

FIRST WORKSHOP ON GRAND UNIFICATION

University of New Hampshire, April 1980

C L I N E E L L I S F R A
M P T O N G E O R G I G L
A S H O W G O L D M A N L
A N G A C K E R M A R S H
A K M O H A P A T R A P A
T I R A M O N D R E I N E
S R U E G G R U J U L A S
A L A M S E C C O S L A N
S K Y S T E I G M A N S T
E I N B E R G S U L A K T
U R N E R W A L I W E I N
B E R G W I L S O N W I N
N W I T T E N Y I L D I Z

Editors: Paul H. Frampton,
Sheldon L. Glashow,
Asim Yildiz.

LIE GROUPS: HISTORY, FRONTIERS
AND APPLICATIONS
VOLUME XI

FIRST WORKSHOP ON GRAND UNIFICATION

New England Center
University of New Hampshire
April 10-12, 1980

Editors
Paul H. Frampton
Sheldon L. Glashow
Asim Yildiz

MATH SCI PRESS
1980

Copyright © 1980 by Robert Hermann
All rights reserved

Library of Congress Catalog Card Number:

ISBN: 0-915692-31-7

Library of Congress Cataloging in Publication Data

Workshop on Grand Unification, 1st, University of
New Hampshire, 1980.

First workshop on grand unification.

(Lie groups ; v. 11)

Organized by P. Frampton and others.

Includes bibliographical references.

1. Nuclear reactions--Congresses. 2. Unified
field theories--Congresses. I. Frampton, Paul H.
II. Glashow, Sheldon L. III. Yildiz, Asim.

IV. Title.

QC793.9.W66 1980

539.7

80-25294

ISBN 0-915692-31-7

MATH SCI PRESS

53 JORDAN ROAD

BROOKLINE, MASSACHUSETTS 02146

Printed in the United States of America

LIE GROUPS: HISTORY, FRONTIERS AND APPLICATIONS

1. Sophus Lie's 1880 Transformation Group Paper, Translation by M. Ackerman, Comments by R. Hermann
2. Ricci and Levi-Civita's Tensor Analysis Paper, Translation and Comments by R. Hermann
3. Sophus Lie's 1884 Differential Invariants Paper, Translation by M. Ackerman, Comments by R. Hermann
4. Smooth Compactification of Locally Symmetric Varieties, by A. Ash, D. Mumford, M. Rapoport and Y. Tai
5. Symplectic Geometry and Fourier Analysis, by N. Wallach
6. The 1976 Ames Research Center (NASA) Conference on: The Geometric Theory of Non-Linear Waves.
7. The 1976 Ames Research Center (NASA) Conference on Geometric Control Theory.
8. Hilbert's Invariant Theory Papers, Translation by M. Ackerman, Comments by R. Hermann
9. Development of Mathematics in the 19th Century, by Felix Klein, Translated by M. Ackerman, Appendix "Kleinian Mathematics from an Advanced Standpoint," by R. Hermann
10. Quantum Statistical Mechanics and Lie Group Harmonic Analysis, Part A, by N. Hurt and R. Hermann
11. First Workshop on Grand Unification by P. Frampton, S. Glashow and A. Yildiz

EDITORS' INTRODUCTION

This workshop held at the New England Center provided a timely opportunity for over 100 participants to gather in a unique environment and discuss the present status of the unification of strong and electroweak forces. One reason for the timeliness was perhaps that experiments of the seventies had already lent confirmation to the separate theories of strong and of electroweak forces, so that for the eighties it now seems especially compelling to attempt the grand unification of these two forces. Also, the planned experiments to search for proton decay and the new experiments which are suggestive, though not yet conclusive, of non-zero neutrino rest masses add further stimulus to the theory. Thus, the workshop provided an ideal forum for exchange of ideas amongst active physicists.

The presentations at the workshop covered the present status of both theory and experiment with a strong interplay. Also, there were presentations from the discipline of astrophysics which is becoming very intertwined with that of high-energy physics especially when in the latter one is addressing energies and temperatures that were extant only in the first nanosecond of the universe.

