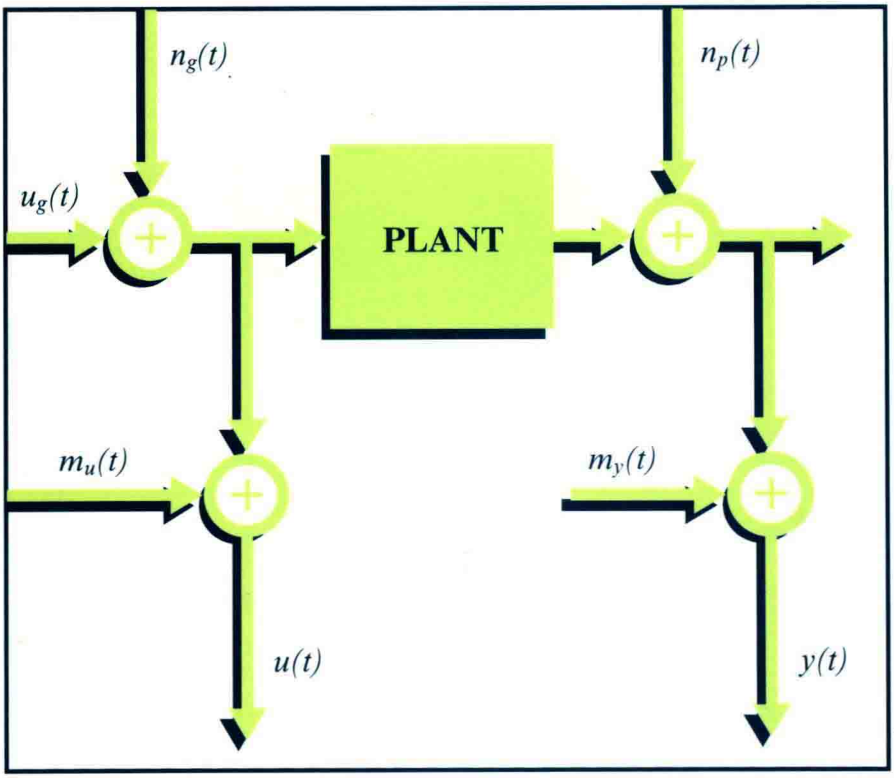
Second Edition

SYSTEM IDENTIFICATION

A Frequency Domain Approach

Rik Pintelon • Johan Schoukens



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To my grandchildren, Lamine, Aminata, Alia-Rik, Malick, Khadidiatou, Babacar, Ibrahima, Khadidiatou and those to be born

Rik

To Annick, Maarten, Sanne, and Ine

Johan

Preface to the First Edition

Identification is a powerful technique for building accurate models of complex systems from noisy data. It consists of three basic steps, which are interrelated: (1) the design of an experiment; (2) the construction of a model, black box or from physical laws; and (3) the estimation of the model parameters from the measurements. The art of modeling lies in proper use of the skills and specialized knowledge of experts in the field of study, who decide what approximations can be made, suggest how to manipulate the system, reveal the important aspects, and so on. Consequently, modeling should preferably be executed by these experts themselves. Naturally, they require relevant tools for extracting information of interest. However, most experts will not be familiar with identification theory and will struggle in each new situation with the same difficulties while developing their own identification techniques, losing time over problems already solved in the literature of identification.

This book presents a thorough description of methods to model linear dynamic time-invariant systems by their transfer function. The relations between the transfer function and the physical parameters of the system are very dependent upon the specific problem. Because transfer function models are generally valid, we have restricted the scope of the book to these alone, so as to develop and study general purpose identification techniques. This should not be unnecessarily restricting for readers who are more interested in the physical parameters of a system: the transfer function still contains all the information that is available in the measurements, and it can be considered to be an intermediate model between the measurements and the physical parameters. Also, the transfer function model is very suitable for those readers looking for a black box description of the input-output relations of a system. And, of course, the model is directly applicable to predict the output of the system.

In this book, we use mainly frequency domain representations of the data. This opens many possibilities to identify continuous-time (Laplace-domain) or discrete-time (z-domain) models, if necessary extended with an arbitrary and unknown delay. Although we advocate using periodic excitations, we also extend the methods and models to deal with arbitrary excitations. The "classical" time-domain identification methods that are specifically directed toward these signals are briefly covered and encapsulated in the identification framework that we offer to the reader.

