Fracture Mechanics and Crack Growth

Naman Recho

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First published 2012 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

ISTE Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

Library of Congress Cataloging-in-Publication Data

Recho, Naman.
Fracture mechanics and crack growth / Naman Recho.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-84821-306-7
1. Fracture mechanics. 2. Materials--Fatigue. I. Title.
TA409.R44 2012
620.1'126--dc23

2011051809

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN: 978-1-84821-306-7

Printed and bound in Great Britain by CPI Group (UK) Ltd., Croydon, Surrey CR0 4YY

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FSC® C013604
FSC www.fsc.org

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Preamble

How to Comprehend This Work?

The aims of fracture mechanics are twofold: on one hand they concern the description of mechanical fields in the neighborhood of the tip of the crack and the energies that are associated therewith; and on the other hand, they deal with the evaluation of the harm of a crack in terms of its propagation.

Two fields of study constitute the structure of this work. The first one is relative to the modeling of the singularity induced by the crack tip that is described in Part I entitled *Stress Field Analysis Close to the Crack tip*. It deals with the modeling of mechanical fields at the crack or singularity tip. The second part, entitled *Crack Growth Criteria*, deals with crack initiation and propagation under monotonic and cyclic loadings.

In Part I, an introduction to continuum mechanics is given. Then, an approach which consists of studying the way which enables us to see how to introduce a singularity in a continuum is developed. To that end, two methods are detailed:

– a *local* one, which describes the stress (or even strain) functions as being continuous everywhere except at the crack tip and which introduces the free boundary conditions relative to the lips of the crack; this forms the asymptotic analysis. In a two-dimensional linear elastic medium, the asymptotic analysis leads to the following description of the stress field:

$$\sigma_{ij}(r,\theta)=\frac{K_I}{\sqrt{2\pi\, r}}f_{ij}(\theta)+\frac{K_{II}}{\sqrt{2\pi\, r}}g_{ij}(\theta)$$

with K_I and K_{II} being two load and geometry functions, describing the amplitude of the stress fields, called stress intensity factors. $f_{ij}(\theta)$ and $g_{ij}(\theta)$, which are two exclusive functions of θ. r and θ, respectively, represent the distance to the crack tip of the volume element studied to the crack tip and its orientation with respect to the axis of the crack.

– For the second method, we can evaluate G, which is the release rate of potential energy, W_{pot}, subsequent to an infinitesimal increase Δa in the crack length:

$$G = -\frac{\partial W_{pot}}{\partial a} = -\frac{\Delta W_{pot}}{\Delta a}$$

In a linear elastic plane medium we can, easily link G to the stress intensity factor in crack-opening mode K_I:

$$G = \frac{K_I^2}{E}, \text{ in plane stress}$$

$$G = \frac{K_I^2}{E}\left(1 - \nu^2\right), \text{ in plane strain}$$

where E is Young's modulus and ν is Poisson's ratio.

Part II, entitled *Crack Growth Criteria* deals with the propagation and extension criteria of a crack in elastic and elastic-plastic media under constant and dynamic loads (*fatigue fracture*).

The analytical solutions obtained cannot be used in the structures with variable geometry and boundary conditions, so we need to use methods of numerical analysis, and in particular the finite element method. Two chapters deal with these numerical applications: one, in Chapter 5 relative to the introduction to the finite element calculations of cracked structures and the other, in Chapter 7 dealing with the forecast of the failure by crack growth of elements of steel structures subjected to fatigue.

Each phenomenon studied is dealt with according to:

– its conceptual and theoretical modeling;

– its use in the criteria of fracture resistance; and

– its implementation in terms of feasibility and numerical application.

The reader is warned that the bulk of the developments in this book concern metal materials. The extension of the conclusions to composite, elastomeric or plastic materials is unreliable.

Preface

> "The fact is that there is no opposition between constraint and liberty, and that, on the contrary, they support each other – since all liberty attempts to overturn or overcome a constraint and every constraint has cracks or points of least resistance that invite liberty to pass through."
>
> *The View from Afar*, Claude Lévi-Strauss (p. 17, Plon, Paris, 1983)

What are the reasons that have led to the writing up of the present work? When re-reading the first French edition of September 1995[1], I noticed that as a conclusion and a final paragraph of the conclusion, I had written:

> "Presently, such a complete analysis (of crack growth in structures, developed in the book) still faces two kinds of difficulties; technical for the determination of certain variables, and economical because of the relatively high cost of numerical analyses and experimental measurements."