On experiment, we heard a comprehensive coverage of the four United States Proton decay experiments. The Brookhaven-Irvine-Michigan experiment in the Morton Salt Mine at Fairport Harbor, Ohio was discussed by LARRY SULAK, while DAVID WINN talked on the Harvard-Purdue-Wisconsin effort in the Silver King Mine, Utah. MARVIN MARSHAK and RICHARD STEINBERG described respectively the Soudan Mine, Minnesota, and the Homestake Mine, South Dakota, experiments. The first three experiments are in construction while the Homestake Mine was able to quote a new lower limit on the proton lifetime of approximately 8×10^{30} years.

Another baryon-number violating effect predicted within some theoretical frameworks is neutron-antineutron mixing and RICHARD WILSON explained his ideas on how to detect this effect.

On the question of neutrino masses and oscillations, the experiment at the Savannah River reactor, South Carolina, was discussed by FRED REINES. [Due to illness, Reines' did not present verbally his written contribution, included in these proceedings; in his place, we heard short contributions from JOHN LO SECCO, PIERRE RAMOND, and MAURICE GOLDBABER]. A survey of related neutrino experiments was given by DAVID CLINE.

The general phenomenological picture was painted by SHELDON GLASHOW who emphasized the increasing importance of passive-type high-energy experiments, as opposed to accelerator experiments; also, the key predictions of the

various models were summarized.

The subject of grand unification overlaps naturally with cosmology especially the big-bang model for the universe. GARY STEIGMAN described work on neutrinos in cosmology and MICHAEL TURNER outlined the work of the Chicago group on the baryon asymmetry of the universe. The avoidance of superheavy monopoles in the early universe was explained by PAUL LANGACKER. Limits on heavy unstable particles were imposed by PAUL FRAMPTON.

The nature of symmetry breaking is a central issue in grand unification schemes. For unitary groups this was treated by HENRI RUEGG while the intricacies of exceptional groups were covered in FEZA GURSEY's lecture. Further aspects of symmetry breaking were presented in turn by RICHARD SLANSKY, LEONARD SUSSKIND, HOWARD GEORGI, and TERRY GOLDMAN.

One shortcoming of the simplest grand unification schemes is that they include only one family of quarks and leptons. In a larger symmetry group, the multiple families may receive some more justification. An example based on $SU(7)$ was given by ASIM YILDIZ. An additional related contribution was made by KAMESHWAR WALLI. JOHN ELLIS, appealing to extended supergravity, advocated an $SU(8)$ structure.

The theory of massive neutrinos is of great interest because of the recent experimental indications of possible neutrino oscillations. The concept of a Majorana mass was explicated by EDWARD WITTEN while the cosmological ramifications were dramatized by ALVARO DE RÚJULA. PIERRE RAMOND and RABINDRA MOHAPATRA explained how neutrino masses are accommodated in grand unification models.

The most prevalent unification schemes are based on a picture of color confinement. An alternative model with non-confined color was the basis of the talk by JOGESHI PATI, whose theory leads to predictions for proton decay which differ in detail from those of the conventional theory.

The expected properties of baryon and lepton nonconserving processes were outlined by STEVEN WEINBERG. His analysis is based on general considerations of the structure of effective field theories and the strong and electroweak gauge symmetries. He also explained his general formalism for deriving effective Lagrangians by integrating out superheavy particles, and discussed the implications of the survival of a cosmic baryon number.

Taken all together, we believe the presentations at the workshop gave a faithful and comprehensive representation of the state of the art in the physics of unification of strong and electroweak forces. There were, of course, many informal interchanges between the participants, in addition to the items contained in this book.

We would like to thank the participants whose attendance contributed so much to the success of the workshop.

Last but not least, we must thank Mrs. JOYCE CASH and her staff for tireless efforts both during, and for several months preceding, the workshop.

These efforts, together with the cooperation of the University of New Hampshire and of the staff of the New England Center, were essential to the smooth running of the workshop.

