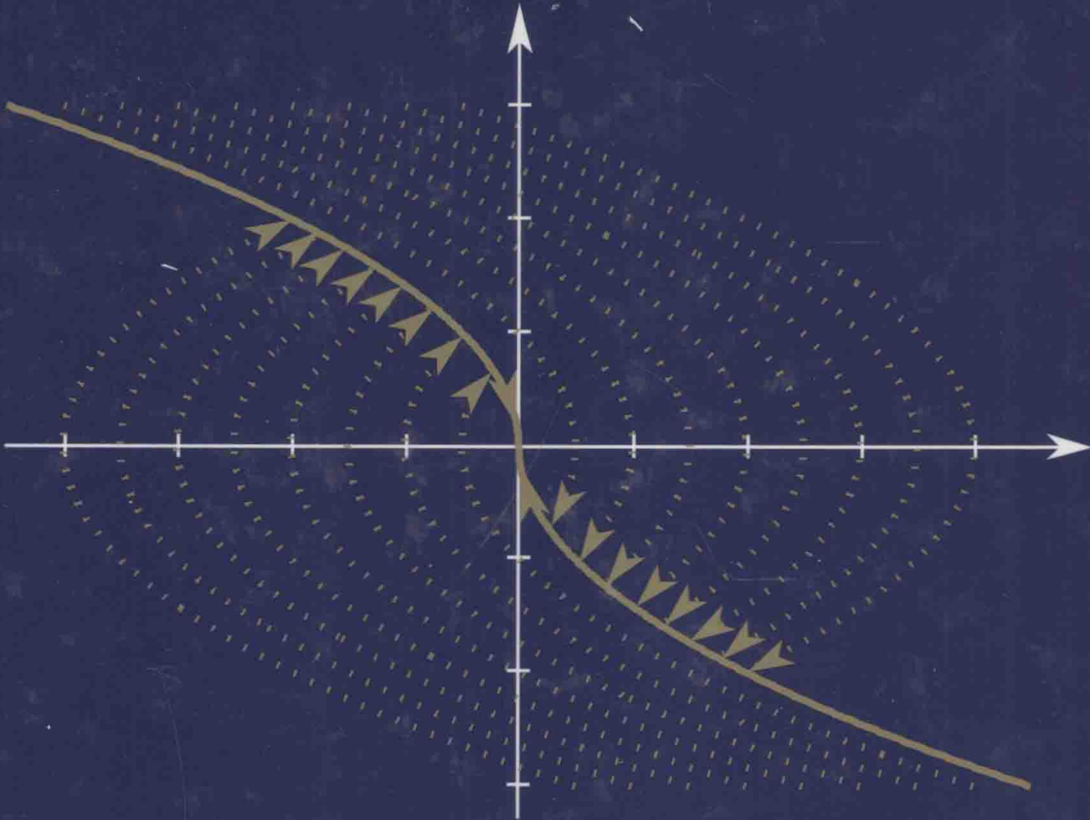
Primer on Optimal Control Theory



Jason L. Speyer
David H. Jacobson

Advances in Design and Control

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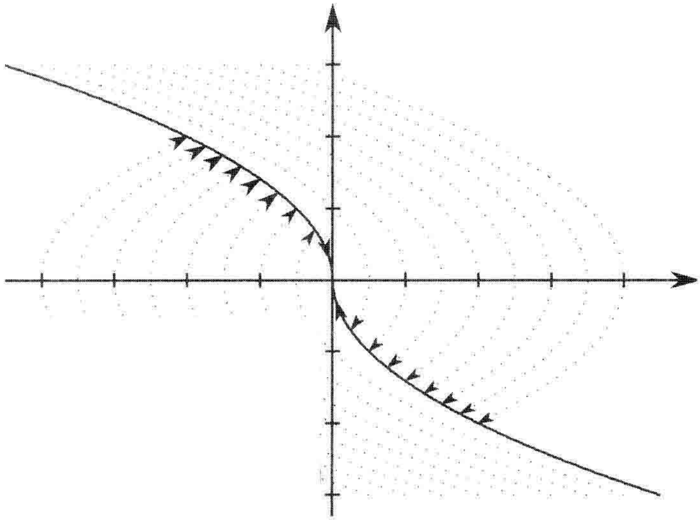

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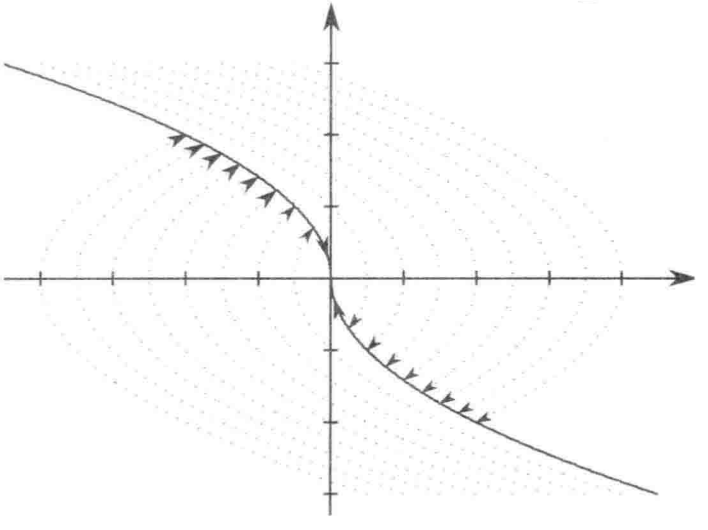

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To Barbara, a constant source of love and inspiration.
To my children, Gil, Gavriel, Rakhel, and Joseph,
for giving me so much joy and love.

