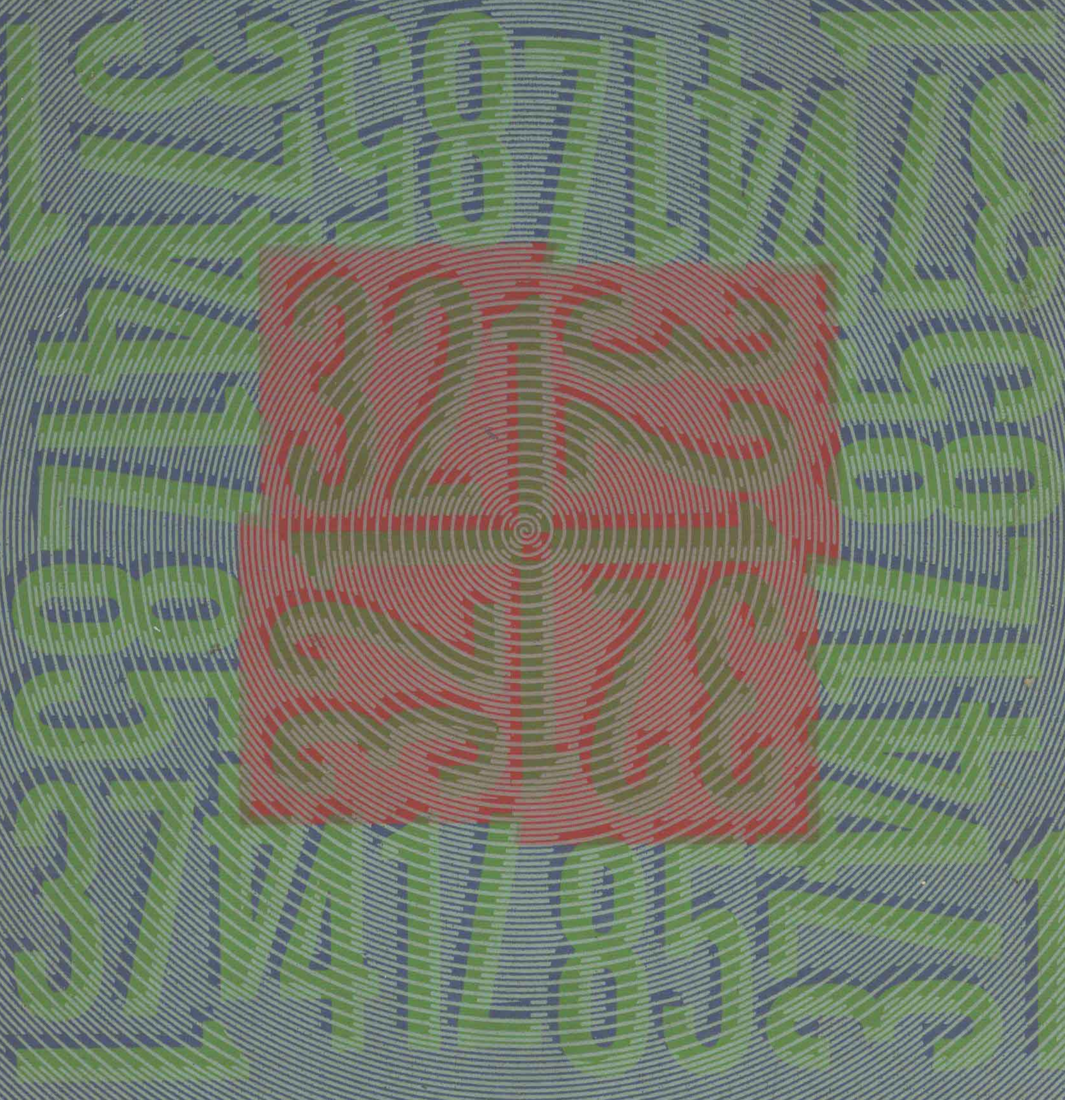
finite mathematics: a liberal arts approach



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finite mathematics: a liberal arts approach

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finite mathematics: a liberal arts approach

This book is dedicated, with love, to
{Mom}, {'Cile}, {Dot}, {Pam and Eric},
{Lance and Connie}, and the set of friends
who appear in the problems

preface

One of the happy, unexpected consequences of writing books is the flood of letters from readers all over the world. Some of these letters contain problems to be solved, some contain constructive criticism very useful in revisions, and some contain suggestions for further books.

