TAN

Applied Calculus

For the
Managerial,
Life,
and Social
Sciences

6TH EDITION

APPLIED CALCULUS

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

SIXTH EDITION

S. T. TAN

STONEHILL COLLEGE

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PREFACE

Applied Calculus for the Managerial, Life, and Social Sciences, Sixth Edition, is suitable for use in a two-semester or three-quarter introductory calculus course for students in the managerial, life, and social sciences. As with the previous editions, our objective in *Applied Calculus for the Managerial, Life, and Social Sciences* is twofold: (1) to write a textbook that is readable by students and (2) to make the book a useful teaching tool for instructors. We hope that with the present edition we have come one step closer to realizing our goal. The sixth edition of this text incorporates many suggestions by users of the earlier editions.

Features

The following list includes some of the many important features of the book:

- **Coverage of Topics** This book contains more than enough material for the usual applied calculus course. Optional sections have been marked with an asterisk in the table of contents, thereby allowing the instructor to be flexible in choosing the topics most suitable for his or her course.
- **Approach** A problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to amplify each new concept or result in order to facilitate students' comprehension of the material. Figures are used extensively to help students visualize concepts and ideas.
- **Level of Presentation** Our approach is intuitive, and we state the results informally. However, we have taken special care to ensure that this approach does not compromise the mathematical content and accuracy. Proofs of certain results are given, but they may be omitted if desired.
- **Applications** The text is application oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. Some of these applications have their source in newspapers, weekly periodicals, and other magazines. Applications are found in the illustrative examples in the main body of the text as well as in the exercise sets. In fact, one of the author's goals is to include at least one real-life application in each section (whenever feasible).
- **Sources** We have included sources for those applications that are based on real-life data.
- **Exercises** Each section of the text is accompanied by an extensive set of exercises containing an ample set of problems of a routine, computational nature that will help students master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test students' mastery of the topics. A large number of these applications are based on mathematical models (functions) that the author has constructed using data drawn from various sources. In these exercises, the source has been cited at the end of each exercise. In Functions and Mathematical Models (Section 2.3), students are asked to sketch the graphs

of functions describing real-life data (HMO membership and the market for cholesterol-reducing drugs). In the Using Technology subsection that follows, students are actually shown how to construct a function describing real-life data (Indian gaming industry) using a graphing calculator. This is followed-up by exercises where students are asked to construct mathematical models from real-life data.

- **Self-Check Exercises** Every section has self-check exercises, with solutions, to help students monitor their own progress.
- **Portfolios** These interviews are designed to convey to the student the real-world experiences of professionals who have a background in mathematics and use it in their professions.
- **Group Discussion Questions** These are optional questions, appearing throughout the main body of the text, that can be discussed in class or assigned as homework. These questions generally require more thought and effort than the usual exercises. They may also be used to add a writing component to the class. Complete solutions to these exercises are given in the *Complete Solutions Manual.*

Technology

Exploring with Technology Questions

These optional questions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Complete solutions to these exercises are given in the *Complete Solutions Manual.*

Using Technology Subsections

The Using Technology pages contain optional material and are placed at the end of the sections for which their use is appropriate. The subsections are written in the traditional example–exercise format, with answers given at the back of the book. Illustrations showing graphing calculator screens are extensively used. Step-by-step instructions (including keystrokes) for many popular calculators are given at the Web site. These subsections may be used in the classroom if desired or as material for self-study by the student.

As many up-to-date and relevant applications have been introduced in these subsections, they provide students with an opportunity to interpret results in a real-life setting.

