Lecture Notes in Physics

301

G. Ferenczi F. Beleznay (Eds.)

New Developments in Semiconductor Physics

Proceedings, Szeged, Hungary 1987

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Proceedings of the Third Summer School
Held at Szeged, Hungary
August 31 – September 4, 1987

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PREFACE

The semiconductor branch of the Hungarian Physical Society organized its third Summer School at Szeged, Hungary from August 31 to September 4, 1988 with the intention of discussing the most recent developments in semiconductor physics. (Proceedings of the previous meetings are published in the Lecture Notes in Physics series as Volume 122 and Volume 175.) 84 participants from 18 countries attended the meeting, which included 15 invited talks covering the areas of multilayer growth technology, theory of electron states, transport theory, defect related effects and structural properties of semiconductors. 33 contributed papers, most of them closely related to the invited talks, gave an exciting insight into the research in this field.

The present volume is a selection of the most interesting papers presented at the Summer School, and its format follows that of the meeting: invited papers are accompanied by related contributions. As the table of contents indicates, we found that transport theory and defect-related effects are the most widely researched subjects in contemporary semiconductor physics. Other topics are, nevertheless, well represented.

At the closing session it was suggested that, since the meeting had been so successful, similar events held at regular intervals would be welcome. First steps are being taken to organize future Schools as joint ventures of the Physical Societies of neighbouring countries.

The editors are grateful to Eva Nemeth for her expert help in preparing this volume.

Budapest, Hungary
January 1988

George Ferenczi
Program Chairman

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INTEGER QUANTUM HALL EFFECT

- Present state of the theory -

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The basic experimental facts concerning the integer quantum Hall effect (QHE) are summarized and confronted with the prediction of standard transport theory. The different ideas and approaches to explain the QHE are reviewed and commented upon. The phenomenological model which simulates localization in additive disordered systems at zero temperature by bound states is considered for two system-geometries. For a finite cylinder a spectral stability condition for the QHE is formulated and shown to be sufficient for current compensation. For a torus the topological quantization of the Kubo Hall conductivity in mobility gaps is outlined. Some problems facing current and future research are pointed out.

1. Facts

At high magnetic fields ($B \sim 10$ T) and low temperatures ($T \sim 1$ K) the Hall resistance R_H of a silicon MOSFET as a function of the gate voltage U_G shows characteristic plateaux. Since U_G is proportional to the carrier density n this observation contradicts the established theory which predicts $R_H \sim 1/n$.

In 1980 Klaus von Klitzing discovered that the plateau values of R_H are entirely independent of the properties of the sample, and are given by

$$R_H\,(\text{plateau}) = \frac{1}{k}\frac{h}{e^2}\,,\quad k = 1, 2, \ldots \tag{1.1}$$

/1/. Furthermore, in the plateau regimes of R_H the (longitudinal) resistivity R practically vanishes. Shortly thereafter this integer quantum Hall effect (QHE) was also observed in GaAs heterostructures (in this case n is constant and B is varied). The plateaux are centered (approximately) around the corresponding integer values $\eta = k$ of the filling factor

$$\eta = (2\pi l^2)\, n \tag{1.2}$$

Here l is the magnetic length, $l^2 = \hbar/eB$. Figs. 1 and 2 show some typical experimental data. At present the experimental accuracy of the

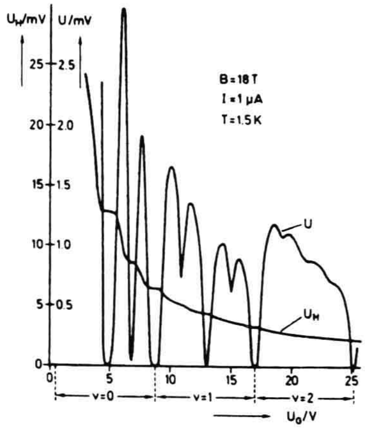

Fig. 1: The quantum Hall effect /1/

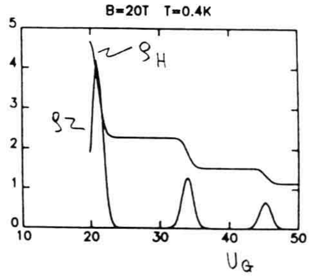

Fig. 2: QHE /2/. Both data refer to Si MOSFETs. $U_G \sim n$

quantization (1.1) is better than 10^{-7}. Thus, the QHE provides a high precision measurement of the Sommerfeld fine structure constant e^2/hc.

