MATHEMATICAL THINKING IN A QUANTITATIVE WORLD

Revised Printing

LINDA R. SONS • PETER

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To Our Parents

To the Teacher

Educated people are often confronted with quantitative thinking in newspapers and magazines, in personal business and government issues, in political rhetoric and campaign literature, in issues regarding their own safety and health or that of loved ones, and in other ways as well. Students who have had a year of high school algebra have often mastered only a level of computational skill. They have never or seldom put to work these skills on problem settings in real life or ordinary "word problems". They have not yet learned how to read quantitative information, to realize that often problems are "solved" through an estimate or to recognize when there is not enough information available to solve a problem. Since this "real use" of mathematics is so foreign to their experience, you will need considerable energy to overcome their reactions to the newness they will encounter. Yet you can be amply rewarded by those students who see the value of what they are learning and strive to develop their thinking.

The intent for the particular choice of content can be learned by study of the objectives for each of the sections which appear in the appendices. Choices made are vehicles for the overarching theme of learning problem solving. For example, students are expected to learn the nature of a statistical test - not to master chi-square testing. The content is chosen to expose students to aspects of quantitative reasoning from a variety of perspectives; for example, you will see estimation, interpretation, and prediction occur throughout the text. Through this exposure, rather than in depth teaching of a more limited content, students are to acquire the skills in quantitative thinking that will enable them to attack heretofore unseen problems, to recognize important questions to ask, and to make better decisions in many settings.

Preface to the First Edition

These text materials are intended for a college course in quantitative reasoning. Consequently, for most students it will be a different experience from their previous mathematics classes. It will differ both in how material is presented as well as what is expected of the student.

The thrust of these materials is towards helping the student develop the use of mathematical thinking in his/her life as an educated person. The course for which these materials are intended seeks to develop in a student a competency in problem solving and analysis which is helpful in personal decision-making; in evaluating concerns in the community, state and nation; in setting and achieving goals; and in continued learning.

The text presupposes two years of college preparatory high school mathematics. One year of college preparatory high school algebra is assumed along with a second year of mathematics which consists of college preparartory geometry or a second year of college preparatory algebra. Elementary geometry is also assumed. Thus, students are expected to have knowledge of solving linear equations and inequalities and systems of these; of solving quadratic equations and using the quadratic formula; of factoring polynomials; of taking square roots and doing operations with radicals; and of using functions and their graphs. Some of these are reviewed in the text.

It is assumed that the student has a hand calculator which can do basic arithmetic, exponents (a y^x key), and square roots.

The student should be cautioned that a text is merely a tool. To be helpful, it needs to be thoroughly used – to be studied with diligence and discipline. It should be read with paper, pencil, and calculator in hand. Examples and exercises should be worked, reworked, and digested.

Exercises in this book are likely to be different from those which have dominated the student's experience in previous mathematics study. Indeed there are some drill problems intended to provide an opportunity for the student to practice a particular skill or mode of thinking. But ordinarily the exercises will appear more in the fashion that they might be encountered in life situations. Thus, some exercises will have too much information, or not enough information, to respond precisely to the question posed. The "best" solution to a problem may be merely an estimate – an approximation to the solution desired. The student will need to learn to verify a solution in terms of its reasonableness and as to whether it is the "best" one can do. Some exercises may demand clarification or interpretation; these are intended to help the student to learn to formulate problems. Other exercises are intended to help the student formulate problem solving strategies.

As the students work through the text, they should discover the repetition of certain ways of thinking and different facets of a number of mathematical ideas. In particular, Chapter V should bring together and deepen ideas exposed in earlier chapters.

The text consists of five chapters - I. Statistics (6 sections); II. Logical Statements and Arguments (4 sections); III. Geometry in Problem Solving (3 sections); IV. Estimation, Approximation, and Judging the Reasonableness of Answers (5 sections); V. Problem solving (4 sections). Section 5 of IV and Section 3 of V were written by Peter Nicholls.

These text materials were developed as a tool for use in the course Core Competency in Mathematics developed by an ad hoc committee of the Department of Mathematical Sciences at Northern Illinois University in the fall of 1984. The committee was composed of John Selfridge, Stanley Trail, Robert Kuller, Peter Nicholls and Linda Sons.

The First Edition consists of a substantial revision of some sections from earlier editions, the correction of some errors, and the addition of a number of exercises. Further, the text now includes answers to selected routine problems. Improvements in the text are a result of suggestions made by students and faculty who have used the preliminary editions in the period 1988-1992. The excellent typing of the early editions was done by Sara Clayton.

