MECHANICS: DYNAMICAL SYSTEMS

Wanda Szemplińska-Stupnicka

The Behavior of Nonlinear Vibrating Systems

Volume I

Fundamental Concepts and Methods: Applications to Single-Degree-of-Freedom Systems

KLUWER ACADEMIC PUBLISHERS

The Behavior of Nonlinear Vibrating Systems

Volume I:
Fundamental Concepts and Methods:
Applications to Single-Degree-of-Freedom Systems

WANDA SZEMPLIŃSKA-STUPNICKA

Institute of Fundamental Technological Research,
Polish Academy of Sciences, Warsaw, Poland

Kluwer Academic Publishers
Dordrecht / Boston / London

Library of Congress Cataloging in Publication Data

Szemplińska, Wanda.
The behaviour of nonlinear vibrating systems / by Wanda Szemplińska.
p. cm. – (Mechanics, dynamical systems; 12–13)
Bibliography: p.
Includes index.
Contents: v. 1. Fundamental concepts and methods. Applications to single degree-of-freedom systems – v. 2. Advanced concepts and applications to multi-degree-of-freedom systems.
ISBN 0-7923-0368-7 (U.S.: v. 1). – ISBN 0-7923-0369-5 (U.S.: v. 2).
ISBN 0-7923-0370-9 (U.S.: set)
1. Vibration. I. Title. II. Series.
TA355.S974 1990
620.3–dc 20 89–15272

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands

Kluwer Academic Publishers incorporates the publishing programmes
of D. Reidel, Martinus Nijhoff, Dr W. Junk, and MTP Press.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands

Printed on acid-free paper

Printed in The Netherlands

Table of Contents of Volume II:

Advanced Concepts and Applications to Multi-Degree-of-Freedom Systems

Preface

The purpose of this book is to provide students, practicing engineers and scientists with a treatment of nonlinear phenomena occurring in physical systems. Although only mechanical models are used, the theory applies to all physical systems governed by the same equations, so that the book can be used to study nonlinear phenomena in other branches of engineering, such as electrical engineering and aerospace engineering, as well as in physics.

The book consists of two volumes. Volume I is concerned with single-degree-of-freedom systems and it presents the fundamental concepts of nonlinear analysis. Both analytical methods and computer simulations are included. The material is presented in such a manner that the book can be used as a graduate as well as an undergraduate textbook. Volume II deals with multi-degree-of-freedom systems. Following an introduction to linear systems, the volume presents fundamental concepts of geometric theory and stability of motion of general nonlinear systems, as well as a concise discussion of basic approximate methods for the response of such systems. The material represents a generalization of a series of papers on the vibration of nonlinear multi-degree-of-freedom systems, some of which were published by me and my associates during the period 1965–1983 and some are not yet published. It also reflects some experimental and computational work on the vibration of complex structures performed for industry and my experience in teaching courses on linear and nonlinear vibrations at the Technical University of Warsaw, Institute of Fundamental Technological Research of the Polish Academy of Sciences, and later at Cornell University in the USA.

There is a long list of people to whom I owe a great deal and whose books, papers, and personal remarks played a significant role in my earlier studies and in writing this text. Here I want to express my deep appreciation and admiration of the three persons whose points of view and way of attacking problems played a particular role, namely, J.J. Stoker, R.M. Rosenberg and C. Hayashi.

In the Introduction to the book *Nonlinear Vibrations in Mechanical and Electrical Systems*, J.J. Stoker asked the question "why should one be interested particularly in the nonlinear problems in mechanics...?". In answering it, he wrote,

> One of the most attractive features of the subject of nonlinear vibration is existence of the surprisingly wide variety of distinctly new phenomena, and what is perhaps still more surprising, these phenomena can be treated by methods which are instructive in themselves, without being difficult, and which do not require the use of sophisticated mathematics.

Although 30 years have passed, since the book appeared in print, a book devoted exclusively to single-degree-of-freedom systems, this point of view and method of attacking the nonlinear vibration problems are not only still alive, but they have been a source of stimulus and inspiration for a new generation of researchers.

Another book devoted exclusively to single-degree-of-freedom systems is *Nonlinear Oscillations in Physical Systems* by C. Hayashi. The book is being enjoyed by students, researchers and others interested in applying analytical methods to concrete physical systems, and has affected to a large extent the mathematical formalism used in this text. My idea was to write a text placing emphasis on the relationship between theoretical analysis and experimental evidence, and one not aimed at a rigorous mathematical treatment of nonlinear differential equations, thus leaving some points open for discussion from the mathematical point of view. I found this a most appealing idea when attempting to explore the complex area of nonlinear vibrations in multi-degree-of-freedom systems.

