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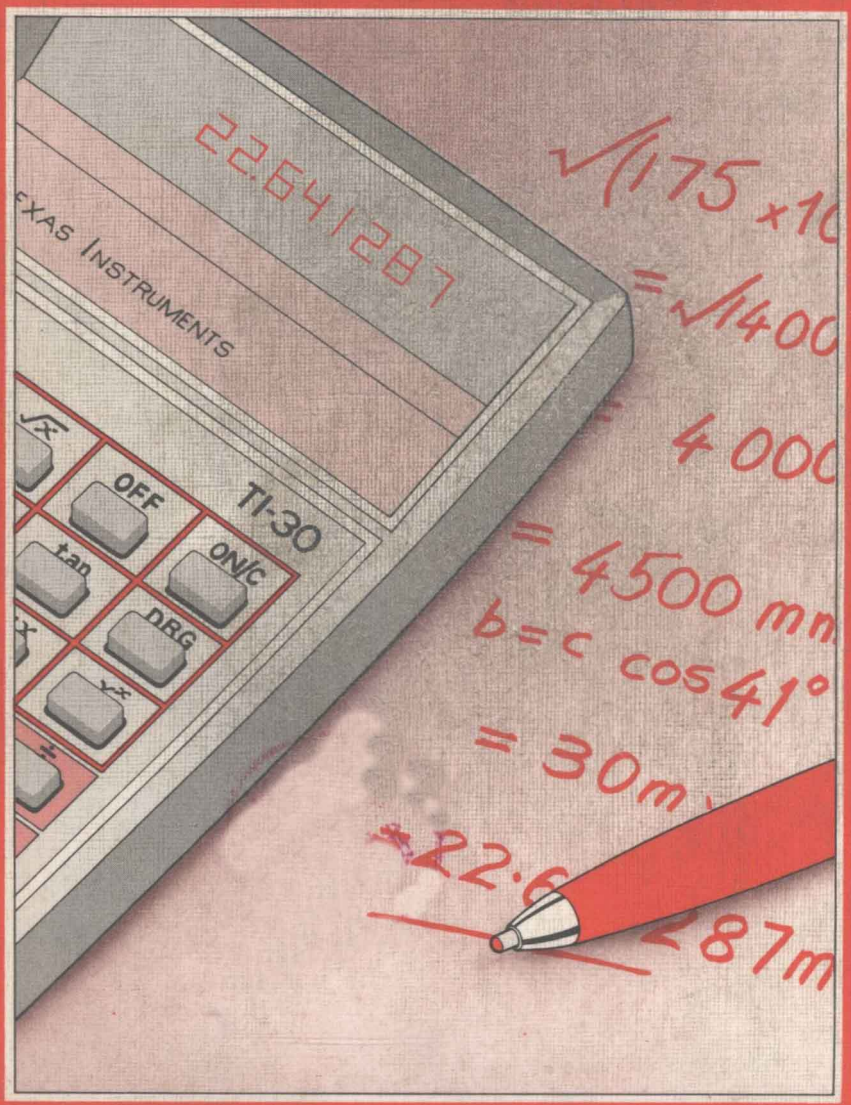
TECHNICIAN MATHEMATICS 1



M. G. PAGE

Technician Mathematics 1

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Preface

The task facing authors writing textbooks for programmes of study leading to the awards of the Technician Education Council is formidable to say the least. The syllabus content of the majority of Technician Courses of the past was relatively well defined, particularly those leading to the awards of the City and Guilds of London Institute with the associated external examinations. The majority of the remainder were associated with guide syllabuses and internal examinations, externally assessed, and in many cases the guide syllabuses were used with little or no modification.

Circumstances have now changed. T.E.C. programmes of study are now virtually all unit based, and theoretically there is no set content for a T.E.C unit which bears a particular title. The content of a unit is primarily a matter for agreement between an educational institution and its local industry, although before a programme can commence, approval is required from the T.E.C., who, when considering whether or not approval is given, takes note of the views made by validating sub-committees of appropriate T.E.C. Programme Committees.

