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Introduction to Physics

ИЗДАТЕЛЬСТВО · МИР ·

А. И. КИТАЙГОРОДСКИЙ

Введение в физику

ИЗДАТЕЛЬСТВО «НАУКА» МОСКВА

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Introduction to Physics

Translated from the Russian
by
O. Smith and L. Levant

MIR PUBLISHERS MOSCOW

First published 1963
Second printing 1968
Third printing 1976
Second edition 1981

На английском языке

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CHAPTER 1

The Fundamental Law of Mechanics

Sec. 1. KINEMATICS

Equations of Motion of a Particle. If the dimensions and shape of a body are of no consequence in the consideration of a particular phenomenon, we can conceive of the body as being represented by a point. This approximate representation of a body by a material (i.e., mass) point is not only justified when the dimensions of the body are small relative to other distances considered in the problem, but is permissible whenever we are only interested in the motion of the centre of the mass of the body.

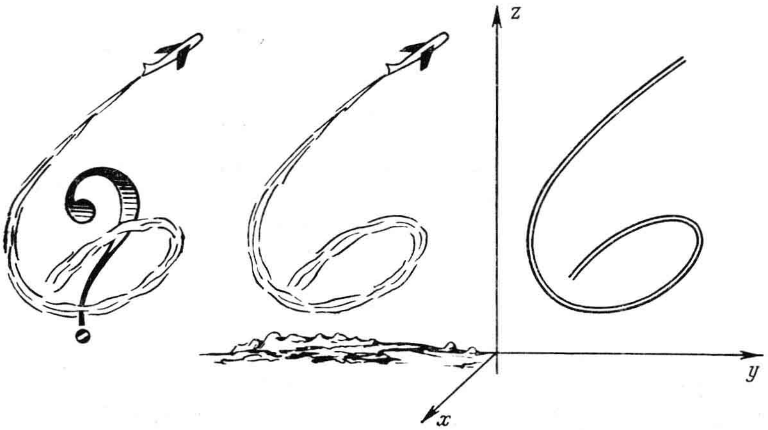

Fig. 1

In order to describe the motion of a particle, one must indicate through which points in space the particle has passed and the instants of time during which it was located at one or another point of the path. For this purpose, it is necessary, in the first place, to select a coordinate frame of reference (Fig. 1). The location of a point in such a coordinate system, which in its simplest form is right-angled, is determined by the three coordinates x, y, z, or by the so-called radius vector $\boldsymbol{r}$, drawn from the origin of the coordinate system to the given point* (Fig. 2).

Thus, motion in space can be roughly described in the form of a table of values for $\boldsymbol{r}$ (each value being given by three quantities!) for the instants of time t_1, t_2, etc.; or accurately described in the form of a continuous function $\boldsymbol{r} = f(t)$ [in essence, three functions, e.g., $x = f_1(t)$, $y = f_2(t)$, $z = f_3(t)$; or $r = \varphi_1(t)$, $\alpha = \varphi_2(t)$, $\beta = \varphi_3(t)$; etc.].

The vector equation $\boldsymbol{r} = f(t)$ or, what amounts to the same, the three equivalent scalar equations are called *the equations of motion*.

Average Velocity. Let us consider AB, a portion of the path. Assume that at the instant of time t the moving particle was at A, and at the instant of time

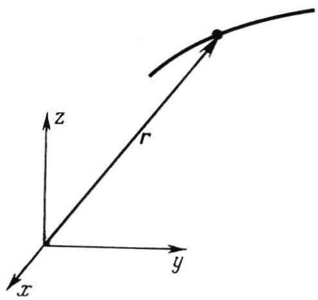

Fig. 2

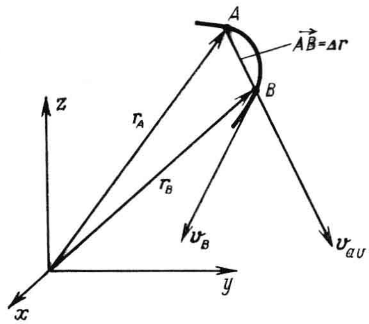

Fig. 3

$t + \Delta t$ at B (Fig. 3). Let us introduce the radius vectors $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. We know that during the interval of time Δt, the particle moved from A to B. It is therefore natural to call the vector $\overrightarrow{AB}$ the particle displacement vector.

Vectors may be added by the parallelogram method. From Fig. 3, we see that

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \overrightarrow{AB} \quad \text{or} \quad \overrightarrow{AB} = \boldsymbol{r}_B - \boldsymbol{r}_A = \Delta \boldsymbol{r},$$

i.e., the particle displacement vector is the vector difference of the radius vectors. The curvilinear motion is determined by the displacement vector $\Delta \boldsymbol{r}$ for time Δt, whereby the smaller $\Delta \boldsymbol{r}$ the greater the accuracy.

The average speed for the path AB is given by the relation

$$v_{av} = \frac{AB}{\Delta t}.$$

This is the speed at which the body would have traversed the distance AB in uniform and rectilinear motion during the interval of time Δt.

Thus, motion over the path AB may be specified by giving the direction of the vector $\overrightarrow{AB} = \Delta \boldsymbol{r}$ and the speed v_{av}. In place of this, we introduce the vector

$$\boldsymbol{v}_{av} = \frac{\overrightarrow{AB}}{\Delta t} = \frac{\Delta \boldsymbol{r}}{\Delta t},$$

* The radius vector $\boldsymbol{r}$ is given by its magnitude, $r = \sqrt{x^2 + y^2 + z^2}$, and the angles it forms with the coordinate axes: $\cos\alpha = \frac{x}{r}$, $\cos\beta = \frac{y}{r}$ and $\cos\gamma = \frac{z}{r}$. Thus, it is determined by three quantities: x, y and z; or r, α and β; or r, α and γ; etc. (two angles determine the third, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$).

which is equal in magnitude to the average speed and whose direction is that of the displacement vector. We can now say that the motion of the body over the path AB is determined by the average velocity.