New and revolutionary ideas were advanced in a series of papers by R.M. Rosenberg and his associates, and summarized in 1966 [180]. The idea consisted of exploring new properties of multi-degree-of-freedom nonlinear systems by means of distinctly new methods and by breaking with the traditional way of thinking in terms of rigorously defined concepts in linear theory. These ideas gave me the courage to undertake a critical analysis of the existing and well-accepted simplifying assumptions used to approximate analytical solutions. Because the new properties were determined by means of a rigorous qualitative analysis, the question arose immediately as to how they are reflected in approximate analytical solutions, or whether an approximate solution reflects them to a satisfactory extent.

Finally, the book *Nonlinear Vibrations* by A.H. Nayfeh and D.T. Mook, covering a large amount of research on multi-degree-of-freedom systems, has provided me with a deeper insight into the phenomenon of internal resonance.

The major features of the present book are as follows:

– From a physical point of view, the main interest lies in exploring distinctly new phenomena arising in nonlinear multi-degree-of-freedom systems, apart from 'internal resonance' effects, and in particular phenomena that cannot take place in single-degree-of-freedom systems.

– From the point of view of theoretical analysis, attention is concentrated on the question of how to use the existing analytical methods, and what modifications are needed to make the simplest first-approximation solution adequate for a given problem, in the sense that it reflects the essential properties of multi-degree-of-freedom systems. A comparison of computer results with numerical results calculated by several methods is aimed at, providing the reader with hints as to which approach might be suitable for a particular problem. Rigorous mathematical foundation for the analytical methods in not pursued here, and some questions concerning the almost-periodic, multi-frequency solutions remain.

– The relationship between the theoretical approximate solution and the results of computer simulations of the behavior of the equations of motion is used as a basis for evaluating the adequacy and accuracy of the applied analytical procedure. The computer simulations also serve as a source of 'experimental' material, which in some cases provides hints as to how to refine an analytical procedure.

It should be stressed here that the list of references does not cover all the recent literature in the field. Only publications regarded as representative of certain trends, ideas and approaches, and related directly to the material of this book, have been included.

This book has its beginnings in 1982, during my visit in the Department of Theoretical and Applied Mechanics at Cornell University; its writing has continued at the Institute of Fundamental Technological Research of the Polish Academy of Sciences in Warsaw, Poland.

The gracious assistance of Professor Antoni Jakubowski and the contribution of Paula and Joel Berg, all from Virginia Polytechnic Institute and State University in the USA, to the preparation of the final version of the book is gratefully acknowledged. Special thanks are also extended to J. Bajkowski, P. Niezgodzki and J. Rudowski from the Institute of Fundamental Technological Research, Warsaw, Poland, for their help with illustrations and computer simulations.

Introduction to Volume I

The significance of nonlinear effects in the field of physical system vibration analysis is now well recognized. Crucial advances in the theoretical treatment of nonlinear differential equations have enabled multi-degree-of-freedom vibration systems to be analyzed. Thus, although it is nearly 100 years since the initial studies of nonlinear vibration by Poincaré, it is only during the last 20 years that major advances have been made in the understanding of phenomena associated with nonlinear systems with multiple-degrees-of-freedom. In well-known books such as those by Krylov and Bogoliubov [B23], Stoker [B40], Minorski [B32], Andronov, Vitt and Khaikin [B1], Bogoliubov and Mitroploski [B6], Cunningham [B11], Kauderer [B22], Hayashi [B19], and Blaguiere [B2], attention was concentrated on, and in some cases was exclusively confined to, systems having a single-degree-of-freedom. In chapters devoted to multiple degree-of-freedom systems, the considerations were primarily based on the so called 'single frequency' solution and hence were confined to the same type of phenomena that were encountered in single degree-of-freedom systems. It is only during the last few years that books have been published that concentrate primarily on the new phenomena inherent only to systems with multiple degree of freedom. Books deserving special mention are those by Evan-Iwanowski [B13], and Nayfeh and Mook [B34].

It should be emphasized that the terms 'single' and 'multiple' degree-of-freedom systems refer merely to different mathematical models of the same real physical system. If we accept the simplifications that a single coordinate can, to a satisfactory degree of accuracy, describe the displacement, and so the vibratory motion, of the physical system under consideration, then we are dealing with the problem of a single-degree-of-freedom system. If we do not confine ourselves to this assumption, and use more than one coordinate to describe its motion, we are faced with the problem of a multiple-degree-of-freedom system.

The complexity of the analysis of nonlinear differential equations resulted in a trend towards confining attention to the simplest model, with a single coordinate. This approach gave an excellent opportunity to explain a great variety of surprisingly interesting new effects that may result from nonlinear characteristics. However, the further development of theoretical methods and experimental material stimulated the investigation of more complex models.