When the T.E.C. issued its *Policy Statement*, educational institutions became aware that an immense amount of work would be necessary when constructing units, particularly when it became evident that the T.E.C. favoured units being written in terms of objectives. As could be expected, the task of compiling units was attacked in a variety of ways. Some individuals and institutions accepted the freedom with avidity. Others saw some advantage in making better use of resources by some form of collective activity, while the balance, in the main, felt it advisable to wait for developments from the individual Programme Committees of the T.E.C.

As time progressed, the Programme Committees proceeded with their deliberations, which were assisted by the provision of suggested units which emanated from a working party drawn from the various curriculum development groups which had appeared. These were referred to as Standard Units, but perhaps it would have been better had they have been referred to as proposals for standard units. The Programme Committees received these proposals with various degrees of satisfaction. Some were accepted without modification, others with reservations, and in certain cases, were found to be totally unacceptable. In the latter case, certain Programme Committees decided to construct their own units.

In the case of mathematics, the author supports strongly the point of view expressed in many quarters that there is a body of study in mathematics at Level 1 which is relevant to the needs of all interested parties, and there should be relatively little difficulty in constructing a unit entitled Mathematics I which could be studied by all technician students, irrespective of the programme being undertaken. At the time of compiling the manuscript for this book, the T.E.C. had commenced some deliberations on common units, in which the author was participating, but no firm proposal had appeared. With a significant number of T.E.C. Courses starting in September 1976, and a substantially increased amount commencing in September 1977, the need for textbooks written specifically for T.E.C. courses became a matter of considerable urgency. At the time that the manuscript of this book was compiled, all that any author could do would be to survey the immediate position and make a considered judgement of an appropriate content.

Consequently, this book has been written with the aim of satisfying completely the objectives of the proposed Standard Unit T.E.C. U75/005, in the state it was in January 1977. In addition, some topics have been added which reflect the author's personal viewpoint of items which may well appear should a Common Unit entitled Mathematics 1 eventually be published. The opinion is far from a guess, as the author has had opportunities of studying proposals for units entitled Mathematics 1 which have emanated from a variety of sources, and he wishes to acknowledge, with considerable gratitude, the assistance of his colleagues in providing the information.

The style follows that of the author's previous work, *Mathematics for Mechanical Technicians* Book 1, which, judging from its sales, has been found to be welcome. However, the problems and examples have been the subject of considerable revision, to ensure that as far as is reasonably possible, the problems are relevant to technician students in general rather than to a specific group.

The text closely follows the sequence of the presentation of the topics in unit T.E.C. U75/005, the only significant variation being statistics, which has been included in the first chapter, which deals with computation. The remaining chapters deal with algebra, diagrammatic representations, geometry and trigonometry. Particular attention has been given to the fact that the pocket-size electronic calculator is now a recognised working tool of the typical student.

Students enter programmes from varying patterns of previous education and with various degrees of competency. The book has been written on the assumption that it is necessary to commence from basic fundamentals, and for many readers certain topics will be merely a consolidation of previous knowledge. A further advantage will accrue for colleges who adopt a policy of placing a considerable proportion

of a new entry into a diagnostic stream. Because the book commences from basic fundamentals, and is intended for students who progress to technician courses, it will also be found suitable for a considerable proportion of craft courses. It will be particularly valuable for those students who have used the book on craft courses, and who change to technician courses at some later stage.

It is the intention of the author that the present book will be the first of a series of textbooks in mathematics for technicians, and fortunately there is some evidence to suggest that the content of units at Levels 2 and beyond will be in a more settled position than was that of Mathematics Level 1 when the manuscript of the present book was written.

It will be interesting to compare the content of this book with that of a common unit in Mathematics Level 1, if and when it appears. In the opinion of the author this is a virtual certainty. One of the delights he has enjoyed as a result of his writings is the kindly help which has been given by his colleagues which has resulted in improvements, and he sincerely trusts this assistance will continue. In the first instance, correspondence should be directed via the publisher.