The conductivity layers in MOSFETs and heterostructures behave like two-dimensional systems. At sufficiently weak currents the conduction properties of an homogeneous and isotropic rectangular system (with area $Ar = L_x L_y$) in a perpendicular magnetic field are described by the linear relations between currents and voltages,

$$\begin{aligned} U_x &= R I_x + R_H I_y \\ U_y &= -R_H I_x + R I_y \end{aligned} \tag{1.3}$$

Introducing the electric field $\underline{E}$, $E_x = U_x/L_x$, $E_Y = U_Y/L_Y$ and the current density $\underline{j}$, $j_x = I_x/L_Y$, $j_Y = I_Y/L_x$ we get

$$\begin{aligned} E_x &= \varrho j_x + \varrho_H j_y \\ E_y &= -\varrho_H j_x + \varrho j_y \end{aligned} \tag{1.4}$$

with resistivities

$$\varrho = R(L_y/L_x), \quad \varrho_H = R_H \tag{1.5}$$

Notice that in two dimensions R_H is independent of the size of the sample. Inverting (1.4)

$$\begin{aligned} j_x &= \sigma E_x - \sigma_H E_y \\ j_y &= \sigma_H E_x + \sigma E_y \end{aligned} \tag{1.6}$$

with conductivities

$$\sigma = \frac{\rho}{\rho^2 + \rho_H^2} , \quad \sigma_H = \frac{\rho_H}{\rho^2 + \rho_H^2} \tag{1.7}$$

Notice that (if $\rho_H \neq 0$) σ and ρ vanish simultaneously. In the usual Hall measurement $I_y = 0$. Since in the plateau regimes $R = 0$ ($U_x < 10^{-14}$V) there $\sigma = 0$ and

$$\sigma(\text{plateau}) = k\frac{e^2}{h} , \quad k = 0, 1, 2, \ldots \tag{1.8}$$

In 2d the physical dimension of the conductivity is

$$[\sigma] = \frac{[\dot{\gamma}]}{[E]} = \frac{\text{charge}\cdot\text{velocity/area}}{\text{voltage/length}} = \frac{\text{charge/time}}{\text{energy/charge}} = \frac{(\text{charge})^2}{\text{action}} \tag{1.9}$$

- in accordance with (1.8) and (1.1). The plateau values of σ_H are integer multiples of the atomic unit e^2/h of the 2d conductivity.

2. Hints

According to the simple classical kinetic model the Hall conductivity of free electrons is

$$\sigma_H^0 = \frac{en}{B} \tag{2.1}$$

or with (1.2)

$$\sigma_H^0 = \eta\frac{e^2}{h} \tag{2.2}$$

Notice that, at integer filling, $\eta = k$, the measured values of the Hall conductivity of (rather complex) real 2d systems coincide with the corresponding values calculated for the (fictitious) free electron system by using the most simple classical model (cf. Fig. 3). In fact this is even true for the Hall conductivity of disordered systems calculated by perturbation-theoretical evaluation of the Kubo formula (cf. Fig. 4). The stability of the Hall conductivity at integer filling($\sigma_H = k\, e^2/h$) is, perhaps, not so surprising because the states corresponding to completely filled Landau levels are highly resistent against perturbation.

