E. S. Wentzel

OPERATIONS RESEARCH

A Methodological Approach

ИЗДАТЕЛЬСТВО · МИР ·

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FOREWORD

The aim of this book is to present in a widely appealing form the subject matter and methods of operations research (OR), a science acquiring in the latest years ever wider applicational arenas. It is of a relatively recent origin and so far its boundaries and contents can hardly be deemed rigorously defined.

As a taught discipline, it is included in many curricula though what is covered is not always the same. In professional quarters, it lacks unanimity in much the same way. Some OR professionals treat it merely as the mathematical methods of optimization such as linear, nonlinear and dynamic programming. Others on the contrary, do not include these techniques in OR, approaching it generally from the viewpoint of game theory and statistical decision making. A third group tends to deny that OR exists as an independent discipline preferring to include it with cybernetics—a term inappropriately defined and therefore variously understood. There are even those who tend to endow OR with an enormously wide sense claiming it to be a key science.

Time, obviously, will tell which direction this relatively young discipline will take, which of the methods generally accepted as its constituents will hold, and which will branch off into self-contained disciplines. A particular question that remains concerns the future relation between OR and systems theory, a widely discussed topic of recent times.

Beyond doubt, however, is the point that whatever the area of activity may be—production planning and inventory control, transportation, military operations and weapon system development, personnel management, social services, health services, communication systems, computer networks and information systems, to name but a few—the problems they pose with ever increasing rate are similarly formulated, can be identified by several features they have in common and, last but not least, can be solved by similar methods. These problems are therefore conveniently grouped under the common heading of OR problems.

A typical situation giving rise to an OR problem is as follows. An undertaking (a system of actions) having a clearly defined objective may be organized in several alternative ways by making a decision based upon choosing from a set of possible alternatives. Each choice offers its own advantages and disadvantages, so that in a complex situation the decision maker might not be able to make a preferable option at once and quickly decide why he should prefer one alternative and not another. To clarify the situation and compare the alternatives in several aspects, OR suggests a series of mathematical operations. Their aim is to analyze the situation critically and thus prepare a decision for those bearing the responsibility for a final choice.

Without being confined to an exclusive area of practice, the discussion in the book will be focused on the methodological aspects which are common to all OR problems wherever they might appear. Therefore the main emphasis will be placed on such methodological treatises as problem formulation, model development, and assessment of computational results, rather than on mathematical rigor. The experience collected in the field suggests that it is these evaluation-and-analysis steps (not manipulatory skill in transformations and computations) that might present major difficulties to an inexperienced practitioner when he will try to implement the mathematical methods of decision making to practical problems of value.

In writing this text the author employed her many-year experience in applying OR to various practical arenas. The standpoint, into which the experience would have inevitably crystallized, was attempted to be delivered in the simplest possible form.

The relevant mathematics is simple and requires some expertise in probability theory. Whenever the text goes beyond this simplicity frame, the needed explanations are immediately provided. As to the crest points of main ideas and methodology, they are treated with the highest rigor, calling for the attention and intense thinking of the reader.

The author tended to avoid as possible an entertaining presentation since an inavoidable curiosity dressing' would believingly distract the reader from, rather than help him to, the point. It does not imply, however, that the author would stick to stiffly formal, vapid fashion—a notorious attribute of mathematical texts. Rather, the adopted spirit is fairly jovial and even slipping occasionally to formulations which are not perfectly correct and, hence, open to criticism. The excuse is that the book is devoted primarily to the practitioners who are novices to the subject, rather than to experts. Ample reservations, should they be made to perfect the rigor, would only repell the reader, making it hard for him to grasp the essence of the matter.

The chapters of the book are not equal in difficulty of comprehension and the amount of mathematics involved. The reader who would like to get a general acquaintance with the subject, problems and possibilities of OR may confine himself to attentive reading through Chapters 1 and 2 and initial sections of the rest chapters. Beyond these introductory chapters, an intelligent reader will find what to calculate, thus having a closer look at the methods of OR and making himself familiar with the relevant mathematics to feel at ease if met with it in specialized issues.

