just the basics, please

a quick review of math for introductory statistics

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To the Student

Congratulations on your purchase of this book. This tells us that you are serious about brushing up on those areas of mathematics that are considered prerequisite to the statistics course in which you are currently enrolled.

We prepared this book with you in mind. We anticipate that you fit one of three student profiles that we have encountered in our statistics classes:

--- You took the prerequisite math course(s) a *long time* ago and you feel certain that you have forgotten most of the material. We're talking years since you had a math course. You can probably remember all the words to a popular song that was playing when you took that math course; but, do you remember the math?

--- You took the prerequisite math course(s) recently, but you did not have a good experience with the class(es). Either you didn't connect with your instructor(s) or you feel you just didn't learn anything for a variety of reasons.

--- You are feeling a bit insecure about your math background. You may or may not have taken the prerequisite math course(s). However, math has never been your favorite subject, so you're not sure if you're really prepared for this statistics course or not.

Now that we have established that your math skills are not as sharp as you'd like, what can we do about it? Together we hope you will work through this book at your own pace to fulfill your special needs. You may not need to study every chapter in this book. If you have the time to work through the whole book, we strongly encourage you to do so. If not, focus on the one or two chapters in this book that most relates to your current statistics material. In a very rough sense the chapters in this book fall into one of two categories: fundamental arithmetic or algebra topics or intermediate topics that directly tie into almost all statistics courses. In terms of the chapters included within each category consider the following flow chart:

Fundamental
Chapters 1, 4, 8

↓

Intermediate
Chapters 2, 3, 5, 6, 7

Everyone should read and study the fundamental chapters (1, 4, and 8). The remaining intermediate chapters (2, 3, 5, 6, and 7) could be read on a just-in-time basis.

At the beginning of the book is a chart that cross references the chapter material in this book with the material in your current statistics book. You will have to bear with us a bit as we can't know exactly which chapter in your statistics book corresponds to our material. (There are a lot of statistics books out there!) We've tried to show you the tie-in using major topical headings. It's up to you to translate the topical headings into a chapter number in your text.

To use the material in this book we recommend several approaches - Plans A, B, and C. Each chapter opens with a pretest and closes with a posttest; the answers to all questions are in the back.

Plan A involves starting the chapter by taking the pretest to see how much you really know. Point values for each question and a time limit are included with each pretest. Each test is self-graded; don't be too easy on yourself! Only give yourself a point if you get the correct answer. To be up to speed with the chapter material, you should either have a perfect score on the pretest or miss at most 1 point.

If you score this well, immediately go to the posttest at the end of the chapter and take it. Again, if you make a perfect score (or are within a point of a perfect score), then you should skip the chapter because you are about as proficient with that chapter material as you can get.

On the other hand, if you don't ace the pretest and/or the posttest, then there is room for improvement. We suggest Plan B. If you have a tendency to be a bit impatient, or you don't wish to read all of the text in each chapter, look for the Summary Examples in each chapter. Read and study each of these Summary Examples and then try to solve the chapter exercises. The answers are in the back. Work as many exercises as needed until you feel comfortable with the material (and you consistently get the correct answers to the exercises). To validate your competency with the material you might take the Posttest again. Presumably you should earn a higher score this time around.

If you still don't feel really sure of yourself, then go to Plan C, which means you should read the entire chapter -- text and all boxed material -- and work all the chapter exercises before tackling the Posttest.

In summary, here are the options for tackling each chapter:

HOW TO STUDY THE MATERIAL IN EACH CHAPTER

PLAN	IF YOU ARE THIS TYPE OF PERSON	DO THIS	HOW TO JUDGE YOUR MASTERY OF THE MATERIAL
A	Impatient	Take the chapter pretest and posttest.	Earn a perfect score (or close to it) on each test.
B	Prone to make careless errors	Take the chapter pretest. Then read all the Summary Examples and work some of the chapter exercises. Take the posttest.	You get the correct answers for most of the chapter exercises and you ace the posttest.
C	Slow but steady	Take the chapter pretest. Then read and work through the entire chapter and the chapter exercises. Take the posttest.	You ace the posttest.

HAVE FUN AND GOOD LUCK!