Due to the disorder the free electron Landau levels

$$\varepsilon_\alpha^0 = \varepsilon_\nu = \hbar\omega_c(\nu + \tfrac{1}{2}) \tag{2.3}$$

$\omega_c = eB/m$, $\nu = 0, 1, 2, \ldots$, are broadened to energy bands. In lowest order

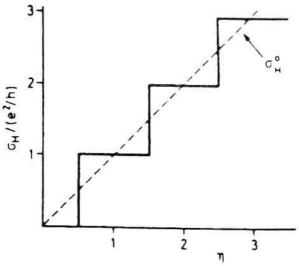

Fig. 3: QHE - extrapolated to zero temperature. At integer filling, $\eta = k$, σ_H and σ_H^o coincide, both being $k\ e^2/h$.

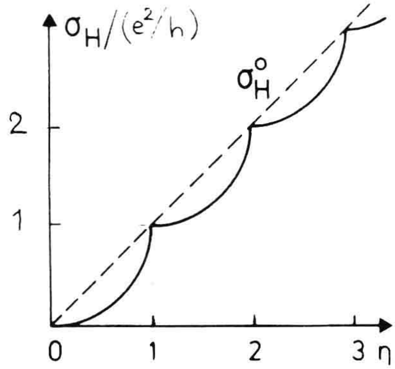

Fig. 4: σ_H calculated by perturbation theory /3/ (schematic).

cumulant approximation for a white-noise correlated random potential, the free electron density of states

$$n^o(\varepsilon) = \frac{1}{2\pi l^2} \sum_\nu \delta(\varepsilon - \varepsilon_\nu) \tag{2.4}$$

is replaced by

$$n(\varepsilon) = \frac{1}{2\pi l^2} \frac{\sqrt{2/\pi}}{\Gamma} \exp\left[-2\left(\frac{\varepsilon - \varepsilon_\nu}{\Gamma}\right)^2\right] \tag{2.5}$$

with

$$\Gamma = \frac{2}{\pi} \hbar\omega_c \hbar/\tau \tag{2.6}$$

where τ is the B = 0 relaxation time (in Born approximation) /4/. Each band contains $1/2\pi l^2$ states per unit area (the degree of degeneracy of the Landau levels). For high mobility samples and sufficiently high magnetic fields, such that $\omega_c\tau \gg 1$, the overlapping of bands is insignificant. The density of states can be used to express the chemical potential ξ in terms of n,B and T,

$$n = \frac{1}{Ar} \sum_\alpha f_\alpha = \int f(\varepsilon)\, n(\varepsilon)\, d\varepsilon \tag{2.7}$$

where $f_\alpha = f(\varepsilon_\alpha)$ is the Fermi distribution function

$$f(\varepsilon) = \left\{ \exp\left[(\varepsilon - \xi)/k_B T\right] + 1 \right\} \tag{2.8}$$

For free electrons, with (2.4)

$$n = \frac{1}{2\pi l^2} \sum_\nu f(\varepsilon_\nu) \tag{2.9}$$

and, with (2.2),

$$\sigma_H^0(\xi) = \frac{e^2}{h} \sum_\nu f(\varepsilon_\nu) \tag{2.10}$$

In the limit $T \to 0$ $\xi = \varepsilon_F$ and

$$\sigma_H^0(\varepsilon_F) = \frac{e^2}{h} \sum_\nu \theta(\varepsilon_F - \varepsilon_\nu) \tag{2.11}$$

Thus $\sigma_H^0(n)$ is a straight line but $\sigma_H^0(\varepsilon_F)$ is a step function (Fig. 5). (This is all right since $n = n(\varepsilon_F)$ is a step function as well.) In contrast to this, the observed Hall conductivity extrapolated to T = 0 (shown in Fig. 3), is a step function on the n scale. Assuming, however,

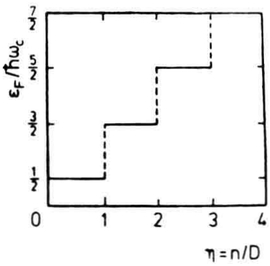

Fig. 5: Fermi energy as a function of the filling factor for free electrons. $D = 1/2\pi l^2$