Student Resources on the Web

Students and instructors will now have access to these additional materials at the Brooks/Cole World Wide Web site: http://series.brookscole.com/tan

- Review material and practice chapter quizzes and tests
- Group projects and extended problems for each chapter
- Instructions, including keystrokes, for the procedures referenced in the text for specific calculators (TI-82, TI-83, TI-85, TI-86, and other popular models)
- Coverage of additional topics such as Indeterminate Forms and L'Hopital's Rule

New in the Sixth Edition

- More than 100 new real-life applications exercises have been added. Examples that include sourced data are Starbuck's annual sales, the prevalence of Alzheimer's patients, working mothers, the alternative minimum tax, surgeries in

physicians' offices, socially responsible funds, women's soccer, the population growth in Clark County, the rising median-age, credit card debt, the sales of functional food products, annual college costs, and the oil production shortage. Examples of other new applications include aircraft structural integrity (the so-called bathtub curve), the profit from a vineyard, the charter revenue of a cruise ship, racetrack design, reaction time of a motorist, and the birthrate of an endangered species. Many new rote problems have also been added.

- In Chapter 2, the introduction to Section 2.2 has been rewritten and new charts have been added for the application of the difference of two functions (Budget Deficit and Surplus). In Section 2.3, the subsection on mathematical modeling has been rewritten with an emphasis on the process of modeling. A new application (The Market for Cholesterol Reducing Drugs) has been added to illustrate the process.
- In Chapter 5, two new examples showing how logarithms can be used to solve for r (rate) and t (time) in interest rate problems have been added. Related exercises have been added to the Exercise Set 5.3. A new Using Technology subsection—Analyzing Mathematical Models—has been added to the section on Exponential and Logarithmic Functions (Section 5.6). Here, again, real-life applications are presented in the accompanying exercise set for students to analyze.
- In Appendix A, a more extensive treatment of inverse functions is given.
- Many new questions have been added to the application exercises that ask students to interpret their results. The emphasis here is to help students understand the meaning of numerical results.
- More than 30 technology screens have been added to the Using Technology sections. These screens show the actual input used in solving these problems.
- The answers to all of the review questions (both even and odd problems) are now given in the answer section. Additionally, answers to problems are now given in both simplified and un-simplified form where appropriate.
- Other changes: In Chapter 3 the discussion of revenue functions has been rewritten and the application involving inflation has also been rewritten. In Section 7.2, a new example on using the table of integrals has been added. In Section 8.3, the discussion of relative extrema has been rewritten and a new figure has been added. In Section 8.6, the artwork for the discussion of double integrals has been redone. In Section 10.2, a new subsection on Joint Probability Density Functions has been added.

Teaching Aids

- *Complete Solutions Manual* includes solutions to all exercises. ISBN 0-534-46506-4
- *Instructor's Suite CD* contains complete solutions to all exercises, along with Microsoft® PowerPoint® slide presentations and test items for every chapter, in formats compatible with Microsoft Office. ISBN 0-534-46505-6
- *Printed Test Bank,* by Tracy Wang, is free to adopters of the book. ISBN 0-534-46504-8
- *BCA/iLrn Testing*, available online or on CD-ROM, browser-based *BCA/iLrn Testing* is fully integrated testing and course management software. With no need for plug-ins or downloads, *BCA/iLrn* offers algorithmically generated problem values and machine-graded free response mathematics. ISBN 0-534-46508-0

Learning Aids

- *Student Solutions Manual*, available to both students and instructors, includes the solutions to odd-numbered exercises. ISBN 0-534-46507-2
- *WebTutor Advantage for WebCT & Blackboard,* by Larry Schroeder, Carl Sandburg College, contains expanded online study tools including: step-by-step lecture notes; student study guide with step-by-step TI-89/92/83/86 and Microsoft Excel explanations; a quick check interactive student problem for each online example, with accompanying step-by-step solution and step-by-step TI-89/92/83/86 solution; practice quizzes by chapter sections that can be used as electronically graded online exercises, and much more. ISBN for WebCT 0-534-41903-8 & ISBN for Blackboard 0-534-41902-X
- *Succeeding in Applied Calculus: Algebra Essentials,* by Warren Gordon, Baruch College—City University of New York, provides a clear and concise algebra review. This text as written so that students in need of an algebra refresher may have a convenient source for reference and review. This text may be especially useful before or while taking most college level quantitative courses, including applied calculus and economics. ISBN 0-534-40122-8

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S. T. Tan

CONTENTS

**Sections marked with an asterisk are not prerequisites for later material.*

APPLIED CALCULUS

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1 Preliminaries

What sales figure can be predicted for next year? In Example 10, page 39, you will see how the manager of a local sporting goods store used sales figures from the previous years to predict the sales level for next year.