This book provides answers to questions at different levels, such as: What is identification and why do I need it? How to measure the frequency response function of a linear dynamic system? How to identify a dynamic system? All these are very basic questions, directly focused on the interests of the practitioner. Especially for these readers, we have added guidelines to many chapters for the user, giving explicit and clear advice on what are good choices in order to attain a sound solution. Another important part of the material is intended for readers who want to study identification techniques at a more profound level. Questions on how to analyze and prove the properties of an identification scheme are addressed in this part. This study is not restricted to the identification of linear dynamic systems; it is valid for a very wide class of weighted, nonlinear least squares estimators. As such, this book provides a great deal of information for readers who want to set up their own identification scheme to solve their specific problem.

The structure of the book can be split into four parts: (1) collection of raw data or nonparametric identification; (2) parametric identification; (3) comparison with existing frameworks, and guidelines; (4) profound development of theoretical tools.

In the *first part*, after the introductory chapter on identification, we discuss the collection of the raw data: How to measure a frequency response function of a system. What is the impact of nonlinear distortions? How to recognize, qualify, and quantify nonlinear distortions. How to select the excitation signals in order to get the best measurements. This nonparametric approach to identification is discussed in detail in Chapters 2 to 5, and 7.[1]

In the *second part*, we focus on the identification of parametric models. Signal and system models are presented, using a frequency and a time domain representation. The equivalence and impact of leakage effects and initial conditions are shown. Nonparametric and parametric noise models are introduced. The estimation of the parameters in these models is studied in detail. Weighted (nonlinear) least squares methods, maximum likelihood, and subspace methods are discussed and analyzed. First, we assume that the disturbing noise model is known; next, the methods are extended to the more realistic situation of unknown noise models that have to be extracted from the data, together with the system model. Special attention is paid to the numerical conditioning of the sets of equations to be solved. Taking some precautions, very high order systems, with 100 poles and zeros or even more, can be identified. Finally, validation tools to verify the quality of the models are explained. The presence of unmodeled dynamics or nonlinear distortions is detected, and simple rules to guide even the inexperienced user to a good solution are given. This material is presented in Chapters 6, and 8 to 12[1].

The *third part* begins with an extensive comparison of what is classically called time and frequency domain identification. It is shown that, basically, both approaches are equivalent, but some questions are more naturally answered in one domain instead of the other. The most important question is nonparametric versus parametric noise models. Next, we provide the practitioner with detailed guidelines to help avoid pitfalls from the very beginning of the process (collecting the raw data), over the selection of appropriate identification methods until the model validation. This part covers Chapters 13 and 14.

The *last part* of the book is intended for readers who want to acquire a thorough understanding of the material or those who want to develop their own identification scheme. Not only do we give an introduction to the stochastic concepts we use, but we also show, in a structured approach, how to prove the properties of an estimator. This avoids the need for each freshman in this field to find out, time and again, the basic steps to solve such a problem. Starting from this background, a general but detailed framework is set up to analyze the properties of nonlinear least squares estimators with deterministic and stochastic weighting.

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1. Chapters 4, 7, and 12 were not in the first edition.

For the special and quite important class of semilinear models, it is possible to make this analysis in much more detail. This material is covered in Chapters 15 to 20.

It is possible to extract a number of undergraduate courses from this book. In most of the chapters that can be used in these courses, we added exercises that introduce the students to the typical problems that appear when applying the methods to solve practical problems.

A first, quite general undergraduate course subject is the measurement of frequency response functions (FRF) of dynamic systems, as discussed in Chapters 2 to 5. Chapter 7[1] deals with advanced techniques for measuring FRFs and is more suited for a graduate course on the topic. To understand Chapter 7 one should first master Chapters 1 to 6.

Another possibility is a first introduction to the identification of linear dynamic systems. Such an undergraduate course should include Chapter 1 and some selected parts of Chapters 6, and 8 to 11. Chapter 12[1] generalizes the results of Chapters 10 and 11 to non-steady state conditions and arbitrary excitations and is more suited for a graduate course on linear system identification. To understand Chapter 12 one should first read Chapters 7, 10, and 11.