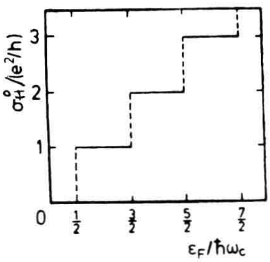

Fig. 6: Hall conductivity as a function of the Fermi energy for free electrons

that in a real system $n = n(\varepsilon_F)$ is a smooth function (as indicated by model calculations) we can conclude that on the ε_F scale the extrapolated values of the measured Hall conductivity is exactly the same as for free electrons, given by (2.11). Thus, a way to characterize the QHE effect is to say that, at T = 0, $\sigma_H(\varepsilon_F)$ is the same as for an ideal free electron gas - inspite of the broadening of the Landau levels to energy bands. This behaviour is obviously radically different from that predicted by the traditional transport theory for disordered systems (perturbation-theoretical evaluation of the Kubo formula).

3. Ideas and Approaches

Shortly after the discovery of the QHE several ideas were developed to explain this suprising phenomenon. Aoki and Ando /5/ pointed out that the QHE may be brought about by localization of electrons in a 2d random potential. In fact, the vanishing longitudinal conductivity in the plateau regimes of σ_H seems to be an obvious indication of localization. Furthermore, localization was shown to occur in the tails of the Landau bands /6/ and, in the limit of very high magnetic fields, everywhere outside of small ranges around the band centers ($\varepsilon = \varepsilon_\nu$) /7/. Since an operative high field transport theory which incorporates localization was - and still is - not available, Aoki and Ando simulated localization by assuming bound states in which the expectation value of the velocity vanishes,

$$\langle \alpha \mid \underline{v} \mid \alpha \rangle = 0 \quad \text{for bound states} \tag{3.1}$$

Connecting this phenomenological description of localization with the Kubo formula they demonstrated that, at T = 0, the longitudinal conductivity vanishes and the Hall conductivity σ_H keeps constant as long as the Fermi energy varies within a regime of bound states. Unfortunately their proof, showing the plateau values of σ_H to coincide with the quantized ones (1.8), seen in experiment turned out to be incomplete. At first glance the proof can be supplemented by reference to perturbation theory /8/ or to the Středa formula /9/ both of which lead, for $\eta = k$, to $\sigma_H = k\, e^2/h$. However both ways of fixing the plateaux at the observed quantized values are rather unsatisfactory. The perturbation theory definitely tails for $\eta \neq k$. The Středa formula which requires spectral gaps between the Landau bands in order to yield the quantized values for integer filling, seems to hold for a confined system only /10/. For such systems, however, due to edge states no band gaps exist. As we shall see (cf. Section 4) considerably more effort is needed to prove quantization within the phenomenological description of localization /10/. In any case, numerical analysis by Ando /11/ has confirmed that the QHE can be explained in terms of independent electrons moving in a random potential.

If the QHE is due to localization, the loss of current (caused by localization) must be compensated by an additive acceleration in the delocalized states - in order to maintain $\sigma_H = k\, e^2/h$ for $\eta = k$. Prange /12/ and subsequently other authors /13,14/ have attempted to demonstrate compensation for some model systems. An elegant way of proof utilizes Levinson's theorem. In the usual formulation this theorem relates the scattering phase shift to the number of bound states brought about by a certain potential. We shall see (cf. Section 4) that the conditions for the QHE in the phenomenological description of localization are sufficient to prove conpensation (and to derive Levinson's theorem) /15/. This is an important fact because localization cannot adequately be explained by potential scattering.