Instantaneous Velocity. If we decrease the interval of time Δt, the point B will approach point A. These points finally merge and the direction of $\overrightarrow{AB}$ then coincides with the tangent to the curve at the point of merger.

As Δt decreases, the ratio $\frac{\overrightarrow{AB}}{\Delta t}$ approaches a limit. The vector v_{inst}, having the direction of the tangent to the curve at the given moment of motion and numerically equal to the limit of the ratio $\frac{AB}{\Delta t}$ as $\Delta t \to 0$, is called *the instantaneous particle* ***velocity***:

$$v_{inst} = \text{limit} \frac{\Delta r}{\Delta t} \quad \text{when} \quad \Delta t \to 0.$$

In other words, the instantaneous velocity is the derivative of the vector r with respect to time:

$$v = \frac{dr}{dt}.$$

It should again be emphasised that it is not absolutely essential to employ vectors in order to describe motion. Instead of using the concept of vector velocity, we could speak of the absolute value of the velocity, $\frac{|dr|}{dt}$,* and indicate the direction of motion. If we did this, however, the same rules and the same experimental facts would require more cumbersome and more wordy formulations. Vector notation corresponds to physical experience, and is moreover concise and expressive. A certain amount of effort, however, is required to become accustomed to it.

Since the projections of the vector r on the coordinate axes are the coordinates of its terminus, x, y and z, the projections of the velocity vector are:

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt} \quad \text{and} \quad v_z = \frac{dz}{dt}$$

Acceleration. To continue our consideration of curvilinear motion, let us draw arrows to represent the instantaneous velocities of the body in passing through the points A and B of its path. If we had not introduced the concept of velocity, we would have to describe the situation as follows: the speed at B is different from that at A; moreover, the direction of motion has changed. Using the concept of velocity, we can state more briefly: the velocity at B is different from that at A.

Velocity can change in magnitude and direction.

If the path AB is rectilinear, the vectors v_A and v_B have the same direction. The change in velocity is obtained by arithmetically subtracting the magnitude of the vector v_A from the magnitude of the vector v_B.

Let us now consider the curvilinear path AB; vectors v_A and v_B differ in magnitude as well as in direction. To determine the increase in the *magnitude* of the velocity, it is necessary, as before, to subtract the magnitude of the vector v_A from the magnitude of the vector v_B:

$$\Delta |v| = |v_B| - |v_A|.$$

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* The vertical bars | | indicate that only the absolute value (modulus) of the vector between the bars is being considered.

However, this quantity does not, of course, completely express the change that has occurred in the motion.

Let us now subtract vector v_A from vector v_B in accordance with the laws for operating on vectors. Fig. 4 shows vector

$$\Delta v = v_B - v_A.$$

Vector v_B, the sum of $\Delta v + v_A$, is the diagonal of the parallelogram constructed on these vectors.

Vector Δv is called the velocity increment. The magnitude of this vector in the case of curvilinear motion is not $\Delta |v| = |v_B| - |v_A|$. From the figure, it is evident that the magnitude of the increment vector $|\Delta v|$ is greater than $\Delta |v|$, the difference in the magnitudes of the velocities. To determine the velocity at point B, one must add velocity v_A and increment Δv by the parallelogram method.

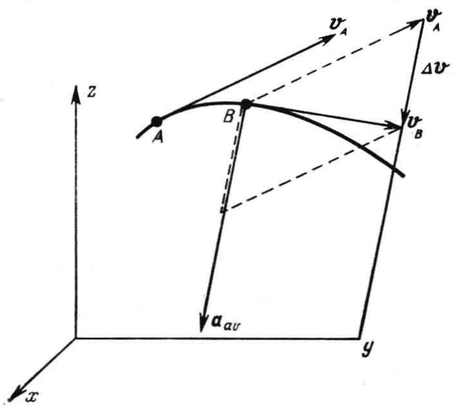

Fig. 4

We can now determine the acceleration for curvilinear motion as follows. The ratio of the velocity increment to the interval of time during which this increment takes place is called *average acceleration*:

$$a_{av} = \frac{\Delta v}{\Delta t}.$$

When the interval of time Δt is decreased, this ratio approaches a limit. The vector

$$a_{inst} = \text{limit} \frac{\Delta v}{\Delta t} \quad \text{when} \quad \Delta t \to 0$$

is called *the instantaneous acceleration* of a body at a given moment of motion. In other words, acceleration is the derivative of velocity:

$$a = \frac{dv}{dt}$$

and

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}.$$

The acceleration vector uniquely determines the nature of the change in the velocity of the body.

Generally speaking, the acceleration vector can form any angle with the curve. This angle determines the nature of the acceleration and the curvature of the path as follows. Through the point of the curve that is being considered, a circle is drawn that has a common tangent with the path of motion at this point, and for the given portion of the curve most accurately approximates it. This circle is called *a tangential circle** and its radius ρ is called *the radius of curvature* at the given point. The acceleration vector is always directed into this circle. If the motion is accelerated, the vector a forms an acute angle with the curve (i.e., with the tangent to the path at the given point). If the motion is retarded, this angle will be obtuse. Finally, if the magnitude of the velocity does not change, the acceleration vector is directed normal to the curve.

* The tangential circle and the calculation of radius of curvature is studied in detail in courses on differential geometry.