A final course, at the graduate level, is an advanced course on identification based on the methods that are explained in Chapters 17 to 20. This gives an excellent introduction for students who want to develop their own algorithms.

A MATLAB® toolbox, which includes most of the techniques developed in this book, is available. It can be used with a graphical user interface, avoiding most problems and difficult questions for the inexperienced user. At the basic level, this toolbox produces almost autonomously a good model. At the intermediate or advanced level, the user obtains access to some of the parameters in order to optimize the operation of the toolbox to solve dedicated modeling problem. Finally, for those who want to use it as a research tool, there is also a command level that gives full access to all the parameters that can be set to optimize and influence the behavior of the algorithms. More information on this package can be obtained by sending an E-mail to one of the authors: rik.pintelon@vub.ac.be or johan.schoukens@vub.ac.be

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1. Chapters 4, 7, and 12 were not in the first edition.

Preface to the Second Edition

During the 10 years since the first edition appeared, frequency domain system identification has evolved considerably. In the second edition we have added new material that reflects our personal view on this development. The book has been updated and new sections and chapters have been added. These mainly deal with arbitrary excitations; periodic excitations under non-steady state conditions; discrete-time and continuous-time parametric noise modeling in the frequency domain; the detection, quantification, and qualification of nonlinear distortions; the best linear approximation of nonlinear systems operating in feedback; and multi-input, multi-output systems. Finally, a large number of new experiments have been included throughout the chapters. In the sequel, we explain these extensions in more detail.

In the first edition the emphasis was strongly put on the use of periodic excitations because, at that time, it was the only way to obtain nonparametric noise models in a pre-processing step, which considerably simplifies the system identification task. Although very successful, this approach has a number of shortcomings: (i) it does not account for the noise leakage that increases the variability of the frequency response function (FRF) estimate and introduces a correlation among consecutive signal periods; (ii) it is sensitive to plant transients that introduce a bias in the FRF and the nonparametric noise models; and (iii) it cannot handle arbitrary excitations. Solutions for these problems are presented in Chapters 7 (nonparametric models) and 12 (parametric transfer function models) of the second edition. These new methods have lead to new insights and, hence, also to new guidelines for the user (see Chapter 14).

The first edition studied the properties of the best linear approximation of a nonlinear plant operating in open loop for the class of Gaussian excitation signals. It was unclear whether these results could be extended to nonlinear systems operating in closed loop configuration and to non-Gaussian inputs. These issues are handled in Section 3.5.2 (non-Gaussian excitations) and Sections 3.8, 7.2.8, and 7.3.5 (nonlinear plants operating in closed loop) of the second edition.

In the first edition odd-odd (only every second odd harmonic is excited) random phase multisines were proposed to detect, qualify, and quantify the level of the nonlinear distortions in FRF measurements. The level of the nonlinear distortions at the non-excited harmonics (= detection lines) was used for quantifying roughly the level of the nonlinear distortions at the nearest excited harmonics. No theoretical justification was given for this extrapolation. In Chapter 4 of the second edition it is proven that the extrapolation is asymptotically (for the number of excited harmonics going to infinity) exact if the detection lines (non-excited) harmonics are randomly distributed over the frequency band of interest. It results in the so-called full or odd random phase multisines with random harmonic grid (see Section 4.2).

The first edition handled the frequency domain identification of parametric discrete-time noise models under the restriction that the DFT frequencies cover the unit circle uniformly. In the second edition (see Section 10.9) this is generalized to continuous-time and discrete-time noise models, identified on (a) part(s) of the imaginary axis or unit circle, respectively. The link with the classical time domain prediction error framework is also discussed in detail.

The first edition was mostly devoted to single-input, single-output systems. In the second edition a full extension to the multivariable case is made for the design of periodic excitations (see Sections 2.7 and 3.7), the nonparametric frequency response matrix (FRM) measurement using periodic and random excitations (see Section 2.7 and Chapter 7), the detection and quantification of nonlinear distortions in FRM measurements using periodic excitations (see Sections 3.7, and 7.3.5 to 7.3.7), and the parametric transfer function modeling (see Chapter 12).