We would like to recognize the extraordinary efforts of our typist, Cindy McDonald, who created all the beautiful pages in this book; our editor, Curt Hinrichs, who saw the wisdom in publishing a just-in-time book for you; and our production editor, Sheryl Gilbert, and Janis Brown, editorial assistant, who handled all the details to make the book publishing process smooth and trouble free. Also the following reviewers supplied many good tips and suggestions for improvement:

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Bruce Trumbo, California State University -- Hayward
Paul Paschke -- Oregon State University

Thanks, gang!

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CHAPTER 1
Arithmetic

PRETEST #1 - Allow 20 Minutes
1 point each -- maximum score 10

1.1 Evaluate the following expression:

$$1.7 + \frac{.7}{25}(16 - 6) = ?$$

1.2 Convert .105 to a percent.

1.3 Convert 81.3% to a decimal.

1.4 Round-off 345.0149 to three decimal places.

1.5 Second quarter sales were $1.25 million. What is the percent change in sales, relative to first quarter sales of $.85 million? Round to one decimal place.

1.6 Use an inequality sign ($<$ or $>$) to show which number is bigger, -1.23 or -1.32.

1.7 $|-7| = ?$

1.8 Rearrange the following numbers in order from low to high:

$$6, 0, -1, 1, -2, -3$$

1.9 Round the number 509,500 to the nearest thousand.

1.10 True or false: $|-5| < |3|$?

When we add, subtract, multiply, or divide, we are doing *arithmetic*. As elementary school students, we may have learned how to do these operations by hand, but today most of us rely on a hand-held calculator to do the number crunching for us. Unless we put a wrong number into the calculator, we are well-assured that the resulting number in the calculator display is correct. But when we have to perform a calculation that mixes the operations, we cannot be certain that our calculator yields the correct answer. We may have forgotten the correct sequence with which adding, subtracting, multiplying, or dividing should be performed or our calculator may not be programmed to perform the operations in the correct sequence. Let us start with a review of the correct ordering of arithmetic operations.

1.1 Order of Operations

Arithmetic operations, in general, conform to the following priority schedule:

☞ **RULES**

PRIORITY OF ARITHMETIC OPERATIONS

1. *Exponentiation (raising a number to a power)*
2. *Multiplication or division*
3. *Addition or subtraction*

The priority schedule implies that when an expression involves all three operations, we should first concentrate our efforts on the exponents before we multiply or divide. Adding or subtracting comes last. For example, the expression $4 + 2 \cdot 3^2$ would be evaluated first by squaring 3 (to get 9); second, multiplying by 2 (to get 18); and third, adding 4 (to get 22): $4 + 2 \cdot 3^2 = 4 + 2 \cdot 9 = 4 + 18 = 22$.

The only exception to the arithmetical operation priority schedule occurs when parentheses are present in expressions. The operation within the parentheses then takes precedence over any other operation. For example, consider the following:

$$(1 - 5)^2 + (4 - 5)^2 + (10 - 5)^2$$

In this case we will work within the parentheses first and perform the indicated subtractions, then we will square, and finally we will add the squares:

$$\begin{aligned}(1 - 5)^2 + (4 - 5)^2 + (10 - 5)^2 &= (-4)^2 + (-1)^2 + (5)^2 \\ &= 16 + 1 + 25 \\ &= 42\end{aligned}$$

One "trick" of subtraction that we sometimes forget occurs when we subtract a negative number, such as subtracting -7 from 24:

$$24 - (-7) = 24 + 7 = 31$$

The double minus signs turn into a positive sign and we end up adding.

Our need to understand the correct sequence of operations will become apparent when we study descriptive statistics. For instance, when we are finding the median for a grouped frequency distribution, we might encounter a computation like this:

$$1.10 + \frac{.92}{8}(10 - 4)$$

The correct answer of 1.79 is found first by performing the subtraction inside the parentheses to get 6; second, multiplying .92 by 6 and then dividing by 8 to get .69; and third, adding 1.10 to .69.