The book by Malkin [B28], published as early as 1956, may be regarded as the first to present an essential contribution to the exploitation of pertubation methods in multiple-degree-of-freedom systems. However, not until 1961 Yamamoto [275] discover the phenomena of combination resonances in physical systems (a rotating shaft) by both theoretical and experimental analysis.

For the problems of vibrations close to main resonances, it was the paper by Arnold in 1955 which first suggested that the mode shape of vibration might be an important factor [8]. The approach, often called the 'single mode method', which relied upon imposing the linear normal mode for resonant or free vibrations of nonlinear systems, in fact reduced the infinite degree-of-freedom system to that described by a single coordinate (normal coordinate). The procedure suppressed any new effects that might involve variations of mode shape due to nonlinearities. Nevertheless, the single mode approach appeared to be a very effective investigatory tool and made it possible to analyze a variety of interesting phenomena caused by large deflections and other sources of nonlinearity for a wide class of structural elements, such as beams, shells and plates. Numerous examples can be found in the books by e.g. Kauderer [B22], Feretis [B14], Schmidt [B38], Kaliski [42], Bolotin [B8], as well as in scientific journals.

The first warning that the single mode method should be treated with a certain degree of caution came from Bennet and Eisley in 1970 [19]. Considering the harmonic vibrations of a beam with geometric nonlinearities, they assumed an approximate solution in the form of a series including several normal modes. They proved that no single 'resonant normal coordinate' but also other normal coordinates can play a significant role in resonant motion. This approach, called the 'multi-mode approach', was then applied by a number of authors to the investigation of the resonant motion of beams, plates, and shells (e.g. [23, 26, 87, 160, 161]). The problem of the contribution of all normal coordinates to the resonant motion of nonlinear systems was also studied by means of computer simulation and by the Ritz-Galerking method by W. Szemplińska-Stupnicka in 1961 [210]. Natural frequencies were assumed to be incommensurable, so that no internal resonance effects were involved in the study.

Apart from the investigation based on approximate analytical methods, the problems of vibrations of multi-degree-of-freedom systems have been attacked, since 1961, by a fundamentally different approach. In a series of papers summarized in 1966 [180], Rosenberg and his associates applied a geometrical approach which reduced the problems of vibrations of n-degree-of-freedom systems to that of examining a trajectory of its representative point in the configuration space. On exploring the essential qualitative nature of normal and resonant motion of a class of 'admissible systems', they introduced the new concept of 'nonlinear normal modes'.

A significant contribution to knowledge of the distinctly new phenomena that may be encountered in multiple-degree-of-freedom systems is due to A. Tondl. In a series of papers and monographs, he stimulated exploration of various types of combination resonances and internal resonances, and problems of the domain of attraction of secondary resonances. He also opened up a new chapter in the field of self-excited systems, indicating the possibility of the existence of multi-frequency, almost periodic 'limit cycles' [241–254].

Among the many authors who contributed a great deal to the advances in the problems of nonlinear and parametric vibrations of systems having many degrees of freedom, one should certainly mention A.D.S. Barr [15, 16], E.H. Dowell [37, 39], J. Dugundji [40], R.M. Evan-Iwanovski [2, 3, 47], C.S. Hsu [67–76], T.C. Huang [78–90], W.O. Kononienko [101], L.I. Kuzniecow [105], E. Mettler [121, 123], Y.A. Mitropolsky [126, 128], B.J. Mosiejenkov [131, 132], D.E. Newland [137], Y. Nishikawa [139, 140], K. Piszczek [149, 150], Rene van Dooren [166–171], A.M. Samojlenko [185, 186], G. Schmidt [191–196], P.R. Sethna [198–202], M. Urabe [260, 261], K.G. Valeev [262], Nayfeh [135], T. Yamamoto [274–283], and many others.

The recently published book by Nayfeh and Mook [B34], which presents a summary of a large amount of up-to-date research in the field of nonlinear vibrations, deserves particular attention. In the study of free, forced and parametric vibrations of systems with finite and infinite degrees of freedom, the emphasis is laid on the particularly striking new phenomenon known as 'internal resonance' which occurs when two or more natural frequencies of the system are nearly commensurable. Utilization of the multi-scale perturbation technique to seek a first-order approximate solution provides a surprisingly simple tool for the investigation of these complicated phenomena, and gives an insight into the mechanism of the significant qualitative changes in the response of the systems due to 'internal resonance' effects.

This book attempts to focus the reader's attention on all these new phenomena which are inherent to multiple-degree-of-freedom systems, apart from internal resonance effects. To this end, methods other than the first-order approximation perturbation technique must be employed. Hence, the question of which method should be used to solve a given problem is an essential point of the study.