In the first edition the experiments were mainly concentrated in one chapter. This chapter has been deleted and replaced in the second edition by new experiments (see Chapters 4, 7, 10, 12, and 13) that use the new insights and the newly developed identification methods.

To guide the reader through this book a number of "lecture maps" for the following topics are provided: introduction to identification; nonparametric FRF measurements; identification of linear dynamic systems; measurement and modeling of multiple-input, multiple-output systems; measurement and modeling of nonlinear systems; and analysis of the stochastic properties of estimators. These selected topics can be used as undergraduate (u) and/or graduate (g) courses.

Introduction to Identification$^{(u)}$

Chapters 1 and 2; Chapter 5, Section 5.1–5.3; and Chapter 8.

Nonparametric FRF Measurements$^{(u,\, g)}$

Part I$^{(u)}$ — Basics: Chapter 1, Section 1.3; Chapter 2; and Chapter 5, Sections 5.1–5.3.

Part II$^{(u,\, g)}$ — Influence of Nonlinear Distortions: Chapters 3 and 4.

Part III$^{(g)}$ — Advanced Methods for Arbitrary and Periodic Excitations under Non-Steady State Conditions: Chapter 6, Sections 6.1–6.3, and 6.6; and Chapter 7.

Identification of Linear Dynamic Systems$^{(u,\, g)}$

Part I$^{(u)}$ — Basics of Frequency Domain System Identification: Chapter 1; Chapter 6, Sections 6.1–6.3, and 6.5; and Chapters 8, 9, 11, and 20.

Part II$^{(u,\, g)}$ — Estimation with Unknown Noise Model: Chapter 6, Sections 6.7 and 6.8; and Chapters 10 and 13.

Part III$^{(g)}$ — Advanced Methods for Arbitrary and Periodic Excitations under Non-Steady State Conditions: Chapters 7, 12, and 14.

Measurement and Modeling of Multiple-Input, Multiple-Output Systems$^{(g)}$

Chapter 2, Section 2.7; Chapter 3, Section 3.7; Chapter 6, Section 6.6; Chapter 7; Chapter 9, Section 9.21; and Chapter 12.

Measurement and Modeling of Nonlinear Systems$^{(u,\, g)}$

Chapters 3 and 4; Chapter 6, Section 6.8; Chapter 9, Section 9.19; Chapter 10, Section 10.7; and Chapter 12, Section 12.4.

Analysis of the Stochastic Properties of Estimators$^{(g)}$

Chapters 16–19.

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Finally, software support for identifying multivariable systems is freely available at the website

http://booksupport.wiley.com

via MATLAB® m-files (design of multi-input periodic excitations, nonparametric frequency response matrix measurements using periodic and random excitations, detection and quantification of nonlinear distortions in FRM measurements, parametric transfer function modeling using nonparametric noise models, and simultaneous parametric identification of noise and plant models).

Acknowledgments

This book is the culmination of 30 years of research in the identification group of the Department of Fundamental Electricity and Instrumentation (ELEC) of the Vrije Universiteit Brussel (VUB). It is the result of close and harmonious cooperation between many of the workers in the department, and we would like to thank all of them for exchanges of ideas and important discussions that have taken place over the gestation period of this, the end result. We are greatly indebted to the former heads of the department ELEC, Jean Renneboog and Alain Barel, to the R&D Department of the VUB, to the FWO-Vlaanderen (the Fund for Scientific Research–Flanders), to the Flemish government (Concerted Action–GOA, and Methusalem research programs), and to the federal government (Interuniversity Attraction Poles–IAP research programs). Without their sustained support, this work would never have seen the light of day.

We are grateful to many colleagues and coworkers in the field who imparted new ideas and methods to us, took time for discussions, and were prepared to listen to some of our less conventional ideas that did not fit in the mainstream of the "classical" identification. Their critical remarks and constructive suggestions contributed significantly to our view on the field.

Last, but not least, we want to thank Yves Rolain for contributing to our work as a friend, a colleague, and a coauthor of many of our papers. Without his sustained support, we would never have been able to access the advanced measurement equipment we used for the many experiments that are reported in this book.