Chapter 6 dealing with queueing theory considers a series of methodologies almost entirely absent from nonspecial literature; that is the reason for its comparatively large size.

The aim of the book is to provide the reader with a foundation upon which to build a further study of a particular topic. References which cover extended applications and more formal mathematical treatment of the subject matter are provided at the end of the book.

E.S. Wentzel

Chapter 1

THE NATURE AND USE OF OPERATIONS RESEARCH

1.1 Operations Research—What It Is and What It Does

Now that the revolutionary influence of science and technology is felt almost everywhere, ever more weight is put by science on planning, management and control. There are many reasons for this. The modern technology grows more sophisticated, scales of ventures and their aftermath effects spread dramatically, automatic control being implemented in many fields, all this calls for the analysis of complicated, objective-bound processes to be performed from the viewpoint of their structure and organization. The science is required to deliver recommendations of how such processes should be controlled in optimal (judicious) fashion. Far behind are those times when a correct and efficient control had been found by trial-and-error approach. Now that the losses due to possible errors may cost too much to be afforded, the control must be developed scientifically.

Requirements of practice called for special scientific methods which can conveniently be grouped under a common heading, operations research (OR). Referring to this term we shall imply the application of quantitative, mathematical methods to prepare decisions bound to be made in all the fields of objective-bound human activity.

To get a more closer look at what we mean under decision, consider a venture headed towards a certain target. The person (or a group of persons) who has initiated the venture always has a set of alternative courses of action, say, to pick up tools and build up a process in some way or another, to allocate resources according to one or another pattern, etc. The *decision* is the alternative that the manager chooses as his course of action. Decisions, consequently, may be bad or good, thought over or hasty, well-founded or arbitrary.

The need for decision making is as old as the hills. Even at times of wilderness a leader of a tribe going to hunt a mammoth has to make a decision on where to place a trap and how to place the hunters and what must their weapon be, etc. We also would make decisions in our everyday life without even noticing it. Leaving, for example, for the office in the morning, we would make a series of decisions: what to wear? should an umbrella be carried? where to cross the street more conveniently? which urban transport to pick? and so on. A manager responsible for a process is to arrive at a decision every time when he has alternatives, say, in placement of manpower, assignment of a work order, to name only a few.

Does this decision making mean that we are involved in operations research? Not at all. Operations research begins whenever one or another mathematical technique is applied to substantiate the decisions being taken. Obviously, in simple situations decisions are taken without any mathematics invoked, simply by the sound judgement and expertise. To decide what to wear going out or where to cross the street mathematics is hardly needed and undoubtedly will not be in the future. An optimum decision is arrived at as if by oneself, guided by the practical experience. If at times a decision made is not the very best, then what? Mistakes are committed to improve on them.

The decisions we will be concerned with more in this book, however, are those heavily loaded with responsibility. Let, for example, the public transportation network be planned in a new town having its own layout of concentration points such as factories, apartment blocks, etc. Obviously, certain decisions are to be made concerning how to trace the routes and what vehicles should use them, where to place the stops, and what should be the interval in the ordinary and rush hours, etc.

These decisions are much harder to make and, what is more important, many things depend on them. Erroneously chosen they spoil life and have an adverse impact on business activity of a whole town. Certainly, in this situation a decision may well be prepared intuitively from experience and common sense (as is not infrequently done). Yet the decisions are more judicious if backed up by mathematical reasoning. The preparative computations may help to avoid long and costly search of a decision by trial and error.

To expand on this, let us take another example. Let us imagine that we are entrusted with a large scale venture, say, to divert waters from northwardbound rivers to water-hungry arid areas of the south. Should we take an arbitrary, willful decision that might bring about adverse effects on a global scale, or better to make preparative computations with mathematical models? The dilemma could hardly be viewed as ambiguous—the multilevel computations are mandatory.