Dana White/PhotoEdit/PictureQuest

The first two sections of this chapter contain a brief review of algebra. We then introduce the Cartesian coordinate system, which allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. This chapter also covers straight lines. The slope of a straight line plays an important role in the study of calculus.

1.1 Precalculus Review I

Sections 1.1 and 1.2 review some of the basic concepts and techniques of algebra that are essential in the study of calculus. The material in this review will help you work through the examples and exercises in this book. You can read through this material now and do the exercises in areas where you feel a little "rusty," or you can review the material on an as-needed basis as you study the text. We begin our review with a discussion of real numbers.

The Real Number Line

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division.

Real numbers may be represented geometrically by points on a line. Such a line is called the **real number,** or **coordinate, line** and can be constructed as follows. Arbitrarily select a point on a straight line to represent the number 0. This point is called the **origin.** If the line is horizontal, then a point at a convenient distance to the right of the origin is chosen to represent the number 1. This determines the scale for the number line. Each positive real number lies at an appropriate distance to the right of the origin, and each negative real number lies at an appropriate distance to the left of the origin (Figure 1.1).

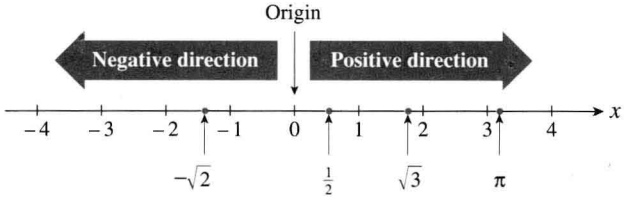


FIGURE 1.1
The real number line

A *one-to-one correspondence* is set up between the set of all real numbers and the set of points on the number line; that is, exactly one point on the line is associated with each real number. Conversely, exactly one real number is associated with each point on the line. The real number that is associated with a point on the real number line is called the **coordinate** of that point.

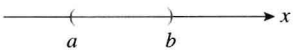
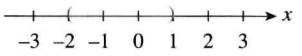
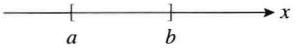
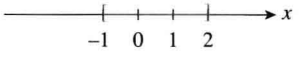
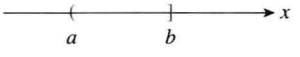
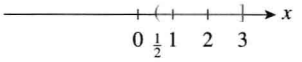
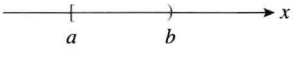
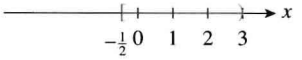
Intervals

Throughout this book, we will often restrict our attention to certain subsets of the set of real numbers. For example, if x denotes the number of cars rolling off a plant assembly line each day, then x must be nonnegative—that is, $x \geq 0$. Further, suppose management decides that the daily production must not exceed 200 cars. Then, x must satisfy the inequality $0 \leq x \leq 200$.

More generally, we will be interested in the following subsets of real numbers: open intervals, closed intervals, and half-open intervals. The set of all real numbers that lie *strictly* between two fixed numbers a and b is called an **open interval** (a, b). It consists of all real numbers x that satisfy the inequalities $a < x < b$, and it is called "open" because neither of its end points is included in the interval. A **closed interval** contains *both* of its end points. Thus, the set of all real numbers x that satisfy the inequalities $a \le x \le b$ is the closed interval $[a, b]$. Notice that square brackets are used to indicate that the end points are included in this interval. **Half-open intervals** contain only *one* of their end points. Thus, the interval $[a, b)$ is the set of all real numbers x that satisfy $a \le x < b$, whereas the interval $(a, b]$ is described by the inequalities $a < x \le b$. Examples of these **finite intervals** are illustrated in Table 1.1.