Rik Pintelon

Johan Schoukens

List of Operators and Notational Conventions

$\mathbb{A}$	outline uppercase font denotes a set, for example, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ are, respectively, the natural, the integer, the rational, the real, and the complex numbers
$\otimes$	the Kronecker matrix product
$*$	convolution operator
Re( )	real part of
Im( )	imaginary part of
$\arg\min_{x} f(x)$	the minimizing argument of $f(x)$
$O(x)$	an arbitrary function with the property $\lim_{x \to 0} \lvert O(x)/x \rvert < \infty$
$o(x)$	an arbitrary function with the property $\lim_{x \to 0} \lvert o(x)/x \rvert = 0$
$\hat{\theta}$	estimated value of θ
$\bar{x}$	complex conjugate of x
subscript 0	true value
subscript Re	$A_{\mathrm{Re}} = \begin{bmatrix} \mathrm{Re}(A) & -\mathrm{Im}(A) \\ \mathrm{Im}(A) & \mathrm{Re}(A) \end{bmatrix}$
subscript re	$A_{\mathrm{re}} = \begin{bmatrix} \mathrm{Re}(A) \\ \mathrm{Im}(A) \end{bmatrix}$
subscript u	with respect to the input of the system
subscript y	with respect to the output of the system
subscript *	limiting estimate
superscript T	matrix transpose

superscript $-T$	transpose of the inverse matrix
superscript H	Hermitian transpose: complex conjugate transpose of a matrix
superscript $-H$	Hermitian transpose of the inverse matrix
superscript $+$	Moore-Penrose pseudoinverse
superscript $\perp$	orthogonal complement of a subspace or a matrix
$\angle x$	phase (argument) of the complex number x
$X_{[i]}(s)$	ith entry of the vector function $X(s)$
$A_{[i,j]}(s)$	i,jth entry of the matrix function $A(s)$
$A_{[:,j]}$	jth column of A
$A_{[i,:]}$	ith row of A
$A_{[n:m,\,p:q]}$	rows n to m and columns p to q of A
$X^{[k]}(s)$	kth realization of a random process $X(s)$
$\lambda(A)$	eigenvalue of a square matrix A
$\sigma(A)$	singular value of an $n \times m$ matrix A
$\kappa(A) = (\max_i \sigma_i(A))/(\min_i \sigma_i(A))$	condition number of an $n \times m$ matrix A
$\lvert x\rvert = \sqrt{(\mathrm{Re}(x))^2 + (\mathrm{Im}(x))^2}$	magnitude of a complex number x
$\lVert A\rVert_1 = \max_{1 \le j \le m} \sum_{i=1}^{n} \lvert A_{[i,j]}\rvert$	1-norm of an $n \times m$ matrix A
$\lVert A\rVert_2 = \max_{1 \le i \le m} \sigma_i(A)$	2-norm of an $n \times m$ $(n \ge m)$ matrix A
$\lVert X\rVert_2 = \sqrt{X^H X}$	2-norm of the column vector X
$\lVert A\rVert_\infty = \max_{1 \le i \le n} \sum_{j=1}^{m} \lvert A_{[i,j]}\rvert$	×-norm of an $n \times m$ matrix A
$\lVert A\rVert_F = \sqrt{\mathrm{tr}(A^H A)}$	Frobenius norm of an $n \times m$ matrix A
$\mathrm{diag}(A_1, A_2, \ldots, A_K)$	block diagonal matrix with blocks A_k, $k = 1, 2, \ldots, K$
$\mathrm{herm}(A) = (A + A^H)/2$	Hermitian symmetric part of an $n \times m$ matrix A
$\mathrm{null}(A)$	null space of the $n \times m$ matrix A, linear subspace of C^m defined by $Ax = 0$
$\mathrm{range}(A)$	range of the $n \times m$ matrix A, linear subspace of C^n that is reachable by making linear combinations of the columns of A ($\mathrm{range}(A) = (\mathrm{null}(A^T))^\perp$)
$\mathrm{rank}(A)$	rank of the $n \times m$ matrix A, maximum number of linear independent rows (columns) of A
$\mathrm{std}(x) = (\mathbb{E}\{\lvert x - \mathbb{E}\{x\}\rvert^2\})^{1/2}$	standard deviation of x