'Score twice before you cut once' a proverb says. Operations research is the very mathematical scoring of future decisions, helping to spare time, efforts, and resources, and to avoid blunders, which are no longer such as to improve anything—the cost of respective correction will be too expensive.

The more complex, expensive and large in scale the designed system is, the less allowable in it are willful decisions and the more gain in importance scientific methods which, when implemented, provide an estimate of each decision's consequences, help discard the unallowable versions and recommend the most successful ones. They help in assessing whether the available information is adequate to prepare a correct decision and, if not, then indicate what data should be additionally collected. It would be extremely risky to be guided solely by intuition, i.e. experience and common sense. Modern science and technology evolve so fast that the experience may simply not have been acquired. To say more, the undertakings initiated are often unique and original. Thus, the experience may be virtually absent, and common sense, if not proven by calculation, will be deceptive. The calculations that make the process of decision making easier are the subject matter of operations research.

We have mentioned already that OR is a relatively young science (the notion of "youth", though, is relative in scientific quarters: many scientific disciplines have been nipped in the bud shortly after their appearance, failing to find an application). The name operational research[1] was first used in the years just before the outbreak of World War II to describe an approach

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[1] The name 'operational research' is still preferred by British scientists to 'operations research' which was coined in the United States—*Translator's note.*

taken by interdisciplinary groups of British scientists summoned to solve strategic and tactical problems for military management. The decisions that were prepared by the groups were primarily concerned with the application of weapons and distribution of forces and facilities among various targets. Similar problems, though with a differently named approach, had been taken elsewhere, in particular, in the Soviet Union. After the war, this approach spread into a variety of practical fields: industry, agriculture, construction, marketing, social services, transportation, communications, health services, pollution control, etc. There hardly exists an area of human enterprise where mathematical models and OR techniques have not been implemented in some form or the other.

To get a better insight into the specifics of the science, consider several typical OR problems. Intentionally taken to represent various fields, they, although simplified in the setting, bring forth fairly well the main idea of OR scope and objectives.

1. Transportation problem. A network of factories consumes certain types of raw materials produced at several sources. The sources are linked with the factories each in its own way, by railway, waterway, motor road, or air, having its own cost per unit of transported load. It is required to design a supply layout, specifying which source should supply what raw material and in what amount, and such that the demand is satisfied at minimum transportation cost.

2. Construction project planning. A section of motor road is being built. The construction involves a certain amount of manpower, construction equipment, repair shops of a known capacity, trucks, and so on. It is required to plan the project, i.e. to schedule the order of works, distribute the equipment and manpower along the construction site, and plan the maintenance and repair works, so that the project is terminated in the shortest term.

3. Seasonal sales movements. The entrepreneur of a marketing firm plans to establish a network of branch off stores to market a certain stock of consumer goods. He is to decide on the number of retailing stores, their location, distribution of inventories and personnel at each of the stores so that the business turns out maximum economic efficiency.

4. Snow fencing of roads. In the Nordic countries snow storms present serious interference to traffic. Any road blocked by snow means losses for the economy. For road fencing, there exist a few alternatives—to construct the road with a suitable profile, to set up fencing facilities beside the road, etc.—each with its own cost for construction and maintenance. The decision maker has at hand the data on prevailing wind directions and frequency and intensity of snowfalls. He is to decide which means of snow control will be best in cost-efficiency terms, i.e. what means of fencing should be assigned to each of the roads subject to the losses from snow blizzards.

5. Search for enemy submarines. An antisubmarine warfare officer receives a message that an enemy submarine has appeared in an area that Coastal Command surveillance. A group of aircraft takes off to find and bomb the submarine. The raid is to be set up in a most rational fashion, that is the routes are traced and altitudes and a pattern of attack are chosen such as to complete the task with the greatest certainty.

6. Sample tests in quality control. To guarantee the quality of produced items at a specified level a factory establishes a sampling test system. A sound