SUMMARY EXAMPLE
Order of Operations

Evaluate $83.2 + \frac{8.4}{54}(34 - 25)$

Do this first:	**Work within parentheses**	$(34 - 25) = 9$
Do this second:	**Multiply/divide**	$\frac{8.4}{54}(9) = 1.4$
Do this third:	**Add**	$83.2 + 1.4 = 84.6 \leftarrow$ **Answer**

Does this statement sound familiar: "I know the rules; I just don't know how to *apply* them to a word problem." Word problems (or story problems, as some people call them) wrap a short story around a specific math skill. Your job is twofold -- understand the story and then decipher which rule or formula applies to the problem. While not all word problems are solved in the same manner, there are some actions you can take to help you get started. Below are recommended problem solving diagnostics for word problems. Each may not apply to every problem.

Problem Solving Diagnostics for Word Problems

1. Underline or highlight key words in the story.
2. Circle all numbers given in the problem and try to attach a symbol or meaning to each number. (Note that some numbers may be meaningless and can be ignored.)
3. Draw a rough picture that captures the image of what is being described, if applicable.
4. Find the sentence with the question mark (?) at the end of it. Read it over and over until you can write down exactly what quantity, symbol, or term you are to find.
5. Write down the appropriate formula and try to apply it, if applicable. If there is no appropriate formula, start making some preliminary calculations. Each time your calculations yield a result, ask yourself "What does this number represent?"

WORD PROBLEM EXAMPLE

As an example, consider the following word problem:

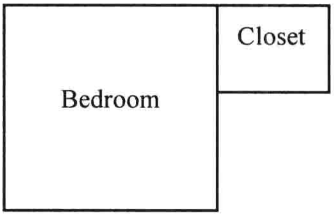
April and Christy share a two bedroom apartment on 3rd Street. A series of recent storms caused the creek behind their apartment to flood, damaging the carpet in Christy's bedroom. The bedroom measures 10 feet wide by 12 feet long, plus a closet that measures 3 feet deep by 5 feet across. Assuming no waste, how many square yards of new carpet do they need to buy?

Let's go through the Problem Solving Diagnostics step-by-step:

1. Key words: bedroom, closet, square yards

2. Numbers: two bedroom (meaningless)
   3rd Street (meaningless)
   10 feet (width of bedroom)
   12 feet (length of bedroom)
   3 feet (width of closet)
   5 feet (length of closet)

3. Picture:



4. Question mark: Find the area of the bedroom plus the closet floor (in square yards).

5. Computations:

$$\text{Area} = \text{length x width}$$
$$= \underbrace{(12 \text{ X } 10)}_{\text{area of bedroom}} + \underbrace{(5 \text{ X } 3)}_{\text{area of closet}}$$
$$= 120 + 15 = 135 \text{ square feet}$$
$$\text{Square yards} = \text{Square feet}/9$$
$$= 135/9$$
$$= 15 \text{ square yards}$$

Notice in step #5 that we *applied* our knowledge of the priority of arithmetic operations: we multiplied first and then added. Also note that we had to convert square feet to square yards, so we had to know that 9 square feet = 1 square yard.

1.2 Decimals, Percents, and Percent Change

A *decimal fraction* is a number between 0 and 1 representing a ratio of two numbers in which the denominator must be 10, 100, 1000, 10000 and so on. For example, .25, .1, and .873 are *decimal fractions* since they can be expressed as the following ratios:

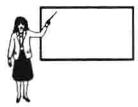
$$.25 = \frac{25}{100},\ .1 = \frac{1}{10},\ \text{and}\ .873 = \frac{873}{1000}$$

In conversation, the term "decimal fraction" is sometimes shortened to the term *decimal*.

Often, we wish to change these decimals into whole numbers (and fractions thereof) by converting them to percents. A *percent* is a number between 0 and 100 representing the numerator of a decimal fraction in which the denominator must be 100. Thus, 25%, 10%, and 87.3% are percents since they are the numerators of the following decimals:

$$\frac{25}{100},\ \frac{10}{100}\ ,\ \text{and}\ \frac{87.3}{100}$$

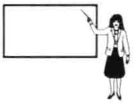
Our interest in percents and decimals is concerned with being able to obtain one from the other. To convert a decimal into a percent, move the decimal point two places to the right and attach the percent symbol (%).