$\text{span}\{a_1, a_2, \ldots, a_m\}$	the span of the vectors $a_1, a_2, \ldots, a_m$ is the linear subspace obtained by making all possible linear combinations of $a_1, a_2, \ldots, a_m$
$\text{tr}(A) = \sum_{i=1}^{n} A_{[i,i]}$	trace of an $n \times n$ matrix A
$\text{var}(x) = \mathbb{E}\{\lvert x - \mathbb{E}\{x\}\rvert^2\}$	variance of x
$\text{vec}(A)$	a column vector formed by stacking the columns of the matrix A on top of each other
a.s.lim	almost sure limit, limit with probability one
l.i.m.	limit in mean square
plim	limit in probability
Lim	limit in distribution
$\mathbb{E}\{\ \}$	mathematical expectation
Prob( )	probability
$b_X = X - \mathbb{E}\{X\}$	bias of the estimate X
$\text{Cov}(X, Y) = \mathbb{E}\{(X - \mathbb{E}\{X\})\overline{(Y - \mathbb{E}\{X\})^H}\}$	cross-covariance matrix of X and Y
$\text{covar}(x, y) = \mathbb{E}\{(x - \mathbb{E}\{x\})\overline{(y - \mathbb{E}\{y\})}\}$	covariance of x and y
cum( )	cumulant
$C_X = \text{Cov}(X) = \text{Cov}(X, X)$	covariance matrix of X
$\hat{C}_X = \frac{1}{M-1}\sum_{m=1}^{M}(X^{[m]} - \hat{X})(X^{[m]} - \hat{X})^H$	sample covariance matrix of M realizations of X
$C_{XY} = \text{Cov}(X, Y)$	cross-covariance matrix of X and Y
$\hat{C}_{XY} = \frac{1}{M-1}\sum_{m=1}^{M}(X^{[m]} - \hat{X})(Y^{[m]} - \hat{Y})^H$	sample cross-covariance matrix of M realizations of X and Y
$CR(X)$	Cramér-Rao lower bound on X
$\text{DFT}(x(t))$	discrete Fourier transform of the samples $x(t),\ t = 0, 1, \ldots, N-1$
$Fi(X)$	Fisher information matrix with respect to the parameters X
I_m	$m \times m$ identity matrix
q	backward shift operator: $qu(kT_s) = u((k-1)T_s)$
$\text{MSE}(X) = \mathbb{E}\{(X - X_0)(X - X_0)^H\}$	mean square error of the estimate X
$R_{xx}(\tau) = \mathbb{E}\{x(t)x^H(t-\tau)\}$	autocorrelation of $x(t)$
$R_{xy}(\tau) = \mathbb{E}\{x(t)y^H(t-\tau)\}$	cross-correlation of $x(t)$ and $y(t)$
$S_{xx}(j\omega)$	Fourier transform of $R_{xx}(\tau)$ (autopower spectrum of $x(t)$)
$S_{xy}(j\omega)$	Fourier transform of $R_{xy}(\tau)$ (cross-power spectrum of $x(t)$ and $y(t)$)
$\hat{S}_{XX}(k) = \frac{1}{M}\sum_{m=1}^{M} X^{[m]}(k)X^{[m]H}(k)$	Sample autopower spectrum of $x(t)$

$\hat{S}_{XY}(k) = \frac{1}{M}\sum_{m=1}^{M} X^{[m]}(k)\, Y^{[m]H}(k)$	Sample cross-power spectrum of $x(t)$ and $y(t)$
$\hat{X} = \frac{1}{M}\sum_{m=1}^{M} X^{[m]}$	sample mean of M realizations (experiments) of X
$\mu_x = \mathbb{E}\{x\}$	mean value of x
$\sigma_x^2 = \mathrm{var}(x)$	variance of the x
$\hat{\sigma}_x^2 = \frac{1}{M-1}\sum_{m=1}^{M} \lvert x^{[m]} - \hat{x}\rvert^2$	sample variance of M realizations of x
$\sigma_{xy}^2 = \mathrm{covar}(x, y)$	covariance of x and y
$\hat{\sigma}_{xy}^2 = \frac{1}{M-1}\sum_{m=1}^{M} (x^{[m]} - \hat{x})(\overline{y^{[m]} - \hat{y}})$	sample covariance of M realizations of x and y

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