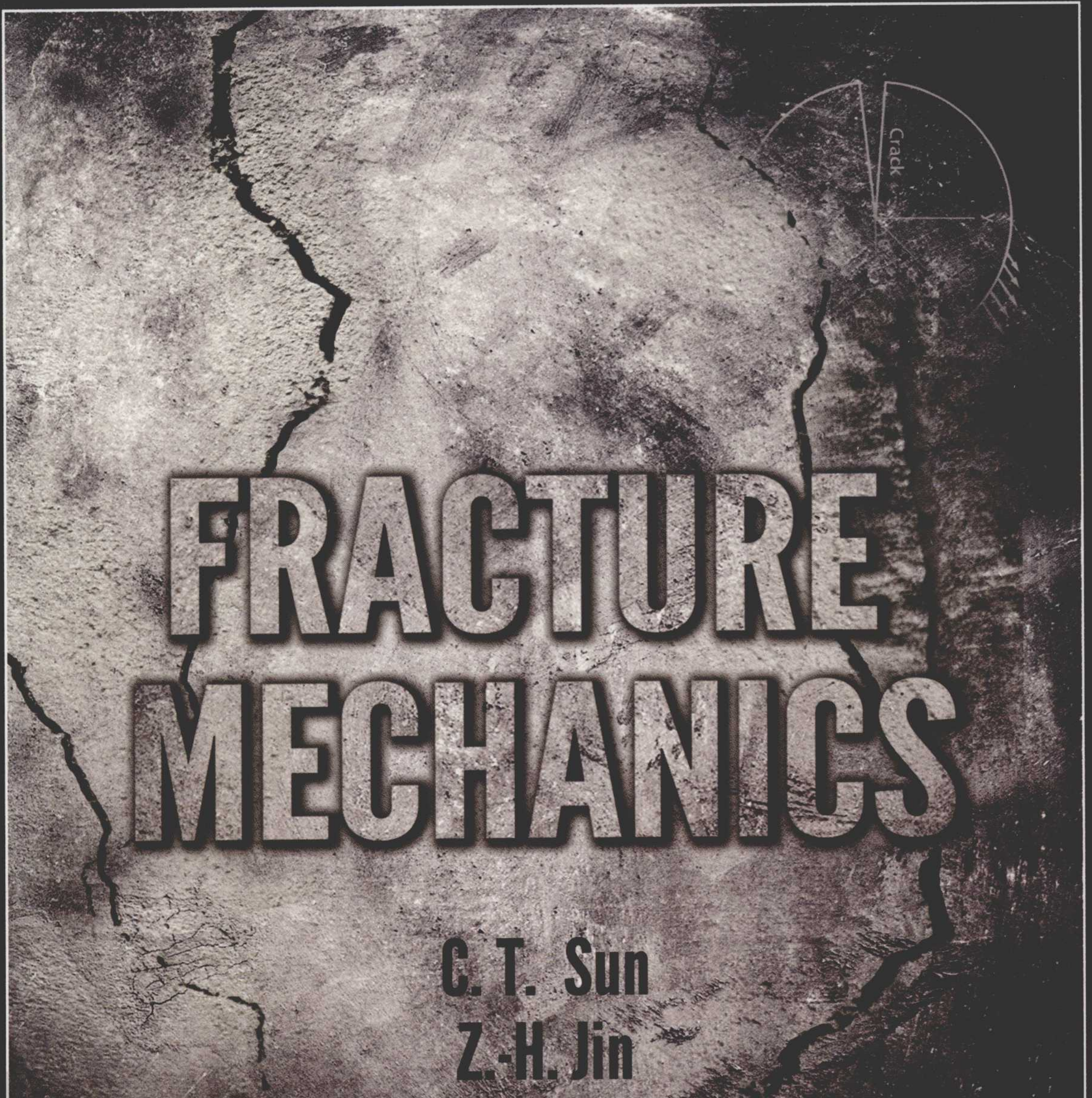

FRACTURE MECHANICS

C. T. Sun

Z.-H. Jin

Fracture Mechanics

C. T. Sun
School of Aeronautics and Astronautics
Purdue University
West Lafayette, Indiana

Z.-H. Jin
Department of Mechanical Engineering
The University of Maine
Orono, Maine

ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Academic Press is an imprint of Elsevier

Academic Press is an imprint of Elsevier
225 Wyman Street, Waltham, MA 02451, USA
The Boulevard, Langford Lane, Kidlington, Oxford, OX5 1GB, UK

© 2012 Elsevier Inc. All rights reserved.

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or any information storage and retrieval system, without permission in writing from the publisher. Details on how to seek permission, further information about the Publisher's permissions policies and our arrangements with organizations such as the Copyright Clearance Center and the Copyright Licensing Agency, can be found at our website: *www.elsevier.com/permissions*.

This book and the individual contributions contained in it are protected under copyright by the Publisher (other than as may be noted herein).