In the 16 years since, numerical methods have evolved in capacity and are now accessible to a larger number of engineers and researchers. Equally, the feasibility of certain experimental measurements has increased with the availability of analysis

1 It corresponds to a book known as: *Rupture par fissuration des structures*, collection Traité des nouvelles technologies, série Matériaux, Hermès, Paris, September 1995.

techniques (fractographic aspects, strain fields, etc.) allowing us to monitor and record the crack growth history of a structure and in particular to measure the mechanical fields in the neighbourhood near the tip of a crack, or even a singularity.

In addition, original works carried out by PhD students have led to significant advances in various aspects covered in the earlier edition, namely:

– the analysis of the reliability of welded components [AP.1.5, AP.1.7];

– the analysis of the effect of overload on the elements of cracked structures [AP.1.2, AP.1.5];

– the follow up of crack growth by numerical methods such as crack bow techniques [AP.1.2];

– the development of specific crack propagation and crack extension criteria [AP.1.1, AP.1.3, AP.1.4];

– the establishment of a new approach to fracture mechanics to find new solutions to problems such as the presence of a crack in an anisotropic elastic material [AP.1.4];

– the analysis of geometric singularities, such as the V-shaped notch [AP.1.1];

– the establishment of new models of coupling – "initiation, propagation" – seen in their local aspects in terms of fracture mechanics ([AP.1.6], and in terms of damage analysis [AP.2.1]), in their global aspects in terms of the S-N curve [AP.1.5, AP.1.9, AP.2.2] or in their numerical aspects in the finite element analysis of an industrial structure [AP.1.2];

– the study of the influence of local effects at the weld toes on the fatigue design of welded joints [AP.1.2, AP.1.10, AP.1.11].

All of these advances have enabled the establishment of a comprehensive global approach, applicable to real structures and not just test specimens, in that the flexibility of a structure, which is affected by the size of the crack, is considered in the calculation.

Finally, the lessons in "fracture mechanics, damage analysis and fatigue design" given at the ETH (engineering schools, Switzerland), the CHEC (*Centre des Hautes Etudes de la Construction*, Paris) and Research Masters and Professional training atthe *Université Blaise Pascal* in Clermont-Ferrand, have provided some subjects for students and teachers who are interested in research.

This edition originated during the establishment of specific scientific seminars given in China, Italy and France ([AP.3.1] to [AP.3.7]) targeted at researchers and engineers. It incorporates the findings of the work done in collaboration with doctoral students since 1995 and published in journals and presented at international and national conferences. This first book treats the theoretical, conceptual and numerical aspects of fracture mechanics and divided into two parts:

Part I: Stress Field Analysis Close to the Crack tip,

Part II: Crack Growth Criteria.

Given the amount of material involved, there may be another book dealing with industrial applications and exercises published in the near future.

Naman RECHO
January 2012

Notations

U = Displacement vector

u = {u, v, w} in Cartesian coordinates

u = $\{ u_r, u_\theta, u_z \}$ in cylindrical coordinates

$\{\Delta\}$ = Displacement fields

= {u, v, w} Cartesian coordinates

= $\{ u_r, u_\theta, u_z \}$ Cylindrical coordinates

$\{ \varepsilon_{ij} \}$ = Strain fields

= $\{ \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{23}, \varepsilon_{31}, \varepsilon_{12} \}$ Cartesian coordinates

= $\{ \varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \varepsilon_{\theta z}, \varepsilon_{zr}, \varepsilon_{r\theta} \}$ Cylindrical coordinates

$\{ \sigma_{ij} \}$ = Stress fields

= $\{ \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{31}, \sigma_{12} \}$ Cartesian coordinates

= $\{ \sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{\theta z}, \sigma_{zr}, \sigma_{r\theta} \}$ Cylindrical coordinates

q = Vectors of displacement variables, see "Displacement vectors"

p = Vectors of stress variables

z = A base Cartesian coordinate for the Hamiltonian transformation

$(\dot{\cdot})$ Partial derivatives relative to the base coordinate $\frac{\partial}{\partial z}$

