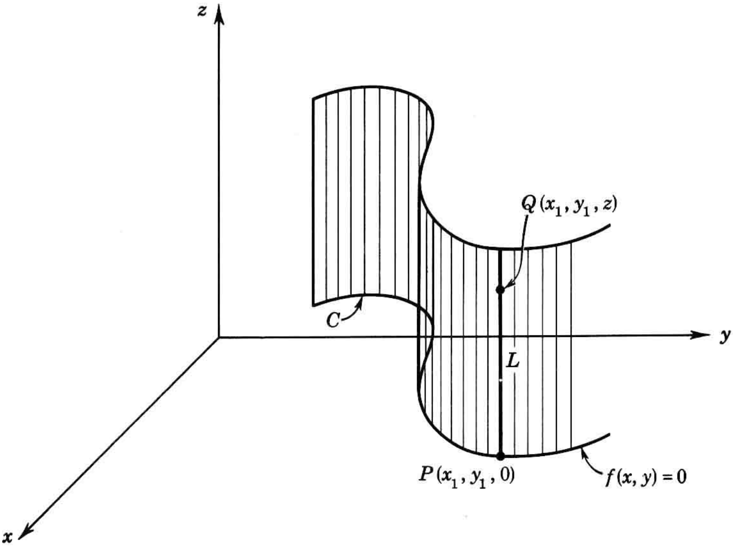

INTRODUCTION TO Mathematics WITH APPLICATIONS

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INTRODUCTION TO MATHEMATICS
with Applications

TO
RUTH AND EDITH

Preface

The purpose of this textbook is to prepare students at the college freshman level for the calculus or to give those who will not go on in mathematics a fundamental understanding of mathematics and some of its applications. It is particularly suitable for students in a scientific or technical program. The choice of topics has largely been influenced by the recommendations of the Committee on the Undergraduate Program in Mathematics.

In keeping with the authors' belief that the understanding of mathematical ideas is strengthened by exhibiting models and concrete interpretations whenever possible, the book provides many applications of the mathematical concepts under study, in both text and exercises. The attempt has been made, however, to keep clear the distinctions between the mathematics, **per se,** and the applications to which it may be put. The authors feel that there is only one mathematics, not well dichotomized as "pure" and "applied," but that this one mathematics is clarified and made increasingly meaningful by frequent encounters with situations most suitably described in mathematical form. Only those situations are considered that can reasonably be expected to be within the experience of the student.

Chapter 1 deals with the often neglected relationship between mathematics and its applications. Problems of measurement and dimensionality are used to exhibit some of the problems involved in this relationship. This chapter will be most useful for beginning science and engineering students; others, however, may choose to omit it.

Chapters 2 through 12 constitute a thorough course in analytic geometry featuring, in addition to the usual topics, material on linear programming, graphic solution of equations, graphic calculus, curve fitting, and vector and matrix methods in coordinate geometry.

In Chapter 9 differentiation and integration of simple polynomial functions have been included. This will meet the needs of freshman students of physics using standard physics texts that employ calculus methods.

Chapter 13 deals with the formal algebra of sets. To make the formal development more natural here, set concepts and notation are used in an informal way in the preceding chapters.

Chapter 14 deals with basic probability concepts. It includes a careful definition of sample space, an examination of some of the basic difficulties involved in deceptively easy problems, and a simple algebra of probability developed in terms of sets.

Chapter 15 on the nature of mathematics is, in a sense, a recapitulation of the relationship between mathematics and its applications. A brief analysis of axiomatic systems is included.

Some novelty is attempted in the treatment of indirect measurement in Section 1.2, of symmetry in Section 4.9, and of a converse theorem concerning the parabola in Section 5.3. Chapter 6, "Other Methods for Describing Sets of Points," utilizes a new approach for deepening the student's understanding of the nature of coordinate systems.