May 1980

P. H. Frampton
S. L. Glashow
A. Yildiz

TABLE OF CONTENTS

EDITOR'S' INTRODUCTION	vii
<i>P.H. Frampton, S.L. Glashow, and A. Yildiz</i>	
WELCOME	1
<i>G. Haaland</i>	
THE NEW FRONTIER	3
<i>S.L. Glashow</i>	
GRAND UNIFIED THEORIES WITHOUT SUPERHEAVY MAGNETIC MONOPOLES	9
<i>P. Langaeker</i>	
SYMMETRY BREAKING PATTERNS FOR UNITARY AND ORTHOGONAL GROUPS	23
<i>H. Ruegg</i>	
THE LIMITS ON UNSTABLE HEAVY PARTICLES	33
<i>P.H. Frampton</i>	
SYMMETRY BREAKING PATTERNS IN E_6	39
<i>F. Gursey</i>	
CHARGE CONJUGATION AND ITS VIOLATION IN UNIFIED MODELS	57
<i>R. Slansky</i>	
BARYON AND LEPTON NON-CONSERVATION, MAJORANA NEUTRINOS AND NEUTRON ($N \leftrightarrow \bar{N}$) OSCILLATIONS	69
<i>R.N. Mohapatra</i>	
POSSIBLE ENERGY SCALES IN THE DESERT REGION	91
<i>T. Goldman</i>	
THE FATE OF GLOBAL CONSERVATION LAWS IN GRAND UNIFIED MODELS	105
<i>P. Ramond</i>	
QUARK-LEPTON UNIFICATION AND PROTON DECAY	115
<i>J.C. Pati and A. Salam</i>	
EVIDENCE FOR NEUTRINO INSTABILITY	149
<i>F. Reines</i>	
REVIEW OF NEUTRINO OSCILLATION EXPERIMENTS	157
<i>J. Lo Secco</i>	

THE IRVINE-MICHIGAN-BROOKHAVEN NUCLEON DECAY FACILITY: STATUS REPORT ON A PROTON DECAY EXPERIMENT SENSITIVE TO A LIFETIME OF 10^{33} YEARS AND A LONG BASELINE NEUTRINO OSCILLATION EXPERIMENT SENSITIVE TO MASS DIFFERENCES OF HUNDREDTHS OF AN ELECTRON VOLT	163
<i>L. Sulak</i>	
A SEARCH FOR BARYON DECAY: PLANS FOR THE HARVARD-PURDUE-WISCONSIN WATER CERENKOV DETECTOR	189
<i>D. Winn</i>	
POSSIBILITIES OF EXPERIMENTS TO MEASURE NEUTRON-ANTINEUTRON MIXING . .	215
<i>R. Wilson</i>	
THE SEARCH FOR NEUTRINO OSCILLATIONS: PRESENT EXPERIMENTAL DATA AND FUTURE EXPERIMENTS	225
<i>D. Cline</i>	
NEUTRINOS IN COSMOLOGY--GOOD NEWS FROM THE BIG BANG	245
<i>G. Steigman</i>	
BIG BANG BARYOSYNTHESIS	257
<i>M. Turner</i>	
SOME COMMENTS ON MASSIVE NEUTRINOS	275
<i>E. Witten</i>	
SUPERGUT	287
<i>J. Ellis</i>	
FERMION MASSES IN UNIFIED THEORIES	297
<i>H. Georgi</i>	
THE SOUDAN MINE EXPERIMENT: A DENSE DETECTOR FOR BARYON DECAY . . .	305
<i>M. Marshak</i>	
THE HOMESTAKE MINE PROTON DECAY EXPERIMENT	313
<i>R. Steinberg</i>	
A MODEST APPEAL TO SU(7)	323
<i>A. Yildiz</i>	
VERTICAL-HORIZONTAL FLAVOR GRAND UNIFICATION	333
<i>K.C. Wali</i>	
NEUTRINO FLUCTUATIONS AND MERGITS: ARE FOSSIL NEUTRINOS DETECTABLE? . .	339
<i>A. De Rújula</i>	
EXPECTATIONS FOR BARYON AND LEPTON NONCONSERVATION	347
<i>S. Weinberg</i>	

PROGRAM 363

ORGANIZING COMMITTEE 365

LIST OF PARTICIPANTS 367

WELCOME

Gordon Haaland

Vice-President for Academic Affairs, University of New Hampshire

My experience with physics was initially limited to a standard college course, but as I pursued my own academic career I became more familiar with physics through the philosophy of science. As I studied psychology, much of the concern that psychology had in terms of both theory and experiment led retrospectively to the literature of the thirties, forties and fifties in areas like the unity of science movement, the whole change in logical positivism, and the impact of contemporary quantum mechanics on the epistemology of science. I was fascinated to find many of the eminent physicists of the day being in the forefront of some of those epistemological issues, particularly the relation between theory and experiment.