TABLE 1.1

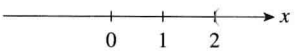
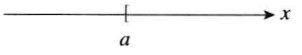
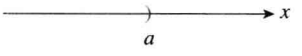
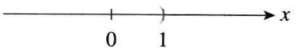
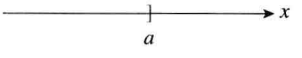
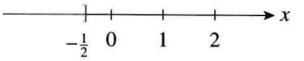
Finite Intervals

Interval		Graph	Example	
Open	(a, b)		$(-2, 1)$	
Closed	$[a, b]$		$[-1, 2]$	
Half-open	$(a, b]$		$\left(\frac{1}{2}, 3\right]$	
Half-open	$[a, b)$		$\left[-\frac{1}{2}, 3\right)$	

In addition to finite intervals, we will encounter **infinite intervals.** Examples of infinite intervals are the half lines (a, ∞), $[a, \infty)$, $(-\infty, a)$, and $(-\infty, a]$ defined by the set of all real numbers that satisfy $x > a$, $x \ge a$, $x < a$, and $x \le a$, respectively. The symbol ∞, called *infinity*, is not a real number. It is used here only for notational purposes in conjunction with the definition of infinite intervals. The notation $(-\infty, \infty)$ is used for the set of all real numbers x since, by definition, the inequalities $-\infty < x < \infty$ hold for any real number x. Infinite intervals are illustrated in Table 1.2.

TABLE 1.2

Infinite Intervals

Interval	Graph	Example	
(a, ∞)		$(2, \infty)$	
$[a, \infty)$		$[-1, \infty)$	
$(-\infty, a)$		$(-\infty, 1)$	
$(-\infty, a]$		$\left(-\infty, -\frac{1}{2}\right]$	

Properties of Inequalities

In practical applications, intervals are often found by solving one or more inequalities involving a variable. In such situations, the following properties may be used to advantage.

Properties of Inequalities

If a, b, and c, are any real numbers, then

		Example
Property 1	If $a < b$ and $b < c$, then $a < c$.	$2 < 3$ and $3 < 8$, so $2 < 8$.
Property 2	If $a < b$, then $a + c < b + c$.	$-5 < -3$, so $-5 + 2 < -3 + 2$; that is, $-3 < -1$.
Property 3	If $a < b$ and $c > 0$, then $ac < bc$.	$-5 < -3$, and since $2 > 0$, we have $(-5)(2) < (-3)(2)$; that is, $-10 < -6$.
Property 4	If $a < b$ and $c < 0$, then $ac > bc$.	$-2 < 4$, and since $-3 < 0$, we have $(-2)(-3) > (4)(-3)$; that is, $6 > -12$.

Similar properties hold if each inequality sign, $<$, between a and b is replaced by $\geq$, $>$, or $\leq$.

A real number is a *solution of an inequality* involving a variable if a true statement is obtained when the variable is replaced by that number. The set of all real numbers satisfying the inequality is called the *solution set.*

EXAMPLE 1 Find the set of real numbers that satisfy $-1 \leq 2x - 5 < 7$.

Solution Add 5 to each member of the given double inequality, obtaining

$$4 \leq 2x < 12$$

Next, multiply each member of the resulting double inequality by $\frac{1}{2}$, yielding

$$2 \leq x < 6$$

Thus, the solution is the set of all values of x lying in the interval $[2, 6)$. ■

EXAMPLE 2 Stock Purchase The management of Corbyco, a giant conglomerate, has estimated that x thousand dollars is needed to purchase

$$100{,}000(-1 + \sqrt{1 + 0.001x})$$

shares of common stock of Starr Communications. Determine how much money Corbyco needs to purchase at least 100,000 shares of Starr's stock.

Solution The amount of cash Corbyco needs to purchase at least 100,000 shares is found by solving the inequality

$$100{,}000(-1 + \sqrt{1 + 0.001x}) \geq 100{,}000$$