The book permits considerable flexibility in the choice of material for a particular course. For instance, for those who are interested in a strong coordinate geometry course (and there appears to be a growing interest in such a course), Chapters 2 through 11 will suffice, augmented perhaps by Chapters 1 and 12. Those interested in a terminal course emphasizing the nature of mathematical structure and models might choose Chapters 1, 2, 4, 6, 12, 13, 14, and 15. Other combinations may be found more suitable for particular needs.

Considerable effort has gone into the design of the graded problem sets in the belief that the student's understanding is best developed by a large number of challenging and interesting problems.

The authors thank Mrs. Paul Hausser for her careful typing of the manuscript. Grateful acknowledgment is made also to Mr. Murray Lieb for working out all of the exercises and to Professor William D. Brower for reading the entire manuscript and working out all the exercises.

H. Barkan
C. Konove

January, 1964

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CHAPTER 1

Measurement, Dimensions and Significant Figures

1.1 Introduction

Mathematics for a long time was regarded as an empirical science dealing with calculations, measurement, and space. Early mathematical investigators among the ancient Egyptians and Babylonians were motivated by practical demands in the fields of agriculture, engineering, and commerce. With the appearance of the Greeks, a new element was introduced into mathematics. That new element was abstract deductive reasoning. Although mathematics has continued to be motivated by practical human needs and interests, there has simultaneously developed in mathematics an increased emphasis on abstraction and the deductive method. This emphasis has led to the now widely accepted definition of mathematics as a set of propositions, all of which are deducible from a given set of statements—called "axioms," or "postulates"—and certain undefined terms. Geometry, as usually studied in the schools, well exemplifies this definition. In this book, however, we will emphasize the view that mathematics is a language, especially well suited to describe and reveal certain aspects of human experience that ordinary language cannot hope to do. For instance, consider the following problem:

> The manager of a certain concern is to receive a bonus of 5% of the net profit after the income tax has been deducted from the gross profit of $100,000. The income tax is known to be 20% of the net profit after the bonus has been deducted. What is the manager's bonus?

A strictly verbal analysis of this problem reveals that it cannot be solved: in order to calculate the bonus, one must first know the income tax; and in order to know the income tax, one must first know the bonus. A clear case of circularity! It is like looking up cosmiginy in the dictionary and finding that it means slizigy, and then looking up slizigy only to be told that it means cosmiginy. On the other hand, if one decided to use the language of mathematics, the problem quickly becomes solvable.

If we let x represent the bonus in dollars, the student can readily verify that the solution of the equation

$$x = 0.05[100{,}000 - 0.20(100{,}000 - x)]$$

will yield the desired information. We are not suggesting that mathematics is always the language of choice. The student on a date with a beautiful girl may find poetry more suitable.

1.2 Measurement

It has frequently been argued that the state of advancement of a science depends largely on the degree to which its fundamental entities are susceptible of precise measurement. From this point of view, physics is usually regarded as the most advanced of the sciences. Recently, many of the relatively new aspiring sciences, such as biology and psychology, have made rapid advances largely attributable to the success of their practitioners in devising appropriate and precise measurements of the fundamental objects of their investigations.

What is involved in measurement is a precisely stated series of operations that yields a number. Thus the measure of the length of an object is a number obtained by counting the number of times a meter stick can be applied to the object, and the measure of the weight of an object on a spring balance is the number to which the indicator points when it comes to rest. These definitions of particular measures, trivial by familiarity, must have taken the human race a long time to devise. One can more readily appreciate the difficulty of defining appropriate measures by considering problems of the infant science psychology in defining measures of hatred, love, fear, pain, and so on.

One of man's most remarkable faiths, confirmed by experience, is that these measured quantities, once defined, will exhibit a relationship among themselves expressible by simple mathematical laws. This faith was perhaps first clearly enunciated by Galileo (1564–1642), the founder of modern science:

> "Philosophy [today we would say science] is written in that vast book which stands forever open before our eyes. I mean the universe: but it cannot be read until we have learnt the language and

become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles, and other geometrical figures, without which means it is humanly impossible to comprehend a single word."

Today we would include any other mathematical symbols, not merely geometrical ones.