Finally, the author wishes to acknowledge the help of Mrs Janet Phelps who typed portions of the manuscript with her usual competence and skill, and above all, patience. Also of the help of Mr Francis Fletcher, B.Sc., A.F.I.M.A., who read the original manuscript and made extremely useful comments, all of which have resulted in improvements to the original draft.

GEORGE PAGE

Wolverhampton 1977

Technician Mathematics 1

This book has been written to provide background material for a student who is engaged upon a programme of studies leading to an award of the Technician Education Council (TEC), and whose programme includes a unit bearing a title of, or similar to, Mathematics Level 1. In particular, it covers all the topics of Standard Unit 005, which has been recommended for adoption in the individual guidelines issued by several Programme Committees of the TEC. The learning objective structure of Unit 005 has been closely followed. There are numerous worked examples and more than sufficient problems (with answers) to satisfy the coursework demands of a typical course of study. The book is the first of a series for Mathematics Units at all levels, and will be immediately followed by Books 2 and 3, which will cover the objectives in Mathematics up to Certificate and Diploma Level.

M. G. Page is Head of the Department of Production Engineering at Wolverhampton Polytechnic. He is a member of the Programme Committee A1 of the T.E.C. dealing with Certificate and Diploma Programmes in General Engineering and common units at Certificate and Diploma level in other engineering disciplines. Mr. Page is also Chief Examiner in Mathematics and Science for the C.G.L.I. Mechanical Engineering Technicians' course (255).

Introduction

It would not come amiss, before commencing a consideration of the topics of Technician Mathematics at Level 1, to discuss the use of pocket-size electronic calculators. The remarkable reductions in the costs of such calculators since their introduction now means that every student can be expected to possess this type of calculator, and this book has been written with that factor kept firmly in view.

The author, in his capacity as one of the Chief Examiners of the City and Guilds of London Institute, has observed, with considerable pleasure, two results of the use of calculators in examinations. The first is the dramatic reduction in the number of arithmetical errors. The second is that due to rapidity with which a calculator provides a result, when compared with traditional arithmetic, more time can be devoted to aspects of mathematics other than numerical computation.

The author will state quite categorically that if the student for whom this book is intended has a calculator available which has a capacity for providing the solution to a particular computational problem, that calculator should be used for that purpose. There is no doubt that this viewpoint may be the subject of criticism by skilled mathematicians, many of whom pass derogatory comments about students who use calculators for the most elementary of computational processes, such as adding together two simple whole numbers. This book is not intended for skilled mathematicians. The use of a calculator for simple processes, which admittedly could be performed more rapidly mentally, has one important advantage. The confirmation from a calculator of a result achieved mentally gives more confidence that the calculator is being used correctly. Furthermore, there are very few computational operations which can be accomplished mentally quicker than by using a calculator, and there is ample evidence to suggest that computations undertaken with a calculator tend to produce less errors than those undertaken mentally, or those made with the aid of what may be termed traditional arithmetic.

What has been said should not be interpreted as an invitation to give scant regard to traditional arithmetic, and especially to mental arithmetic. A person may be placed in a situation where a computational problem has to be solved and no calculator is available. Or, as often proves an embarrassment, to have a calculator available which lacks the necessary motive power. Consequently, it is just as necessary now as it was in the past to be competent with traditional arithmetic, with mathematic tables and with a slide rule.

A further point in favour of calculators, since in general errors occur less frequently when traditional arithmetic is used, is that it will tend to give less support to the viewpoint that the major consideration in solving a problem is to get the method right, any resulting computational work being of lesser importance. It must be agreed that this policy may help a student to satisfy assessments, where quite often a portion of the maximum allocation of marks to a problem is devoted to the method, but students who carry this attitude into a working life quickly realise that accuracy is just as important as methodology. In the scholastic situation a student should be encouraged to consider that the accuracy of a solution is just as important as the method by which that solution was obtained.