Notices

Knowledge and best practice in this field are constantly changing. As new research and experience broaden our understanding, changes in research methods, professional practices, or medical treatment may become necessary.

Practitioners and researchers must always rely on their own experience and knowledge in evaluating and using any information, methods, compounds, or experiments described herein. In using such information or methods they should be mindful of their own safety and the safety of others, including parties for whom they have a professional responsibility.

To the fullest extent of the law, neither the Publisher nor the authors, contributors, or editors, assume any liability for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions, or ideas contained in the material herein.

Library of Congress Cataloging-in-Publication Data
Application submitted.

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

ISBN: 978-0-12-385001-0

For information on all Academic Press publications
visit our Web site at *www.elsevierdirect.com*

Printed in the United States
11 12 13 14 15 10 9 8 7 6 5 4 3 2 1

Working together to grow
libraries in developing countries

www.elsevier.com | www.bookaid.org | www.sabre.org

ELSEVIER BOOK AID International Sabre Foundation

Fracture Mechanics

To my wife, Iris
and my children, Edna, Clifford, and Leslie
C. T. Sun

To my wife, Zhen
Z.-H. Jin

Preface

Fracture mechanics is now considered a mature subject and has become an important course in engineering curricula at many universities. It has also become a useful analysis and design tool to mechanical, structural, and material engineers. Fracture mechanics, especially linear elastic fracture mechanics (LEFM), is a unique field in that its fundamental framework resides in the inverse square root type singular stress field ahead of a crack. Almost all the fracture properties of a solid are characterized using a couple of parameters extracted from these near-tip stress and displacement fields. In view of this unique feature of fracture mechanics, we feel that it is essential for the reader to fully grasp the mathematical details and their representation of the associated physics in these mathematical expressions because the rationale and limitations of this seemingly simple approach are embodied in the singular stress field.

There are already more than a dozen books dealing with fracture mechanics that may be used as textbooks for teaching purposes. With different emphases, these books appeal to different readers and students from different backgrounds. This book is based on the lecture notes that have been used at the School of Aeronautics and Astronautics, Purdue University, for more than 30 years. It is intended as a book for graduate students in aeronautical, civil, mechanical, and materials engineering who are interested in picking up an in-depth understanding of how to utilize fracture mechanics for research, teaching, and engineering applications. As a textbook, our goal is to make it mathematically readable to first-year graduate students with a decent elasticity background. To achieve this goal, almost all mathematical derivations are clearly presented and suitable for classroom teaching and for self-study as well.

In selecting and presenting the contents for this book, we use the aforementioned rationale as a guide. In Chapter 2, the Griffith theory of fracture and the surface energy concept are introduced. Chapter 3 presents the elastic stress and displacement fields near the crack tip and introduces Irwin's stress intensity factor concept. The chapter describes detailed derivations of the stress fields and stress intensity factor K using the complex potential method and Williams' asymptotic expansion approach. Finally, the chapter introduces the fracture criterion based on the stress intensity factor (K-criterion) and discusses the K-dominance concept to make the reader aware of the limitation of the K-criterion.

Chapter 4 is totally devoted to energy release rate in conjunction with the path-independent J-integral. The energy release rate concept is first introduced, and the relationship between the energy release rate G and stress intensity factor ($G - K$ relation) is established followed by the fracture criterion based on the energy release rate (G-criterion). The J-integral is widely accepted because its value is equal to the energy release rate and it can be calculated numerically with stress and displacement fields away from the singular stress at the crack tip. Another simple, yet efficient, crack-closure method has been shown to be quite accurate in evaluating energy release rate. Therefore, a couple of finite element-based numerical methods for calculation of energy release rate and the stress-intensity factor using the crack-closure method are included in this chapter.

In most fracture mechanics books, the near-tip stress field is presented in plane elasticity for Mode I and Mode II loadings and in generalized plane strain for Mode III. In reality, none of these 2-D states exists. For instance, a thin plate containing a center crack is usually treated as a 2-D plane stress problem. In fact, the plane stress assumption fails because of the presence of high stress gradients near the crack tip and a state-of-plane strain actually exists along most part of the crack front. The knowledge of the 3-D nature of all through-the-thickness cracks is important in LEFM. In Chapter 4, a section is devoted to the the 3-D effect on the variation of stress intensity along the crack front.

Under static Mode I loading, experimental results indicate that the direction of crack extension is self-similar. As a result, in determining Mode I fracture toughness of a solid, the crack extension direction is not an issue. The situation is not as clear if the body is subjected to combined loads or dynamic loads. Of course, if the body is an anisotropic solid such as a fiberous composite, the answer to the question of cracking direction is not as simple and is not readily available in general. In view of this constraint, we only consider isotropic brittle solids in Chapter 5. The focus is on the prediction of crack extension direction. From a learning point of view, it is interesting to follow a number of different paths of thinking taken by some earlier researchers in the effort to predict the cracking direction.