The improved "look" of the current edition is largely due to the efforts of my colleague J. B. Stephen. He taught the course, caught its spirit, and recognized how to make the text more readable for the students. Besides doing all the computer text work, he also contributed a number of editorial changes and exercises. His work is acknowledged with profuse thanks and will be greatly appreciated by all those who use this edition.

S.D.G. Linda Sons, N.I.U., May 1992

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Chapter I

Statistics

It is said that we live in the "information age". Through the touch of a button on a computer, or a subscription to a publication such as USA Today, we have at our fingertips a large amount of material describing the day-to-day aspects of the world. In many cases, this information is numerical in nature. Some information is supplied on a daily basis to us, and its computation and interpretation is given to us. For example, we generally understand the following questions and their answers.

1. What is the chance it will rain today?
2. What is the possibility that you will win the State lottery?
3. What is the batting average of your favorite baseball player?
4. What percentage of the population favors a particular political candidate?

Other information requires interpretation or computation on the part of the individual using the data. For example;

1. What does it mean to say that "seat belts save lives"?
2. How can you describe the dangers of acid rain?
3. How do wildlife experts know a species is becoming extinct?
4. What does it mean when the cost of living rises?

Each of the above questions involve some aspect of probability and statistics. This chapter is designed to familiarize students with the use, reliability and construction of statistical arguments.

In practice we are exposed to two types of statistics. The first is **descriptive statistics**. Descriptive statistics, as the term implies, is the science of using numbers to concisely describe data sets. At their best, such statistics can be used to clearly and quickly give an indication of the general qualities of a large set of numbers (such as the purchase price of all houses in the state of Illinois in a given year).

The second type of statistics is **inferential statistics**. Inferential statistics is concerned with predicting behavior (that is, inferring relationships). Typically, one examines a small section of a set (called a sample) and uses numerical information, concerning this sample, to reach conclusions about the whole set.

Knowledge of probability and statistics can help you to describe and understand relationships; improve your decision-making; and deal with uncertainty and change.

As a preliminary to our discussion of statistics, we will investigate the various ways that numerical information is presented.

1 Graphical Presentation of Data

Numerical data is presented in a variety of ways. Among those you will frequently see in newspapers, magazines and on television are line graphs, bar graphs, and pie charts. Sometimes the numerical information is merely listed, or put into a table. In this section we will see a number of examples which illustrate the ways in which data may be represented, and learn the names of these representations. The correspondence between the various methods of representing data is discussed, along with the interpretation of charts and graphs. Finally, we will see that it is possible to manipulate the impact of the data on the reader.

Below are four line graphs taken from January 1992 newspapers.

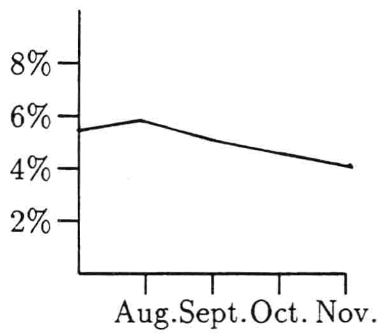

Figure 1.1
Yield on 3 month treasury bills

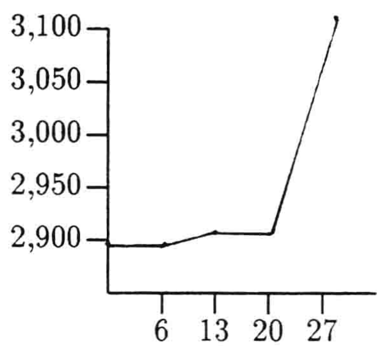

Figure 1.2
Dow Jones Average for dates in December 1991

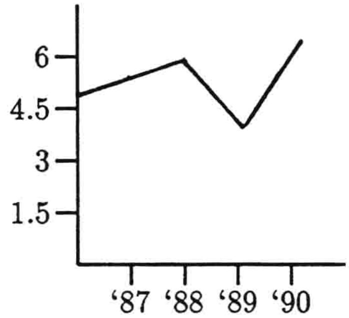

Figure 1.3
IBM Net Annual Earnings (in Billions)

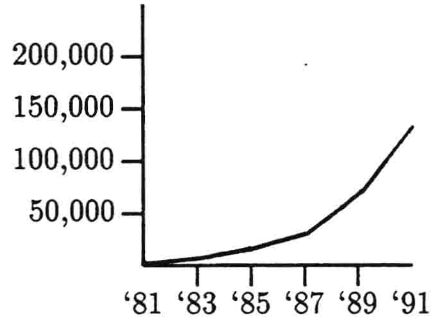

Figure 1.4
Deaths from AIDS

Each of these line graphs shows the variance of some quantity with respect to a unit change in time. In Figure 1.1 time is measured in months, weeks are used in figure 1.2, 1 year intervals are used to measure time in figures 1.3 and 2 year intervals are used in figure 1.4.