Normally, my task is simply to say a few things on behalf of the university and get out of your way, and I don't intend to forget that signal responsibility.

I am fascinated by this conference, and believe that the University of New Hampshire is very honored by your presence here today partly because contemporary physics has continued to be in the forefront of raising the significant epistemological issues of science. It is clear that you are dealing with one of the most significant of our contemporary scientific problems at the forefront of our intellectual endeavors.

This workshop represents one of the real responsibilities and missions of a university, that is to provide a context for discussions on the frontiers of intellectual endeavor. The University of New Hampshire is committed to this type of opportunity. We are interested in, and believe in, the value and importance of a university to provide the opportunities for people such as yourselves to explore the breadth of our world and to explore the ranges of ideas. Consequently, we are committed to research.

We are very pleased that you are here. We hope that you find this to be a productive environment. We hope that the university can provide for you in these next few days the types of resources that will make this an exciting intellectual endeavor for you. We are pleased and honored both by the nature of the topic and the quality and importance of you people as participants.

Welcome to the University of New Hampshire!

We trust that the next several days of this workshop will be fruitful to you. We appreciate your coming. Thank you.

THE NEW FRONTIER

Sheldon Lee Glashow^{*}

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

PARTICLE PHYSICS WITHOUT ACCELERATORS

Pions, muons, positrons, neutrons, and strange particles were found without the use of accelerators. More recently, most developments in elementary particle physics depended upon these expensive artificial aids. Science changes quickly. A time may come when accelerators no longer dominate our field: not yet, but perhaps sooner than some may think.

Important discoveries await the next generation of accelerators. QCD and the electroweak theory need further confirmation. We need to know how b quarks decay. The weak interaction intermediaries must be seen to be believed. The top quark (or the perversions needed by topless theories) lurks just out of range. Higgs may wait to be found. There could well be a fourth family of quarks and leptons. There may even be unanticipated surprises. We need the new machines.

On the other hand, we have for the first time an apparently correct *theory* of elementary particle physics. It may be, in a sense, phenomenologically complete. It suggests the possibility that there are no more surprises at higher energies, at least at energies that are remotely accessible. Indeed, PETRA and ISR have produced no surprises, even at energies many times greater than were previously studied. The same may be true for PEP, ISABELLE, and the TEVATRON. Theorists do expect novel high-energy phenomena, but only at absurdly inaccessible energies. Proton decay, if it is found, will reinforce belief in the great desert extending from 100 GeV to the unification mass of 10^{14} GeV. Perhaps the desert is a blessing in disguise. Ever larger and more costly machines conflict with dwindling finances and energy reserves. All frontiers come to an end.

You may like this scenario or not; it may be true or false. But, it is neither impossible, implausible, nor unlikely. And, do not despair nor prematurely lament the death of particle physics. We have a way to go to reach the desert, with exotic fauna along the way, and even the desolation of a desert can be interesting. The end of the high-energy frontier in no way implies the end of particle physics. There are many ways to skin a cat. In

^{*} Research supported in part by the National Science Foundation under Grant No. PHY77-22864.

this talk I will indicate several exciting lines of research that are well away from the high-energy frontier. Important results, perhaps even extraordinary surprises, may await us. But, there is danger on the way.

The passive frontier of which I shall speak has suffered years of benign neglect. It needs money and manpower, and it must compete for this with the accelerator establishment. There is no labor union of physicists who work at accelerators, but sometimes it seems that there is. It has been argued that plans for accelerator construction must depend on the "needs" of the working force: several thousands of dedicated high-energy experimenters. This is nonsense. Future accelerators must be built in accordance with scientific, not demographic, priorities. The new machines are not labor-intensive, and must not be forced to be so. Not all high energy physicists can be accommodated at the new machines. The high-energy physicist has no guaranteed right to work at an accelerator, he has not that kind of job security. He must respond to the challenge of the passive frontier.

1. CP PHENOMENOLOGY

Here is a small but important enterprise: the search for the electric dipole moment of the neutron. The theorist is confident that the effect does not vanish, but it has not yet been found. One line of thought requires a dipole moment of order 10^{-24} cm. In another, it is expected to be a million times smaller. Which view is correct, if either, will soon be determined by experiment. It is a result of the greatest theoretical interest. In a similar vein is a precision study of the CP violation in K decay. It is essential to know whether or not there are measurable departures from the superweak model. Both of these examples are in the way of loose ends that have been passed over in the push to higher energies. No great surprises await us here, just important and basic physics. There are many other such examples. In this lecture, we are out for bigger game.