Chapters 6 and 7 present the result of the effort in extending the LEFM to treat fracture in elastic-plastic solids. In Chapter 6, plastic zones near the crack tip for the three fracture modes are analyzed. Several popular and simple methods for estimating the crack tip plastic zone size are covered. The initial effort in taking plasticity into account in fractures was proposed by Irwin who suggested using an effective crack length to account for the effect of plasticity. Later, the idea was extended to modeling the so-called *R*-curve during stable crack growth. Another approach that uses the *J*-integral derived based on deformation plasticity theory to model the crack tip stress and strain fields (the HRR field) also has many followers. In addition to Irwin's adjusted crack length and the *J*-integral approach, crack growth modeled by crack tip opening displacement (CTOD or CTOA) is also discussed in Chapter 7.

Interfaces between dissimilar solids are common in modern materials and structures. Interfaces are often the weak link of materials and structures and are the likely sites for crack initiation and propagation. Interfacial cracks have many unique physical behaviors that are not found in homogeneous solids. However, surprisingly, the development of fracture mechanics for interfacial cracks has followed exactly the same path as LEFM. In other words, fracture mechanics for interfacial cracks is all centered on the crack tip stress field. The only difference is in the violently oscillatory behavior of the crack tip stress field of interfacial cracks. Chapter 8 presents a thorough derivation of the crack tip stress and displacement fields. Attention is also focused on the significance of stress oscillation at the crack tip and the nonconvergent nature of the energy release rates of the individual fracture modes.

The cohesive zone model (CZM) has become a popular finite element-based tool for modeling fracture in solids. CZM is often considered by some as a more realistic form of fracture mechanics because it does not employ the idealized singular stresses. Although there are fundamental differences between the two concepts, the purposes of the two are the same. Therefore, it is reasonable to include CZM in this book. In Chapter 9 we make an effort to present the basic formulation of CZM, especially the cohesive traction law. Instead of covering examples of applications of the cohesive zone model, we place greater emphasis on the logic in the formulation of CZM.

Chapter 10 contains brief and condensed presentations of three additional topics, namely, anisotropic solids, nonhomogeneous solids, and dynamic fracture. The reason for including these three topics in this textbook is, perhaps, just for the sake of completeness. For each topic, the coverage is quite brief and with a limited scope and does not warrant a full chapter.

C. T. Sun
Z.-H. Jin

About the Authors

C. T. Sun received his undergraduate education at National Taiwan University. He obtained his M.S. in 1965 and Ph.D. in 1967 from Northwestern University. In 1968 he joined Purdue University, where he is presently Neil A. Armstrong Distinguished Professor in the School of Aeronautics and Astronautics. He has been engaged in composites research for more than forty years. In addition to his work in composites, Professor Sun has published extensively in the areas of fracture mechanics, smart materials, and nanomechanics. He has authored a textbook on aircraft structures published in 1998 with the second edition published in 2006.

Z.-H. Jin is an Associate Professor in the Department of Mechanical Engineering at the University of Maine. He obtained his Ph.D. in Engineering Mechanics from Tsinghua University in 1988. His research areas include fracture mechanics, thermal stresses, mechanical behavior of materials, and geodynamics. He has published more than 70 refereed journal papers and three book chapters.

Fracture Mechanics

Contents

CHAPTER 4 **Energy Release Rate** **77**

CHAPTER

Introduction

1

1.1 FAILURE OF SOLIDS

Failure of solids and structures can take various forms. A structure may fail without breaking the material, such as in elastic buckling. However, failure of the material in a structure surely will lead to failure of the structure. Two general forms of failure in solids are excessive permanent (plastic) deformation and breakage. Plasticity can be viewed as an extension of elasticity for decribing the mechanical behavior of solids beyond yielding. The theory of plasticity has been studied for more than a century and has long been employed for structural designs. On the other hand, the latter form of failure is usually regarded as the strength of a solid, implying the total loss of load-bearing capability of the solid. For brittle solids, this form of failure often causes the body under load to break into two or more separated parts.

Unlike plasticity, the prediction of the strength of solid materials was all based on phenomenological approaches before the inception of fracture mechanics. Many phenomenological failure criteria in terms of stress or strain have been proposed and calibrated against experimental results. In the commonly used failure criteria, such as the maximum principal stress or strain criterion, a failure envelope in the stress or strain space is constructed based on limited experimental strength data. Failure is assumed to occur when the maximum normal stress at a point in the material exceeds the strength envelope, that is,

$$\sigma_1 \geq \sigma_f$$

where σ_1 (> 0) is a principal stress and σ_f is the tensile strength of the solid. The failure envelope has also been modified to distinguish the difference between tensile and compressive strengths and to account for the effects of stress interactions.

In general, the classical phenomenological failure theories predict failure of engineering materials and structures with reasonable accuracy in applications where the stress field is relatively uniform. These theories are often unreliable in the presence of high-stress gradients resulting from cutouts. Moreover, there were many premature structural failures at stresses that were well below the critical values specified in the classical failure theories.

Fracture Mechanics