The student is undoubtedly familiar with indirect measurement in trigonometry. In order to determine the height of a tree, the measured angle of elevation at a measured distance from the base of the tree is given. The height is calculated from the given measured quantities by means of a trigonometric law. More generally, a mathematically expressed physical law permits the calculation ("indirect measurement") of certain quantities from other directly measured quantities. For example, in the study of hydraulics, it can be shown that the amount of water, Q cu ft, which has flowed out of an orifice in a certain tank is related to the elapsed time, t sec, by the following equation:

$$Q = 2.5t - t^2/800.$$

Thus, if the elapsed time is measured, one can calculate the corresponding value of Q without bothering to measure it directly.

A problem arising in the calculation of quantities based on measured quantities is that of determining the precision of the calculated quantities, when the precision of the measured quantities is known. In this presentation we will follow a long-standing tradition of mixed pessimism and optimism and give certain "rules of thumb" that are generally adequate for engineering and scientific calculations. We will assume that the error in any measured quantity will be equal to one half the size of the smallest unit marked off on the scale of the instrument. Thus if a ruler were marked off in eighths of inches, the error will be as great as $\frac{1}{2} \times \frac{1}{8}$, or $\frac{1}{16}$th in. (Many will call this assumption unnecessarily pessimistic.)

1.3 Significant Figures

In order to give the rules of thumb mentioned above, it will be first necessary to define what we mean by "significant figures." The reader should recall that a number, say 756, in the base 10 means $7(10^2) + 5(10) + 6$. Thus each digit counts the number of 1's, 10's, etc., contained in the entire number.

A digit is significant if the maximum error in the number in which it is contained is at most equal to one-half the basic unit (1's, 10's, etc.) that this particular digit counts.

In order to determine by this rule whether or not a particular digit is significant, it is necessary to know the maximum error in the measured quantity. This, in turn, usually implies knowledge of the measuring procedure and the instrument employed. Frequently one encounters measured quantities for which this information is not explicitly available. Under these conditions the following conventions are used:

1. All nonzero digits are considered significant.
2. All zero digits lying between nonzero digits are considered significant.
3. All zero digits lying to the right of nonzero digits, and also to the right of the decimal point, are considered significant.
4. All other zero digits will be considered not significant.

Thus we have:

Number	*Significant Digits*	*Number of Significant Digits*
356	3, 5, 6	3
0.0052	5, 2	2
1.0052	1, 0,0, 5, 2	5
520	5, 2	2
15.20	1, 5, 2, 0	4
14.7 ± 0.05 (0.05 is the maximum possible error)	1, 4, 7	3
14.7 ± 0.03	1, 4, 7	3
14.7 ± 0.06	1, 4	2
14.7 ± 0.005	1, 4, 7, 0*	4

1.4 EXERCISES

1. Complete the chart.

Number	*Significant Digits*	*Number of Significant Digits*
271		
190		
0.0096		
0.00002		
1.004		
27.950		

* The maximum possible error justifies the additional significant figure.

2. Complete the chart.

Number	*Significant Digits*	*Number of Significant Digits*
4.0061		
5.0600		
27,960		
1,800		
3,600.0		
0.000081		

3. Complete the chart.

Number	*Significant Digits*	*Number of Significant Digits*
6.10 ± 0.02		
6.10 ± 0.2		
6.10 ± 2		

4. Complete the chart.

Number	*Significant Digits*	*Number of Significant Digits*
$36{,}000 \pm 10$		
$36{,}000 \pm 1$		
$36{,}000 \pm 1{,}000$		

In rounding off a number to a given number of significant figures, we choose that number with the required number of significant figures that is closest to the given number. When two numbers possessing the required number of significant figures are equally close to the given number, it is customary to round off to the one ending in an *even* digit.

In Exercises 5–7 complete the chart.

5. Round off numbers as indicated.

Number	*5 Significant Figures*	*3 Significant Figures*	*2 Significant Figures*
5.333333	5.3333	5.33	5.3
50.6459	50.646	50.6	51.
60.0045	60.004		
150.546			
0.0068950			