Many of the letters I have received as an author had concrete suggestions for the following topics: slide rule, computing, PERT, management mathematics, linear programming, and the theory of games. At first, the variety of topics seemed to preclude any unified treatment, but after a while they seemed to fall into a pattern. These topics represented, in the main, mathematical solutions of modern problems solved by the methods usually identified with *finite mathematics.*

"Finite mathematics" is not very well defined. *Books* on finite mathematics usually start with a lengthy treatment of symbolic logic, truth tables, sets, and probability and then proceed to problems in linear programming, the theory of games, and other noncalculus (hence *finite*) applications of mathematics to industry, commerce, and business. Unfortunately, most *courses* in finite mathematics do not get beyond the first half of books on the subject so that these courses are, in reality, largely devoted to problems in finite probability somewhat similar in content (if not in method) to the old unit in advanced algebra. The liberal arts aspect of "In how many ways can seven different flags be run up a flagpole?" eludes me.

In this book, I decided to cut to a minimum all those topics that are mainly of professional interest to a mathematics major. (If a mathematics major takes this course, he has probably had both probability and truth tables elsewhere or will take an advanced course in fields that discuss these professionally.) My purpose was to get to topics more relevant to a liberal arts or business student, e.g., linear programming, the theory of games, Markov chains, PERT, and computing.

The book therefore starts with a discussion of *linear programming* in Chap. 1. Included in this chapter are just those concepts of sets and graphs that are essential to the treatment. The *third* chapter deals with *Markov chains,* but in order to explain them it is necessary to teach, in Chap. 2, some of the elements of probability. With this basis, Markov chains are explainable in terms of the Kemeny tree. At the same time, the opportunity was seized to teach an indispensable tool, *matrices,* used in Chaps. 4 and 5 to illuminate the *theory of games* from what I believe to be a novel approach.

Thus ends the first half of the book. As you can see, it was designed to fit together economically and precisely, containing only those topics that are essential for the attainment of my goal: an understanding of the power of mathematics in solving problems by linear programming, Markov chains, and the theory of games.

The second half of the book emphasizes relevant computation of various kinds. I suppose that many teachers will skip Chap. 6 on the *slide rule,* but I predict that students will read it anyhow. Slide rules have their own fascination, and the engineer is more commonly seen at his desk with a "slipstick" than playing with a computer. I admit that the slide rule is not a modern invention, but its use is modern. At any rate, I retained the chapter because there were so many requests and because not one of the five professional reviewers who examined the manuscript objected to it.

Chapters 7 to 9 deal mainly with applications of mathematics to problems in industry and commerce: savings plans, annuities, amortization, PERT, and forecasting. The first three of these cannot be explained intelligently without logarithms, and so I included Chap. 7 on logarithms for students who have not been introduced to them or who, in the opinion of the teacher, need a review. At the present time it is fashionable to underestimate the power of logarithms in computation. Actually, logarithms were a tremendous achievement in the mathematics of computation. The tables for many business problems are logarithmic in nature, and computers use logarithms for computation. At any rate, most students will have had an exposure to logarithms and will need little more than a refresher. The chapter was included to make this convenient. At the same time, the book contains many optional topics so that time saved here can be devoted to some of those.

As for Chaps. 8 and 9, even mathematicians will find problems in *simulation, PERT,* and *forecasting* new, different, and fascinating.

The second half of the book concludes with Chap. 10 on FORTRAN. The programs are based upon the IBM 360/30. At this time in history, a chapter on computing in a book designed for liberal arts and business students needs no defense.

What was omitted? I deliberately omitted advanced set theory, complex problems in probability and statistics, and truth tables and switching circuits. (I, personally, like all these topics.) Truth tables and the application of truth tables to switching circuits, for example, are fascinating to a student of mathematics and the sciences, but experience leads me to believe that the usual short treatment (1) does not appear to be relevant to the liberal arts or business student and (2) is rarely or never carried through the stream of thought in the remainder of the book in which it is introduced.