A simple model which illustrates localization and explains qualitatively the QHE is a system of independent electrons moving in a slowly varying random potential V(x,y) and a strong magnetic field /16,17/. Introducing center and relative coordinates with respect to the cyclotron motion,

$$x = X + v_y/\omega_c \,, \quad y = Y - v_x/\omega_c \tag{3.2}$$

(v_x, v_y) and (X,Y) are pairs of conjugate variables,

$$[v_x, v_y] = i\,(\ell\omega_c)^2 \tag{3.3}$$

$$[X, Y] = i\ell^2 \tag{3.4}$$

The velocity components v_x and v_y are bounded; the expectation values of the relative coordinates in an energy eigenstate is proportional to l, i.e. to 1/B. Consequently, for sufficiently large B we can approximate V(x,y) by V(X,Y). Furthermore, since (according to (2.4)) the limit $B \rightarrow \infty$ is equivalent to the classical limit $h \rightarrow 0$ we can approximate the quantum dynamics by the corresponding classical one. Replacing the kinetic energy $mv^2/2$ by its eigenvalue ε^0_α, we get the slow motion Hamiltonian

$$H = \varepsilon^0_\alpha + V(X,Y) \tag{3.5}$$

The equations of motion

$$\dot{X} = \frac{1}{eB}\frac{\partial V}{\partial Y} \,, \quad \dot{Y} = -\frac{1}{eB}\frac{\partial V}{\partial X} \tag{3.6}$$

describe a reversible 1d motion along equipotential lines V(X,Y) = const. If the space average of V vanishes then, according to percolation theory, in the thermodynamic limit, all equipotential lines in the bulk with energy $\varepsilon \neq \varepsilon^0_\nu$ are closed (localized states), and open equipotential lines (delocalized states) exist only at $\varepsilon = \varepsilon_\nu$ (i.e. at the centers of the Landau bands).

To calculate the current we have to add to V the potential energy

$e\phi$ due to a driving electric field $\underline{E} = -\text{grad}\,\phi$. Obviously, the motion along closed equipotential lines does not contribute to the net current

$$j_y = \frac{-e}{Ar} \sum_{\nu} \iint \frac{dX\,dY}{2\pi \ell^2}\, f\left(-\frac{1}{eB}\frac{\partial U}{\partial X}\right) \tag{3.7}$$

$Ar = L_x L_y$, $U = V + e$. Assuming that all channels of open (percolating) equipotential lines with $\varepsilon_\nu \leqslant \varepsilon \leqslant \varepsilon_{\nu+1}$ are occupied we get for the current at $T = 0$

$$j_y = \frac{e^2}{h} \sum_{\nu} \theta(\varepsilon_F - \varepsilon_\nu) \sum_{\ell} \Delta U_\ell / e L_x \tag{3.8}$$

where $\Delta U_l/e$ is the potential drop across the l^{th} channel. Since $\sum_\ell \Delta U_l/e$ is nothing else but the total potential drop across the sample, (3.8) is equivalent to

$$\sigma_H = k\,\frac{e^2}{h} \tag{3.9}$$

where k is the largest integer for which $k \leqslant \eta$. Unfortunately, this impressingly simple derivation of the desired result (3.9) (Iordansky /16/) does not even exclude corrections of the order l/L. The formulation based on linear response theory (Kubo formula) /17,18/ is subject to the same limitation. Since in experiments, $l/L \approx 10^{-4}$ and the accuracy of quantization is 10^{-8}, the required accuracy of the proof of (3.9) is at least $(l/L)^2$. This may have motivated the aim to combine the high field percolation model with the gauge argument /13/ (see below). The problem, however, is not to prove the stability of (3.9) within the leading order high field model (which is guaranteed by the topologically distinct nature of closed and open equipotential lines) but to determine the accuracy of the model as such. For the relation of the high field percolation model to percolation theory, cf. Trugman /19/. The onset of dissipation is investigated in /20,21/. The high field model also provides an explanation of observed thermoelectric effects analogous to the QHE /22/.