$\{S_{ij}\}$ = Normalized stress fields

$\{S_{ij}\} = \{S_{rr}, S_{r\theta}, S_{\theta\theta}\}$ in polar coordinates

H (p, q, z) = Hamiltonian

$L(\dot{q}, q, z)$ = Lagrangian

H = Hamiltonian operator's matrix

λ, μ = Eigenvalues

Ψ = Eigenvectors

Ω = Hellinger-Reissner energy

$W(\varepsilon)$ = Strain energy of a solid

$w(\varepsilon)$ = Strain energy density

W_{ext} = External work

W_{pot} = Potential energy of a solid

k_t = Stress concentration factor

n = Exponent of plastic behavior law

w_d = Deviatoric energy

$\{s_{ij}\}$ = Stress deviatoric field

E = Young's modulus

μ = Shear Lamé coefficient

ν = Poisson ratio

λ = Lamé coefficient

E_T = Plasticity modulus

σ_e = Elastic limit

σ_y = Yield stress = $\sigma_{e'}$ for initial loading

ε_{ij}^{e} = Elastic strain

ε_{ij}^{p} = Plastic strain

$F(\sigma_{ij})$ = Stress boundaries

$\sigma_I, \sigma_{II}, \sigma_{III}$ = Principal stresses

V = Solid volume

S = Solid surface

S_u, S_F = Surface where displacement or force fields, respectively, are provided

CA = Kinematically admissible displacement field

SA = Statically admissible stress field

$P_{(i)}$ = Virtual power of internal forces

$P_{(e)}$ = Virtual power of external forces

T_i = External forces

T_i^d = Given external forces

W_{co} = Complementary energy

$[\varepsilon_{ij}][\sigma_{ij}]$ = Strain and stress tensors

$\{\varepsilon\}$ $\{\sigma\}$ = Strain and stress vectors

$\{\Delta\}$ = Displacement vectors

$\{\delta\}$ = Nodal displacement vectors

$\{F\}$ = Nodal forces vectors

$\psi(x,y)$ = Airy function

$[N]$ = Serendip function

$[B]$ = Intermediary matrix between the strain and nodal displacement vectors

$[D]$ = Matrix of the behavior law linking $\{\sigma\}$ to $\{\varepsilon\}$

$[K]^e$ = Elementary stiffness matrix

$[K]$ = Global stiffness matrix

Mode I, mode II, mode III are the elementary fracture modes, opening mode in plane shear and out of plane shear modes, respectively

K_I, K_{II}, K_{III} = Stress intensity factor relative to the three modes previously mentioned

M^e, M^P = elastic and plastic mixity index of modes I and II fracture

σ_∞ = Applied stress at infinity (far stress field)

ReZ = Real part of the complex function Z

ImZ = Imaginary part of the complex function Z

γ= Surface energy

G = Energy release rate

(J) = J-contour integral

δ_{CP}, δ_{DP} = Depths of the plastic zone in plane stress and plane strain, respectively

r_y = Radius of the plastic zone in the sense of Irwin

δ = Crack opening displacement (COD)

K_p = Factor equivalent to the stress intensity factor in the elastic–plastic domain

K_{IC} = Stress intensity factor corresponding to brittle fracture

G_{IC} = Critical energy release rate

J_{IC} = Critical *J*-integral

a_c = Critical crack length

σ_c = Critical applied stress

θ_0 = Crack extension angle

da/dN = Crack growth rate

R = Ratio of minimum applied stress to the maximum applied stress = $\sigma_{min}/\sigma_{max}$

ΔK = Variation of the stress intensity factor = $K_{max}-K_{min}$

ΔK_0 = ΔK limit, below which there is no propagation = Threshold

$\Delta\sigma$ = Variation of applied stress

N_R = Number of cycles to failure = Fatigue life

a_0 = Initial crack length

a_f = Final crack length

C,n = Parameters of the Paris propagation law

N_I = Number of cycles to crack initiation

a_i, n_i = Crack length and current cycles during propagation

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