No assumptions have been made about the previous training of the students. As I have pointed out, the book is self-contained in this respect, allowing the teacher to skip "old stuff" and substitute other topics wherever this is possible. There is enough material for a one-semester course or a one-year course.

No effort has been spared to make this book useful in a practical, educational situation. There are a great many illustrative problems completely worked out and explained in detail. Answers to the odd-numbered problems are given in the text, for the convenience of students. The Teachers' Manual contains answers to all problems and complete solutions for certain lengthy ones, for the convenience of the teacher. The Teachers' Manual also contains suggested syllabi for various lengths of courses at various levels and some other material which, it is hoped, will make life easier for the teacher.

The text has been read carefully by many teachers and mathematicians. I am especially grateful to my good friend Frank Hawthorne and to the editors and reviewers of McGraw-Hill for invaluable criticisms and suggestions.

Irving Allen Dodes

* In a short course, the sections preceded by an asterisk may be omitted without loss of continuity.

contents

an introduction to operations research

A cafeteria near a college sells a delicious 80-item lunch (a plate of beans) for 35¢ at a profit of 32¢, and also sandwiches for 25¢ at a profit of 20¢. The storage bin for beans and sandwiches holds no more than 70 sandwiches and 40 plates of beans for each 40-minute lunch period. The manager must have at least 10 plates of beans and 30 sandwiches to be sold each lunch period in order to meet the minimum requirements of the supplier. There are 90 students in the lunchroom each lunch period, and each one may buy either a sandwich or a plate of beans, but not both. In order to increase his profit, the manager puts in a line of 10¢ candy, each piece of which has a 5¢ profit. He discovers that, in each lunch period, each of the three cashiers spends 1 minute collecting for each sandwich and 2 minutes collecting for each plate of beans (the cashier needs the time to count the beans). If there is any time left over, the cashiers are supposed to sell candy. Each piece of candy takes ½ minute to sell. What combination of beans, sandwiches, and candy will bring about the maximum profit for the cafeteria?

By the end of this chapter, you will know how to solve this involved problem by a very simple method (it sounds harder than it really is). It is one of the kinds of problems solved by a method called *linear programming* in a fairly new field of mathematics called *operations research*. Some other problems involved in operations research are:

Nutrition problems: What is the cheapest way to feed livestock to get the best results?

Transport problems: What is the cheapest way to route ships to pick up cargo and make the largest profit?

Production problems: How many toys shall a manufacturer make each month to avoid excessive storage and yet meet the probable demand?

Buying problems: How much perishable stock, such as eggs, should a store keep to avoid loss of goodwill and yet not lose money on leftovers?

Other problems deal not only with business but also with military strategy, logistics, game strategy, science, and the social sciences. Most of these require rather advanced methods, and we shall not be able to illustrate them. However, the problems we shall demonstrate will show you the general thought behind the solution. That is all we want to do.

First, we shall teach you some exact mathematical language and review some not-too-difficult algebra, so that our conversation can continue pleasantly. (If you know the language of sets and how to plot inequalities, you can skip to Sec. 1.5, where we start the discussion of linear programming.)

1.1 THE LANGUAGE OF SETS

Any collection of objects, people, things, or ideas is called a *set.* For example, {Peter, Paul, Mary} is a set. The members of a set are called its *elements,* and the number of elements in a set is called its *cardinal number.* In this case, the cardinal number of the set is 3. This set is said to be a *finite set.*

From the viewpoint of a mathematician, the important property of a set is that we can determine what is in it and what is not in it. For example, if we consider the set of even numbers, written symbolically as {even numbers},[1] we know that 2 is in it, but 3 and 6½ are not in it. (Therefore, it is a set.)

In some cases we can determine the cardinal number of a set by counting. In the case of the set of even numbers, {even numbers}, we can start counting but we can never finish. Mathematicians call this an *infinite set* and say that its cardinal number is $\aleph_0$ (aleph-null). The Hebrew letter with subscript zero is used to represent the cardinal number of sets in which one can count but never finish counting.[2] We shall deal with finite sets and infinite sets in this book, but not in detail.