2. NEW KINDS OF STABLE MATTER

It has been suggested that there exists a very strong but unobserved interaction that sets the scale of weak interaction effects. Associated with these new forces are new particles with masses between 100 GeV and 100 TeV. (In these technicolor scenarios, the lower reaches of the desert are made to bloom.) Some of these particles may be reasonably stable, so that the particles or their effects may be seen today.

With lifetimes shorter than 10^{10} years, the heavy particles will have already in large measure decayed. Relic high energy neutrinos or photons would be their only spoor. With longer lifetimes, we might see them decaying

in real time. Experiments that have been done put severe constraints on the concentration and lifetime of these hypothetical particles. Experiments that will be done can obtain stronger constraints, or perhaps reveal the new particles. Paul Frampton will address this subject in his talk.

Here, I wish to consider the possible existence of new forms of *stable* matter. Imagine that such matter was produced in the big bang, and that it cohabits with us today. To be specific, I shall speak of singly charged heavy particles of subnuclear size with or without nuclear interactions, called X^\pm and with unknown masses somewhere between 100 GeV and 100 TeV. What is the fate of such particles?

The X^- could defend itself against X^+X^- annihilation by binding to a nucleus. Binding to a proton would produce a neutral system. It would be subject to fusion processes to yield ${}^4\text{He } X^-$, which would behave chemically like a super-heavy Hydrogen isotope. Alternatively, X^- could bind to other common nuclei yielding superheavy atoms of Z one less: superheavy Al or Mn are interesting possibilities. On the other hand, X^+ would be expected to form superheavy hydrogen as an end-product unless the X^+ has strong couplings to nucleons. If it did, ${}^4\text{He } X^+$ would be anticipated as a superheavy Lithium isotope. If X^\pm is discovered to exist, its distribution in nuclei will be important both to establish the properties of X-matter and as a key to the nature of the early universe. Meanwhile, these arguments may be used to suggest plausible sites (like Manganese nodules) wherein to discover super-heavy matter. I am aware of no experiments that put useful limits on the terrestrial abundance of X-matter, except for the fact that no one has encountered a nugget of the stuff.

The putative superheavy matter should be easily concentrated by centrifugation. Several possibilities present themselves for detection.

P. Horowitz suggests detection by back-scattered non-relativistic Protons. Back-scattering from heavy nuclei involves a maximum recoil energy loss of 2%. Back-scattering from a superheavy isotope leads to no perceptible energy loss.

Detection may be accomplished by an e/m measurement at a tandem Van de Graaf. Conventional sputter sources should produce superheavy ions effectively. Intervening foils can disrupt polymers and molecules that could mimic superheavies. Detectors must be tuned to the superheavy regime.

Here is an ambitious and risky field of scientific endeavor. What could be more exciting than the discovery of an entirely new kind of stable matter? What enterprise could *seem* less likely to produce a positive result? Do I have any takers?

3. NEUTRINO MASSES AND NEUTRINO MIXING

The only good symmetry is a gauge symmetry. Everything that can happen

does happen. Neutrinos should have masses, and they should mix. The only open question is how big the effect is. Particle physics is controlled by the unifying mass $\sim 10^{14}$ GeV and the weak mass $G^{-1/2} \sim 100$ GeV. Neutrino masses, being a $\Delta I = 1$ weak effect, do not arise in lowest order. They depend upon the unifying mass scale. They are suppressed by $\sim 10^{-12}$ compared to ordinary masses. This loose argument gives very small neutrino masses, $\sim 10^{-3}$ eV at best. Perhaps it is wrong by one or more powers of α . Only experiment can tell. It is not implausible that neutrino masses are 1 eV or larger and that there is substantial mixing. This would provide a variety of experimentally measurable parameters: four angles and three masses in a three fermion family picture. The experimenter needs the challenge, the theorist needs the hints. (Remember: no one has plausibly predicted the top quark mass.)