SUMMARY EXAMPLE
Decimal to Percent

Convert .049 to a percent		
Do this first:	**Move decimal point two places to the *right***	.049 becomes 04.9
Do this second:	**Attach % symbol**	4.9% ← **Answer**

Conversely, to create a decimal from a percent, drop the percent sign and move the decimal two places to the left as indicated in the example.



SUMMARY EXAMPLE
Percent to Decimal

Convert .91% to a decimal		
Do this first:	**Drop % symbol**	.91% becomes .91
Do this second:	**Move decimal point two places to the *left***	.91 becomes .0091 ← **Answer**

A common use of percents is in reporting a percent change from one time period to the next. A *percent change* is the percent difference in two values relative to one of the values. A percent change differs from a percent in that a percent change is <u>not</u> restricted to being a number between 0 and 100. For example, a percent change could be –15% or 200%. In general, to find a percent change we use the "old" value as our point of reference. Here is the basic formula for finding a percent change:

☞ FORMULA

PERCENT CHANGE RELATIVE TO OLD VALUE

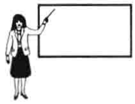
$$\textit{Percent Change} = \frac{\textit{New value} - \textit{Old value}}{\textit{Old value}} \times 100\%$$

For instance, suppose during the first quarter of last year an automaker sold 25,000 vehicles. During the first quarter of this year, the automaker sold 24,000 vehicles. Then the percent change in sales relative to the first quarter of last year is:

$$\textit{Percent change} = \frac{24{,}000 - 25{,}000}{25{,}000} \times 100\%$$

$$= \frac{-1{,}000}{25{,}000} \times 100\% = -.04 \times 100\% = -4\%$$

Sales were down 4 percent from the same period in the previous year.



SUMMARY EXAMPLE
Percent Change

Attendance at last year's show was 16,000 people. This year 16,800 people attended the show. What was the percent change in attendance, relative to last year's show?

Do this first:	***Subtract:*** ***New − Old***	16,800 − 16,000 = 800
Do this second:	***Divide:*** $\frac{New - Old}{Old}$	$\frac{800}{16,000} = .05$
Do this third:	***Multiply:*** $\frac{New - Old}{Old}$ X 100%	.05 X 100% = 5% ↑ **Answer** **Attendance increased 5 percent over last year's figure.**

We will encounter the concept of percent change in the study of time series. For time series applications we will be interested in the percent error in a forecast relative to the actual value. The formula for percent error is:

☞ FORMULA

PERCENT ERROR RELATIVE TO ACTUAL VALUE

$$\textit{Percent error} = \frac{\textit{Actual value} - \textit{Forecasted value}}{\textit{Actual value}} \text{ X } 100\%$$

If, for example, the actual value were 8 and the forecasted value were 7.4 then the percent error in the forecast would be:

$$\textit{Percent error} = \frac{8 - 7.4}{8} X\ 100\% = \frac{.6}{8} X\ 100\% = .075\ X\ 100\% = 7.5\%$$

The actual value was 7.5% higher than the forecasted value.

1.3 Rounding

As we perform computations that involve multiplication, division, and raising numbers to non-integer powers, we may produce decimal fractions with many digits. Although rounding may be necessary, we caution against rounding intermediate calculations. Round only the final answer, as indicated in our round-off rule for decimals below.

☞ **RULES**

ROUNDING OFF DECIMALS

1. *Determine the number of decimal places desired to the right of the decimal.*
2. *Examine the digit in the succeeding decimal place.*
   a. *If the digit is 5 or more, increase the preceding digit by 1 and eliminate all succeeding digits.*
   b. *If the digit is 4 or less, leave the preceding digit as is and eliminate all succeeding digits.*

Let us demonstrate this rule using the number 7.149501. If we wished to round-off to one decimal place, we would examine the digit 4 in the second decimal place and, according to part 2b above, round the number to 7.1. To round to two decimal places, the digit 9 in the succeeding decimal place forces us to increase the 4 in the second decimal place by one, so that the rounded value is 7.15. Rounding to three decimal places yields 7.150: the digit 5 in the fourth decimal place causes us to increase the 9 in the third position by one, which, in turn, increases the 4 in the second position by one.

One variation of our rounding procedure -- *rounding up* -- will occur occasionally in statistics. The round-up rule for decimals is stated next.