The price of a calculator which is capable of providing a particular range of computations has decreased remarkably over recent years, although at the time the manuscript of this book was written the trend of decreasing prices was much less noticeable. The rate of decrease in price was apparently fairly close to that of inflation, and the general level of prices seems now to have 'bottomed-out'. One calculator the author uses was purchased some five years ago at a cost of well over £50. A calculator with virtually the same capabilities could be purchased at the time the manuscript of this book was written, for well under £20, but there seems to be very little possibility of a significant reduction in price in the immediate future. Calculators are very reliable pieces of equipment, and as distinct from purchasing cars, no particular make can be considered superior to others as regards reliability. Once our reader has decided on a specification which satisfies his or her requirements, and any one of a particular group will satisfy that specification, the cheapest of that group should be purchased.

In all probability, the first decision that will have to be made is in respect of the motive power, which can be either a disposable dry battery, a rechargeable battery, mains only, or a facility for either or both of the latter. This is reflected in the price, and if it can be afforded, since the calculator may be used in a variety of situations, a calculator having a rechargeable battery which is supplied with an adaptor for recharging while it is being used from the mains is preferable. If the cheapest possible calculator has to be purchased the motive power will invariably be a disposable dry battery. In which case our reader is urged to have a replacement battery readily available, rather than being placed in the embarrassing position of having a calculator which is devoid of motive power.

The next decision will very likely be in respect of the number of digits available in the display. For the present and immediate future needs of our reader a display of more than eight digits is something of a luxury, as is a facility for computation in vulgar fractions. The display should be capable of indicating values in both ordinary form and in standard

form, the latter giving a display indicating a coefficient and the power of 10 which it multiplies.

The programme of study on which our reader is engaged is constructed from a series of objectives to be attained. In particular, the course is designed to lead to further studies. These factors provide some guidance in the selection of a suitable calculator. It should be chosen to complement the objectives of the immediate course, and, if possible, some attention should be given to future requirements, so that an investment in a calculator does not prove to be unsound. As applied to a calculator, the word function is used to describe a computational process of which the calculator is capable. Most manufacturers will indicate how many functions a particular calculator can provide. A typical statement is that a particular model is a '28 function' calculator. In general terms, the greater the number of functions, the greater will be the price.

The computational requirements of Technician Mathematics at Certificate and/or Diploma level suggest the following functions of a calculator would be to advantage, and are given in some general, but not exact, order of priority:

1. The four basic functions of addition, subtraction, multiplication and division.
2. A square rooting function.
3. A squaring function.
4. A reciprocal function.
5. Logarithmic functions; natural and common.
6. Trigonometrical functions; sine, cosine and tangent.
7. An inverse function; determining antilogs, arcsin, arccos, and arctan.
8. Degree or radian notation for angles.
9. Change sign function, + to −, and vice versa.
10. Power function; determining y^x and $\sqrt[x]{y}$.
11. At least one store for retaining values for subsequent use, usually called a memory.

Of recent times, it has been noticed that calculators which have a capability for the functions previously listed gradually tended to become known as 'scientific' models. To cater for a large number of functions, most of the 'scientific' calculators are provided with a changeover arrangement, so that some keys are allocated to more than one function.

Bearing in mind the preceding information, at the time at which the manuscript of this book was written, the Commodore Scientific Notation Calculator SR 7919D and the Texas Instruments Scientific Model TI 30 would most certainly have evoked the interest of a prospective

purchaser, particularly when the price was compared with the functions available. The author prefers and now uses the TI30.

However, the design of calculators is a rapidly changing technology. Other models introduced since the manuscript for this book was written may well prove a 'better buy'. In any case, once our reader has decided which model to purchase, or to change to, it would be most advisable to 'shop around'. There may be remarkable differences between retailers of the selling price of a particular model.

Contents

1 Computation

1.1
The manipulation of numbers

1.1.1
Directed numbers

Mathematics is that branch of science which has evolved from the discovery that it was possible to represent concepts such as 'how many' and 'what shape' by means of symbols. A typical example is that portion of mathematics which deals with counting, the symbols that are used being called *numbers*. In early days small stones were used as an aid to counting. The Latin word for pebble is calculus, from which is derived the word calculation. Over a period of time the word *calculation* has become associated with the obtaining of a result from data by any mathematical procedure, numerical or otherwise, which does not involve the use of measurement or graphical methods. In more recent times the word *computation* has been adopted to mean calculation with numbers. We shall commence our studies of mathematics with computation.