The text of the book is divided into two volumes. The purpose of Volume I is to present a concise survey of the approximate analytical vibrations in single-degree-of-freedom systems. A new procedure called 'combined Ritz-averaging' or 'combined harmonic-balance-averaging' is also outlined here.

The first chapter is devoted to the mathematical modeling of the vibratory systems to be considered. The basic mathematical model consists of a set of n coupled ordinary differential equations of the second order. The model results from lumped parameter mechanical systems (discrete mechanical model), as well as from the distributed parameter mechanical systems (continuous mechanical model). In the latter case, the original partial differential equations of motion are reduced to a finite number of simultaneous ordinary differential equations by assuming a solution in the form of a truncated series in normal modes, and applying Galerkin's procedure.

Chapter 2 presents fundamental concepts of geometric theory and stability in motion. The stability in the sense of Liapunov, orbital stability, local and global stability and the concepts of the domain attraction of a given solution are the points of major interest. The concepts are first introduced by intuitive arguments based on the application of the phase-plane method and are then defined more rigorously on the basis of variational equations in n dimensional state space. An analysis of various variables, in terms of which the stability is defined and investigated, provides an essential relation between the theory and further applications to be carried out by the approximate analytical methods.

Chapter 3 gives a concise survey of the principal approximate analytical methods for n-degree-of-freedom systems, and aims to reveal practical hints on how to use them, with an indication of their advantages and shortcomings. It is shown that numerous procedures in the recent literature can be placed into two principal categories: (I) methods based on the 'small parameter' idea, which include procedures known as the averaging method, slowly varying coefficient method, the asymptotic method, and a number of other pertubation techniques; (II) methods requiring an initial assumption of solution as a function of time and then minimizing the residuals of the equations of motion.

The averaging and asymptotic methods are then considered in some detail, the other category being represented by the Ritz, Galerkin, and harmonic balance methods. Each method is accompanied by an outline of a procedure for the analysis of the local stability of the approximate steady-state solution under consideration. The new method, which aims to combine the advantageous features of the methods of both categories, and which is called the combined Ritz-averaging method, is also outlined.

In Chapter 4, problems of the vibration of single-degree-of-freedom systems are presented. Attention is focused on a comparison of the results obtained by the various methods described in Chapter 3, and their verification by computer simulation.

In Chapter 5, the periodic, but nonharmonic oscillations of the single-degree-of-freedom system subjected to harmonic and parametric excitation are considered. The general term 'secondary resonances' replaces the more specific terms 'sub', 'ultra', or 'sub-ultra' harmonic resonance. For this class of solutions the significant discrepancies between the results of the Ritz and averaging methods are revealed for the first time. To illustrate this problem, all types of secondary resonances of the Duffing-type system are calculated by means of the two methods and compared with the computer simulation results.

The problem of nonharmonic oscillations of the system with parametric excitation is discussed separately in Section 5.2.

Acknowledgements

The author would like to thank Academic Press Ltd., London, for their permission to include in this publication the figures first published in the *Journal of Sound and Vibration* in the articles mentioned in the list of References at the end of the book, Nos. [14d], [229], and [230], and also Pergamon Press, Oxford, for their permission to include figures first published in the *International Journal of Non-Linear Mechanics* in the articles mentioned in the References, Nos. [215], [216], [225], [232], [233], and [234].

Table of contents

1

The Mathematical Model

In the study of nonlinear vibrating systems, it is common to describe the motion of the system in terms of a finite set of independent coordinates. The corresponding mathematical formulation consists of a set of simultaneous ordinary differential equations. In the process of modeling a real physical system in terms of a system described by a relatively small number of coordinates, when the model is often referred to as discrete, some simplifying assumptions are necessary. The adequacy of a discrete model and its relative simplicity represent contradictory requirements. It should be understood that the discretization process is not unique, and depends not only on the nature of the physical system but also on the nature of the problem to be solved.

Generally, in every analytical discretization procedure two major decisions are involved: the selection of the discretization process itself and the selection of the number of coordinates. With respect to the first decision, two basic discretization processes are recognized. To illustrate the two options, we consider a uniform, thin, elastic rod, as shown in Figure 1.1a. It is obvious that an exact description of a deflection of the rod requires a function of spatial variables $w(x, t)$, i.e., an infinite number of coordinates. To describe the motion of the rod approximately by means of a finite number of coordinates $q_1, q_2, ..., q_n$, we can either reduce the rod to a lumped parameter system, by replacing the rod with a system consisting of n mass points and massless elastic elements, or represent the function $w(x, t)$ by the truncated series:

$$w(x, t) = \sum_{j=1}^{n} \psi_j(x)\, q_j(t), \tag{1.1a}$$