For Celia, Greta, Jonah, Levi, Miles, Thea,
with love from Oupa!

Preface

This book began when David Jacobson wrote the first draft of Chapters 1, 3, and 4 and Jason Speyer wrote Chapters 2, 5, and 6. Since then the book has constantly evolved by modification of those chapters as we interacted with colleagues and students. We owe much to them for this polished version. The objective of the book is to make optimal control theory accessible to a large class of engineers and scientists who are not mathematicians, although they have a basic mathematical background, but who need to understand and want to appreciate the sophisticated material associated with optimal control theory. Therefore, the material is presented using elementary mathematics, which is sufficient to treat and understand in a rigorous way the issues underlying the limited class of control problems in this text. Furthermore, although many topics that build on this foundation are covered briefly, such as inequality constraints, the singular control problem, and advanced numerical methods, the foundation laid here should be adequate for reading the rich literature on these subjects.

We would like to thank our many students whose input over the years has been incorporated into this final draft. Our colleagues also have been very influential in the approach we have taken. In particular, we have spent many hours discussing the concepts of optimal control theory with Professor David Hull. Special thanks are extended to Professor David Chichka, who contributed some interesting examples and numerical methods, and Professor Moshe Idan, whose careful and critical reading of the manuscript has led to a much-improved final draft. Finally, the first author must express his gratitude to Professor Bryson, a pioneer in the development of the theory, numerical methods, and application of optimal control theory as well as a teacher, mentor, and dear friend.

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Introduction

The operation of many physical processes can be enhanced if more efficient operation can be determined. Such systems as aircraft, chemical processes, and economies have at the disposal of an operator certain controls which can be modulated to enhance some desired property of the system. For example, in commercial aviation, the best fuel usage at cruise is an important consideration in an airline's profitability. Full employment and growth of the gross domestic product are measures of economic system performance; these may be enhanced by proper modulation of such controls as the change in discount rate determined by the Federal Reserve Board or changes in the tax codes devised by Congress.

The essential features of such systems as addressed here are dynamic systems, available controls, measures of system performance, and constraints under which a system must operate. Models of the dynamic system are described by a set of first-order coupled nonlinear differential equations representing the propagation of the state variables as a function of the independent variable, say, time. The state vector may be composed of position, velocity, and acceleration. This motion is influenced by the inclusion of a control vector. For example, the throttle setting and the aerodynamic surfaces influence the motion of the aircraft. The performance criterion which establishes the effectiveness of the control process on the dynamical system can take many forms. For an aircraft, desired performance might be efficient fuel cruise (fuel per range), endurance (fuel per time), or time to a given altitude. The performance criterion is to be optimized subject to the constraints imposed by the system dynamics and other constraints. An important class of constraints are those imposed at the termination of the path. For example, the path of an aircraft may terminate in minimum time at a given altitude and velocity. Furthermore, path constraints that are functions of the controls or the states or are functions of both the state and control vectors may be imposed. Force constraints or maximum-altitude constraints may be imposed for practical implementation.

In this chapter, a simple dynamic example is given to illustrate some of the concepts that are described in later chapters. These concepts as well as the optimization concepts for the following chapters are described using elementary mathematical ideas. The objective is to develop a mathematical structure which can be justified rigorously using elementary concepts. If more complex or sophisticated ideas are required, the reader will be directed to appropriate references. Therefore, the treatment here is not the most general but does cover a large class of optimization problems of practical concern.

1.1 Control Example

A control example establishes the notion of control and how it can be manipulated to satisfy given goals. Consider the forced harmonic oscillator described as

$$\ddot{x} + x = u, \quad x(0),\ \dot{x}(0)\ \text{given}, \tag{1.1}$$

where x is the position. The overdot denotes time differentiation; that is, $\dot{x}$ is dx/dt. This second-order linear differential equation can be rewritten as two first-order differential equations by identifying $x_1 = x$ and $x_2 = \dot{x}$. Then

$$\dot{x}_1 = x_2, \quad x_1(0) \text{ given}, \tag{1.2}$$

$$\dot{x}_2 = -x_1 + u, \quad x_2(0) \text{ given}, \tag{1.3}$$

or

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u. \tag{1.4}$$

Suppose it is desirable to find a control which drives x_1 and x_2 to the origin from arbitrary initial conditions. Since system (1.4) is controllable (general comments on this issue can be found in [8]), there are many ways that this system can be driven to the origin. For example, suppose the control is proportional to the velocity such as $u = -Kx_2$, $K > 0$, is a constant. Then, asymptotically the position and velocity converge to zero as $t \to \infty$.