The word number can have many interpretations. The interpretation most familiar to our reader is associated with the use of numbers as a counting system. This enables us to answer problems such as how many bricks there are in a particular pile of bricks. The answer is given by selecting one of the *natural numbers* which we represent nowadays by the symbols 1, 2, 3, 4, 5 and so on. It is convenient in life to refer to a collection of things as a *set*. The set of natural numbers has no limit, there is an infinite amount of natural numbers. If we use numbers as an aid to counting, then we use *zero* (symbolic representation 0) to indicate that there are none of the particular things which have to be counted.

Algebra is a general term used in mathematics to describe the methods of reasoning about numbers by employing letters to represent those numbers and arithmetical signs to represent their relationships. Let us proceed to using numbers as an aid to measurement. Let us imagine we are measuring the vertical heights of features with respect to some basic feature such as ground level. A feature such as the top of a building will be above ground level, the foot of a mine shaft will lie below ground level. Our measurements will be facilitated if we erect a vertical *line of numbers* where distances are proportional to the magnitude of numbers.

We can ease our task of making measurements if we accept the convention that proceeding in a vertically upward direction is known as proceeding in a positive direction, and to use the symbol +, known as the positive sign, or plus, to indicate this upward direction. Conversely, we adopt the convention that proceeding in a vertically downward direc-

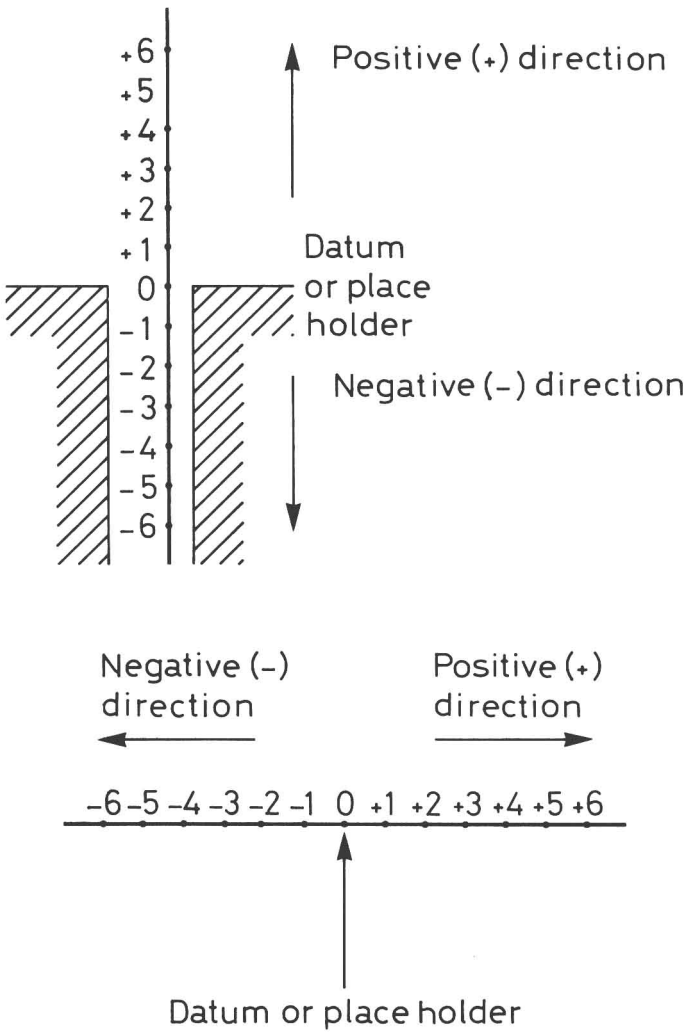

Fig. 1.1