Two or more sets may have some elements in common. For example, consider the sets,

Set A = {Peter, Paul, Mary, Pam, Lance}
Set B = {Pam, Eric, Lance, Dorothy}

Lance and *Pam* are in both sets. We call them *common elements* for sets A and B, and we define a new set

Set C = {Pam, Lance}

which is composed only of the common elements. This is called the *intersection* of A and B. We write

$$C = A \cap B$$

which is read "Set C is the intersection of set A and set B," or, more briefly, "C is A intersection B."

We can also define a new set composed of all the *different* elements of sets A and B:

Set D = {Peter, Paul, Mary, Pam, Lance, Eric, Dorothy}

[1] Braces, { }, are used to abbreviate "the set of."
[2] There are other infinities, such as the number of points on a line, which cannot even be counted. You can read more about infinities in a remarkable book by Richard Courant and Herbert Robbins, "What Is Mathematics?", pp. 77–88, Oxford University Press, Fair Lawn, N.J., 1941.

This new set contains all the elements of sets A and B, but no others. It is called the *union* of A and B. We write

$$D = A \cup B$$

which is read "Set D is the union of set A and set B" or "D is A union B." Notice that the symbol $\cup$ looks like the letter U.

definition: intersection

The *intersection* (set) of two or more sets is the smallest set of all *common* elements.

definition: union

The *union* (set) of two or more sets is the smallest set of all *different* elements.

The five illustrative problems that follow are intended to clarify the ideas presented so far.

illustrative problem 1

Two sets are A = {snakes, hedgehogs, newts, worms} and B = {spiders, beetles, newts, worms, snails}.
(*a*) Draw a diagram for the two sets.
(*b*) What is the intersection set, $A \cap B$?
(*c*) What is the union set, $A \cup B$?
(*d*) What is the cardinal number of each set?

solution

(*a*) See Fig. 1-1.
(*b*) $A \cap B$ = {newts, worms}.
(*c*) $A \cup B$ = {snakes, hedgehogs, newts, worms, spiders, beetles, snails}.
(*d*) The cardinal numbers are A, 4; B, 5; $A \cap B$, 2; $A \cup B$, 7.

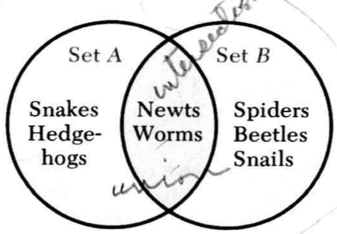


Fig. 1-1 Illustrative Problem 1.

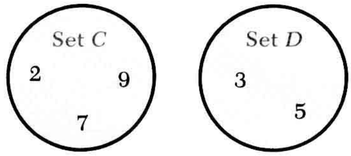


Fig. 1-2 Illustrative Problem 2.

illustrative problem 2

Two sets are $C = \{2,7,9\}$ and $D = \{3,5\}$. Draw a diagram, and find the intersection and union and the cardinal numbers.

solution

(*a*) See Fig. 1-2.
(*b*) $C \cap D = \{\quad\}$. There is nothing in this set. It is called the *null set* or *empty set,* often symbolized by the symbol $\varnothing$. We write $C \cap D = \varnothing$ ("The intersection of sets C and D is the null set").
(*c*) $C \cup D = \{2,7,9,3,5\}$.
(*d*) The cardinal numbers are as follows: C, 3; D, 2; $C \cap D$, 0; $C \cup D$, 5.

illustrative problem 3

Two sets are $F = \{2,4,6,8,10\}$ and $E = \{2,4\}$. Draw a diagram, and find the union and intersection and the cardinal numbers of all the sets involved.

solution

(*a*) See Fig. 1-3.
(*b*) $E \cap F = \{2,4\}$. Note that this is set E, so that $E \cap F = E$.
(*c*) $E \cup F = \{2,4,6,8,10\}$. Note that this is set F, so that $E \cup F = F$.
(*d*) The cardinal numbers are as follows: E, 2; F, 5; $E \cap F$, 2; $E \cup F$, 5.