Note that the system converges for any positive value of K. It might logically be asked if there is a *best* value of K. This in turn requires some definition for "best." There is a large number of possible criteria. Some common objectives are to minimize the time needed to reach the desired state or to minimize the effort it takes. A criterion that allows the engineer to balance the amount of error against the effort expended is often useful. One particular formulation of this trade-off is the *quadratic performance index*, specialized here to

$$J_1 = \lim_{t_f \to \infty} \int_0^{t_f} (a_1 x_1^2 + a_2 x_2^2 + u^2) dt, \tag{1.5}$$

where $a_1 > 0$ and $a_2 > 0$, and $u = -Kx_2$ is substituted into the performance criterion. The constant parameter K is to be determined such that the cost criterion is minimized subject to the functional form of Equation (1.4).

We will not solve this problem here. In Chapter 2, the parameter minimization problem is introduced to develop some of the basic concepts that are used in the solution. However, a point to note is that the control u does not have to be chosen a priori, but the best functional form will be produced by the optimization process. That is, the process will (usually) produce a control that is expressed as a function of the state of the system rather than an explicit function of time. This is especially true for the quadratic performance index subject to a linear dynamical system (see Chapters 5 and 6).

Other performance measures are of interest. For example, minimum time has been mentioned for where the desired final state was the origin. For this problem to make sense, the control must be limited in some way; otherwise, infinite effort would be expended and the origin reached in zero time. In the quadratic performance index in (1.5), the limitation came from penalizing the use of control (the term u^2 inside the integral). Another possibility is to explicitly bound the control. This could represent some physical limit, such as a maximum throttle setting or limits to steering.

Here, for illustration, the control variable is bounded as

$$|u| \leq 1. \tag{1.6}$$

In later chapters it is shown that the best solution often lies on its bounds. To produce some notion of the motion of the state variables (x_1, x_2) over time, note that Equations (1.2) and (1.3) can be combined by eliminating time as

$$\frac{dx_1/dt}{dx_2/dt} = \frac{x_2}{(-x_1 + u)} \quad \Rightarrow \quad (-x_1 + u)dx_1 = x_2 dx_2. \tag{1.7}$$

Assuming u is a constant, both sides can be integrated to get

$$(x_1 - u)^2 + x_2^2 = R^2, \tag{1.8}$$

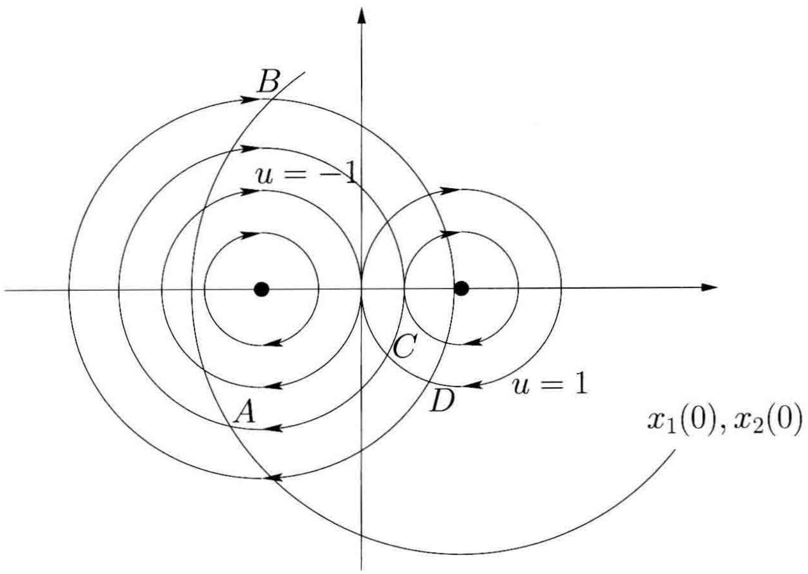

Figure 1.1: Control-constrained optimization example.

which translates to a series of concentric circles for any specific value of the control. For $u = 1$ and $u = -1$, the series of concentric circles are as shown in Figure 1.1 There are many possible paths that drive the initial states $(x_1(0), x_2(0))$ to the origin. Starting with $u = 1$ at some arbitrary $(x_1(0), x_2(0))$, the path proceeds to point A or B. From A or B the control changes, $u = -1$ until point C or D is intercepted. From these points using $u = +1$, the origin is obtained. Neither of these paths starting from the initial conditions is a minimum time path, although starting from point B, the resulting paths are minimum time. The methodology for determining the optimal time paths is given in Chapter 4.

1.2 General Optimal Control Problem

The general form of the optimal control problems we consider begins with a first-order, likely nonlinear, dynamical system of equations as

$$\dot{x} = f(x, u, t), \quad x(t_0) = x_0, \tag{1.9}$$

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