There are limits of various kinds on neutrino mixing effects. There are even some indications of an effect. Many kinds of experiment are relevant:

(1) *Accelerator Experiments*: The original experiment of Lederman, Schwartz, and Steinberger established the existence of two neutrinos, and put weak limits on the mixing. Better experiments have been done and still better ones can be done. A beam enriched in energetic electron neutrinos can search for $\nu_e - \nu_\tau$ mixing. A recent BNL experiment using a beam of essentially pure ν_μ can look more closely for $\nu_\mu - \nu_e$ mixing. FNL bubble chamber experiments put limits on $\nu_\mu - \nu_\tau$ mixing. Beam dump experiments seem to yield curious and unexpected results. Much remains to be done, and many good experiments suggest themselves.

(2) *Reactor Experiments*: Whatever mixing $\bar{\nu}_e$ is subject to will show up as an anomalously small cross section for charged current processes. Uncertainties in flux can be compensated by measurement of neutral-current processes. Several experiments are in progress and will be reported on at this conference.

(3) *Meson Factories*: can provide copious sources of neutrinos. These can be used to study $\bar{\nu}_e - \bar{\nu}_\mu$ mixing, or any variety of ν_e mixing. Published data reveal no indication of any effect, but again, more precise work can be done. A marvelous neutrino beam will soon be available at LAMPF.

(4) *Solar Neutrinos*: Here, there is an indication of a large mixing effect. Any neutrino masses greater than 10^{-6} eV can be relevant. More decisive experiments can be done, but they require large quantities of exotic and expensive materials like Gallium or Indium.

(5) *Beta Decay Physics*: Careful experiments have put a limit of 60 eV on the mass of the electron neutrino, from the study of tritium beta decay. It is not impossible that the tau neutrino mass is of order ~ 30 eV, and that there is considerable $\nu_e - \nu_\tau$ mixing. In this case, the endpoint region of the tritium Kurie plot should display a curious bimodal behavior, with a "glitch" occurring 30 eV before the endpoint. The detection of such an effect is difficult, but perhaps possible. It would seem worthwhile to try.

Better experiments are being attempted in Guelph and in Moscow.

Searches for neutrinoless double beta decay have put a limit of ~ 1 KeV on the mass of the electron neutrino. Better experiments may be possible. In particular, we suggest the search for the neutrinoless double K capture process.

(6) *Atmospheric Neutrinos*: Neutrinos produced by interactions of primary cosmic rays in the atmosphere produce a 2/1 admixture of muon and electron neutrinos. This fact has not been verified, and it could be altered by neutrino mixing effects. With upward moving neutrinos taken into account, neutrino masses as small as 3×10^{-3} eV can be detected. Some data will result from planned proton decay experiments. Dedicated neutrino-mixing experiments should also be designed.

(7) *Extraterrestrial Sources*: Other neutrino sources can be imagined. We have mentioned relic neutrinos from the decay of shortlived ($< 10^{10}$ years) superheavy particles. Supernovae in our galaxy are another possibility, but nearby events are not frequent. Continuous observation at several permanent underground stations would be desirable. Let us not miss the next nearby supernova! Time distributions of neutrinos from such a source can shed considerable light on neutrino masses and mixing.

4. ASTROPHYSICAL NEUTRINO PHYSICS

This subject is large, growing, and unfamiliar to me. My remarks are incomplete and perhaps irresponsible. There is a well known solar neutrino problem. It may have an orthodox explanation, or it may be the first indication of neutrino mixing. The Chlorine experiment is sensitive to an energetic minority of solar neutrinos which arise from a minor solar nuclear process. The Gallium experiment, which would be sensitive to better understood low energy neutrinos, requires fifty tons of Gallium. This experiment, or an equivalent one, deserves the highest priority. Solar neutrino mixing is sensitive to smaller neutrino masses than any other conceivably observable process. It is a beautiful example of fundamental physics done in the passive mode. Moreover, the Gallium will survive intact and constitute a National Strategic Reserve.

Another astrophysical argument for neutrino masses is suggested in the universe according to Guth and Tye, who conclude that the universe is neither open nor closed, but is flat. (This is an appealing possibility quite independently of Guth and Tye). The visible mass of the universe is insufficient for this: something like ten times more mass is needed. Suppose this missing mass resides in relic neutrinos.

The relic black-body radiation consists of photons, gravitons, and neutrinos. There are about 100 neutrinos/cm³ of each species. Should the