The gauge argument asserts that the QHE is due to a particular symmetry property: for a 2d system on the surface of a cylinder, the change of axial flux by a unit flux quantum, together with the transfer of a unit charge from one edge of the cylinder to the other (which are at different potentials) is a symmetry transformation of the system if, at $T = 0$, the Fermi energy lies in a mobility gap (Laughlin /23/) or, more generally, the ground state of the system is non-degenerate and separated from the rest of the energy spectrum /24/. The gauge argument requires some interpretation. This is provided, for instance, by the topological approach to the QHE /25,26/ which was initiated by the observation that for an electron in an ideal 2d lattice the Hall conductivity defined by the Kubo formula is topologically quantized and equal to an integer multiple of e^2/h if, at T = 0, the Fermi energy lies in an energy gap /27/. (The same result follows also from the Středa formula /28/). The characteristic feature of the topological approach is a double-periodic Hamiltonian (2d system on a torus). As we shall see (cf. Section 4) the Kubo Hall conductivity for such a system can be proved to be topologically quantized if, at T = 0, the Fermi energy lies in a mobility gap - the localization being simulated by bound states /29/.

Of course, a theory of the QHE as a localization phenomenon is only acceptable if it explains rather than assumes localization. Confronted with this requirement, the state of the art in QHE theory is rather unsatisfactory. Following the lines of the B = 0 self-consistent localization theory Ono /30/ obtained for the high field longitudinal conductivity exponential delocalization at the band centers ($\varepsilon = \varepsilon_\nu$). Perturbation theory yields qualitatively the same result /31/. Similar investigations for the Hall conductivity have not yet been reported. For the time being the only localization theory which treats σ and σ_H on equal grounds is the field theory by Levine, Libby and Pruisken /32/. In this theory the relevant long range modes are described by the Lagrangian

$$\mathcal{L} = \sigma^{(0)} \mathcal{L}_1 + \sigma_H^{(0)} \mathcal{L}_2 \tag{3.10}$$

where $\mathcal{L}_2$ is due to the axial symmetry breaking by the magnetic field and the coupling constants are the mean field values of the longitudinal and the Hall conductivity respectively. For finite action field configurations (instantons) $\mathcal{L}_2$ is a topological invariant /33/. COnsequently, the two parameter scaling space decomposes into equivalent sectors. Levine et al /32/ argue that for $T \to 0$, $\sigma^{(0)}$ and $\sigma_H^{(0)}$ are renormalized to 0 and $k\ e^2/h$ respectively. Although this seems rather plausible a formal proof (solution of renormalization group equations) is still missing.

Still not clarified is the role played by the Coulomb interaction between the electrons. As pointed out recently the Hall conductivity of a 2d free electron system depends strongly both on the potential distribution of the driving force ($\underline{E}$) /34/ and on the system size (in the direction of the field) /35/. It is believed that the Coulomb interaction substantially reduces these effects and, thus, acts to set up the classical free electron Hall conductivity (2.1).

4. Spectral stability

4.1 Finite cylinder

We consider a 2d system of independent electrons in a random potential V(x,y) on a rectangular part of the (x,y)-plane in a perpendicular homogeneous magnetic field B. In the x direction the electrons are assumed to be confined by a potential $V_c(x)$ to an interval of finite length L_x. In order to get a non-vanishing current, say in the y direction, we impose the periodic boundary condition

$$\Psi(x,y+L_y) = \Psi(x,y) \tag{4.1}$$

on the wave functions. By (4.1) the geometry of the system is - with respect to connectivity - equivalent to the surface of a cylinder R x S'. Unfortunately, the usual coordinate representation of quantum mechanics cannot be implemented on S'. The reason is that (4.1) prevents both the existence of a global multiplication operator corresponding to y and the unitary equivalence of the operators $-i\hbar\,\partial/\partial y$ and $-i\hbar\,\partial/\partial y$ + const. One way to overcome this difficulty is to apply the U(1) bundle theory. Another, essentially equivalent, way is to repeat the system periodically in the y direction

$$V(x,y+L_y) = V(x,y) \tag{4.2}$$

and apply the usual coordinate representation

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial y} + eBx\right)^2 + V(x,y) + V_c(x) \tag{4.3}$$

