in particular, the characterization of the force-producing elements can be quite disparate. One can easily imagine that rubber tires, steel wheels, boat hulls, or airplane wings act in quite different ways to influence the motions of vehicles operating on land, on water, or in the air.

On the other hand, all vehicles have some aspects in common. They are all usefully described for many purposes as essentially rigid bodies acted upon by forces that control their motion. Some of the forces are under control of a human operator, some may be under active control of an automatic control system, but all are influenced by the very nature of the force-generating mechanisms inherent to the particular vehicle type. This means that not very many people claim to be experts in the dynamics of a large number of types of vehicles.

When describing the stability of vehicle motion, however, the treatment of the various types of force-generating elements exhibits a great deal of similarity. In stability analysis, it is often sufficient to consider small deviations from a steady state of motion. The basic idea is that a stable vehicle will tend to return to the steady motion if it has been disturbed while an unstable vehicle will deviate further from the steady state after a disturbance. The mathematical description of the vehicle dynamics for stability analysis typically uses a linearized differential equation form based on the nonlinear differential equations that generally apply. The linearized equations show more similarity among vehicle types than the more accurate nonlinear equations. Thus, a focus on stability allows one to appreciate that there are interesting similarities and differences among the dynamic properties of a variety of vehicle types without being confronted with the complexities of nonlinear differential equation models.

In this chapter, stability analyses will be performed for two extremely simplified vehicle models to illustrate the approach. In later chapters more realistic vehicle models will be introduced and it will become clear that despite some analogous effects among vehicle types, ultimately the differences among the force-generating mechanisms for various vehicle types determine their behavior particularly under more extreme conditions than are considered in stability analyses. Complicated mathematical models are routinely used in the design of many vehicle types and are studied using computer simulation. Such models often contain so many parameters that it is not easy to see how to solve dynamic problems that may arise. In this book we will restrict the discussion mainly to relative simple but insightful models that are particularly good at illuminating stability problems.

In this introductory chapter, the two examples that will be analyzed require essentially no discussion of force-generating elements such as tires or wings. They do, however, introduce the basic ideas of vehicle stability analysis. The first example is actually kinematic rather than dynamic in the usual sense, since Newton's laws are not needed. The second example is