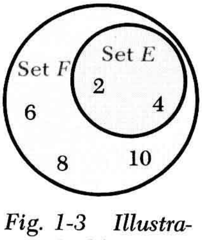


Fig. 1-3 Illustrative Problem 3.

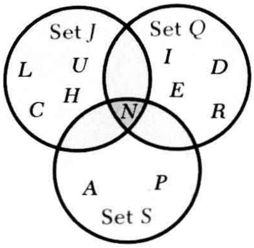


Fig. 1-4 Illustrative Problem 4.

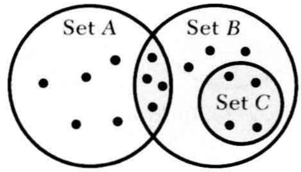


Fig. 1-5 Illustrative Problem 5.

illustrative problem 4

Three sets are $J = \{L,U,N,C,H\}$, $Q = \{D,I,N,E,R\}$, and $S = \{N,A,P\}$. Draw a diagram, and find the intersection and union and the cardinal numbers of the sets.

solution

(*a*) See Fig. 1-4.
(*b*) $J \cap Q \cap S = \{N\}$.
(*c*) $J \cup Q \cup S = \{L,U,N,C,H,D,I,E,R,A,P\}$.
(*d*) The cardinal numbers are J, 5; Q, 5; S, 3; $J \cap Q \cap S$, 1; $J \cup Q \cup S$, 11.

illustrative problem 5

In Fig. 1-5, each dot represents a different element of a set. Find the cardinal number of each set and of the union and intersection of the sets.

solution

Set A has 9 elements. Set B has 11 elements. Set C has 4 elements. The union, $A \cup B \cup C$, has 16 elements. The intersection, $A \cap B \cap C$, is the null set. It has no elements, and its cardinal number is zero.

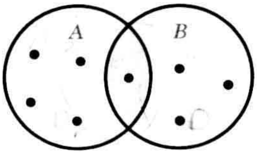
PROBLEM SECTION 1.1

Part A. Draw diagrams, and find the union, and the intersection, of the following sets.

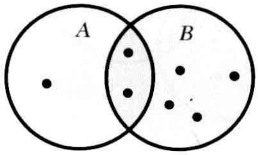
1. $\{A,B,C\}$, $\{B,C,D\}$
2. $\{F,X,L\}$, $\{X,L,P\}$
3. $\{A,B\}$, $\{C,E,M\}$
4. $\{P,Q\}$, $\{D,R,S\}$
5. $\{A,B,C\}$, $\{C,B,A\}$
6. $\{M,N,R\}$, $\{R,N,M\}$
7. $\{A,B,C,D\}$, $\{B,C\}$
8. $\{R,S\}$, $\{P,Q,R,S\}$
9. $\{A,B\}$, $\{B,C\}$, $\{C,D\}$
10. $\{M,N,R\}$, $\{N,R,S\}$, $\{R,S,T\}$

Part B. In each of the following, find the number of elements in each set (including the union, and intersection sets). Each dot represents an element.

11.



12.



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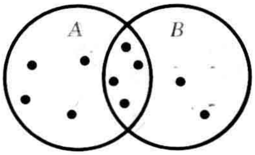

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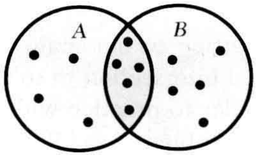

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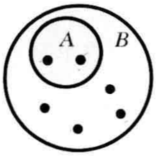

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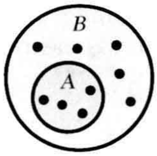

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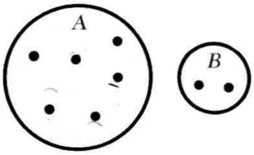

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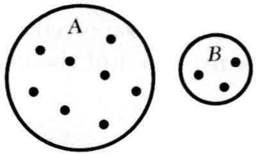
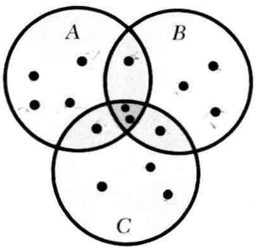


Part C. In each of the following, find the number of elements in each set (including the union, and intersection sets). Each dot represents an element.

19.



20.