Owing to (4.2) the Bloch theorem applies in the y direction

$$H\,\phi_n(x,y;\vartheta) = \varepsilon_n(\vartheta)\,\phi_n(x,y;\vartheta) \tag{4.4}$$

$$\phi_n(x,y;\vartheta) = e^{i\vartheta y/\hbar}\,u_n(x,y;\vartheta) \tag{4.5}$$

$$u_n(x,y+L_y,\vartheta) = u_n(x,y;\vartheta) \tag{4.6}$$

being the Bloch wave number. The Bloch factors u_n satisfy

$$H(\vartheta)u_n(x,y;\vartheta) = \varepsilon_n(\vartheta)\,u_n(x,y;\vartheta) \tag{4.7}$$

with

$$H(\vartheta) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial y} + \hbar\vartheta + eBx\right)^2 + V(x,y) + V_c(x) \quad (4.8)$$

Thus, $u_n(x,y;\vartheta)$ can be interpreted as the wave function $\psi_n(\vartheta)$ corresponding to a representation

$$p_y : p_y(\vartheta) = -i\hbar\frac{\partial}{\partial y} + \hbar\vartheta, \; \vartheta \in \frac{2\pi}{L_y}(\mathbb{R}/\mathbb{Z}) \quad (4.9)$$

and restricted to $y \in [0, L_y[$.

By restricting ϑ to the first Brillouin zone, $\vartheta = \vartheta_0 \in [-\frac{\pi}{L_y}, \frac{\pi}{L_y}[$ we get the usual energy bands $\mathcal{E}_n(\vartheta_0)$ labeled by the band index n. In the case of free electrons ($V = 0$) with Dirichlet boundary conditions $\psi_n = 0$ for $x = \pm L_x/2$ replacing the confinement potential (Teller-model) the energy spectrum of $H(\vartheta = 0)$ depends on the eigen values $\hbar k_n$, $k_n = \frac{2\pi}{L_y} n$, $n \in \mathbb{Z}$ of the conserved momentum $p_y = -i\hbar\, \partial/\partial y$ as well as on the Landau quantum number $\nu = 0,1,2,\ldots$ (Fig. 1). The energy bands $\mathcal{E}_n(\vartheta_0)$ are related to the spectrum $\mathcal{E}_\nu(k_n)$ by

$$\mathcal{E}_n(\vartheta_0) = \mathcal{E}_\nu(k_{n'} + \vartheta_0) \quad (4.10)$$

and can be joined together to form continuous energy branches

$$\mathcal{E}_\nu(\vartheta) = \mathcal{E}_\nu(k_n + \vartheta_0), \; \vartheta \in \frac{2\pi}{L_y}\mathbb{R} \quad (4.11)$$

In the Landau model ($V_c = 0$, $L_x \to \infty$) these energy branches are the familiar equidistant degenerate Landau levels $\mathcal{E}_\nu(\vartheta) = \mathcal{E}_\nu = \hbar\omega_c(\nu + \frac{1}{2})$ In the Teller model the energy branches depend on ϑ. For sufficiently large values of L_x/l the lifting of degeneracy is significant at the edges $\vartheta \simeq \pm L_x/2l^2$ only. For the Hall effect in the Teller model, cf. the recent work by Ono and Kramer /36/.

In a disordered system ($V \neq 0$) both extended and localized states may exist. In the phenomenological model /5/ to be adopted in this and in the following section, the localization is simulated by bound states in which the expectation value of the velocity $v_y = \frac{i}{\hbar}[H, y]$

$$\langle \phi_n(\vartheta) | v_y | \phi_n(\vartheta) \rangle = \langle \psi_n(\vartheta) | v_y(\vartheta) | \psi_n(\vartheta) \rangle = \frac{1}{\hbar}\frac{\partial \mathcal{E}_n(\vartheta)}{\partial \vartheta} \quad (4.12)$$

with

$$v_y(\vartheta) = \frac{1}{\hbar}\frac{\partial H}{\partial \vartheta} \quad (4.13)$$