Line graphs may arise as convenient means for summarizing data and seeing relationships between variables.

For example, the correspondance in the table of salaries with the number of employees in a company receiving each salary, in the table below, can also be represented by the line graph in Figure 1.5.

Salary in thousands of dollars	Number of employees
10	20
12	15
14	40
16	30
18	5
20	20

Table 1.1

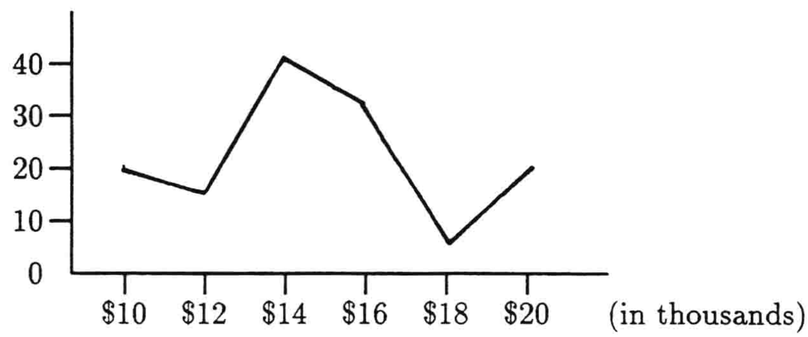

Figure 1.5

Table 1.1 is called a **frequency distribution** and the line graph in Figure 1.5 is called a **frequency polygon**. The frequency distribution in Table 1.1 shows the frequency (number) of employees receiving a given salary. The frequency polygon plots the salary along the horizontal axis and the frequency along the vertical axis. The table and the line graph display the **same** information; they are merely different ways of presenting this information.

Ordinarily the choice of how to present a data set depends on what the person presenting the data feels will result in the best interpretation of the data. We now show six ways one data set can be presented. Three of these, namely, Tables 1.2, 1.3 and 1.4, are frequency distributions. Tables 1.3 and 1.4 arise by grouping the data in Table 1.2.

The data below are the average June temperatures in degrees Fahrenheit for Ohio during the period 1890-1939:

73.3	72.0	69.8	69.2	65.9	66.8	68.9	73.0	70.0	67.4
71.0	69.5	70.9	69.8	70.9	64.7	73.4	65.9	70.8	70.3
73.0	68.1	66.9	65.6	66.6	66.9	70.9	64.6	71.0	69.7
70.6	71.9	64.4	69.2	69.8	68.8	71.0	65.0	74.4	68.5
71.3	71.5	68.4	70.1	71.1	74.2	68.0	67.1	76.0	72.5

Source: Hendricks and Scholl (1943)

This listing certainly illustrates the drawbacks of presenting numerical information in a "raw" form. For example, it is difficult to see that about half of the years had an average June temperature of 70 degrees or higher. The following representations will organize the data in various ways which will permit interpretation with greater ease.

A table listing the values with their frequency of occurrence may be a means for communicating this data so that it can be more readily interpreted. For example, we could represent the above data in a table as follows:

Temperature	Frequency	Temperature	Frequency	Temperature	Frequency
64.4	1	68.4	1	71.0	3
64.6	1	68.5	1	71.1	2
64.7	1	68.8	1	71.3	1
65.0	1	68.9	1	71.5	1
65.6	1	69.2	2	71.9	1
65.9	2	69.5	1	72.0	1
66.6	1	69.7	1	72.5	1
66.8	1	69.8	3	73.0	2
66.9	2	70.0	1	73.3	1
67.1	1	70.3	1	73.4	1
67.4	1	70.6	1	74.2	1
68.0	1	70.8	1	74.4	1
68.1	1	70.9	3	76.0	1

Table 1.2

Table 1.2 is a frequency distribution, but it is not as convenient a means for communicating this data set as Table 1.1 was for the data it represented. In Table 1.2

the data are too dispersed (spread out) for convenient interpretation. Consequently one might group the data in class intervals of, say, one unit or two units. Such groupings would be:

Grouping by one unit

Temperature	Frequency
64 - 64.9	3
65 - 65.9	4
66 - 66.9	4
67 - 67.9	2
68 - 68.9	6
69 - 69.9	7
70 - 70.9	7
71 - 71.9	8
72 - 72.9	2
73 - 73.9	4
74 - 74.9	2
75 - 75.9	0
76 - 76.9	1

Table 1.3

Grouping by two units

Temperature	Frequency
64 - 65.9	7
66 - 67.9	6
68 - 69.9	13
70 - 71.9	15
72 - 73.9	6
74 - 75.9	2
76 - 77.9	1

Table 1.4

Table 1.4 could be displayed as a frequency polygon by using midpoints of the temperature intervals for one coordinate and the frequency for the intervals as the second coordinate. The horizontal axis depicts temperature, and the vertical axis gives frequency for that temperature. So we plot the points (65, 7), (67,6), (69, 13), (71, 15), (73, 6), (75, 2), and (77, 1). The frequency polygon would then appear as below:

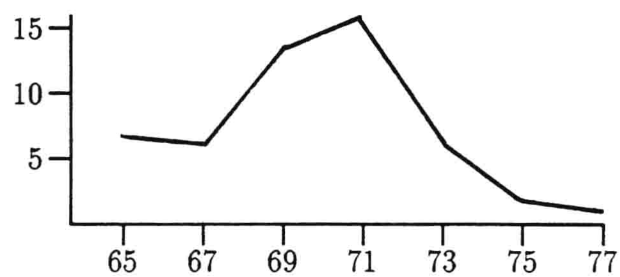

Figure 1.6

Another representation for the same data can be given using a bar graph as below:

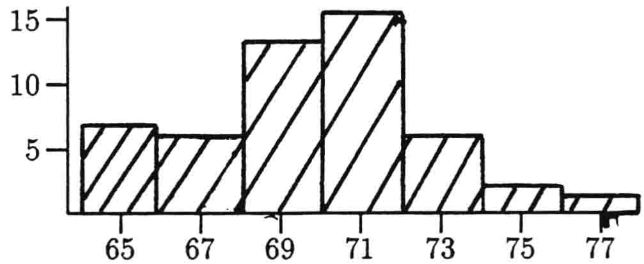

Figure 1.7

Figure 1.7 is a **bar chart** of a frequency distribution - which is also called a **histogram.** The bars are of equal width and correspond to equal class intervals with the height of each bar corresponding to the frequency it represents. Thus the area of a bar above each class interval is proportional to the frequencies represented in that class.

Although bar graphs may arise from grouped data, they need not, and may simply be viewed as a more effective way to present some data than a line graph. Two other charts taken from January 1986 newspapers illustrate this point.

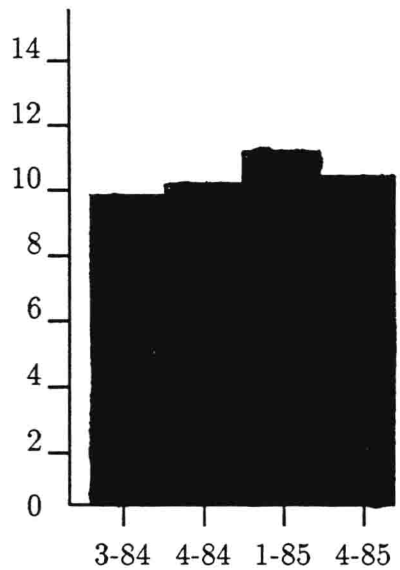

Figure 1.8
Auto sales Monthly Average (in thousands)

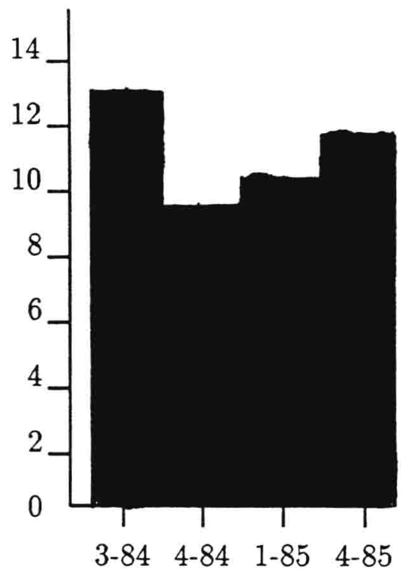

Figure 1.9
Help-wanted
Monthly average
(in thousands)

Another common example of a bar chart is the depiction of the Dow Jones average, as illustrated on some news casts.