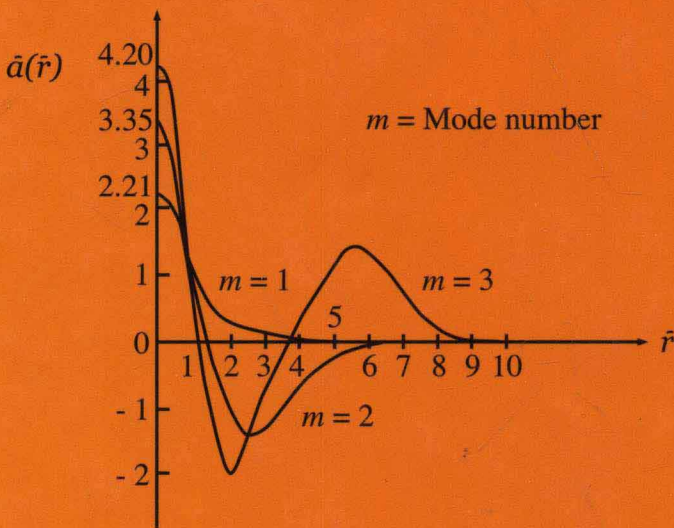


# Nonlinear Optics

Theory, Numerical Modeling,  
and Applications



Partha P. Banerjee

# Nonlinear Optics

**Theory, Numerical Modeling,  
and Applications**

**Partha P. Banerjee**

*University of Dayton  
Dayton, Ohio, U.S.A.*



MARCEL DEKKER, INC.

NEW YORK • BASEL

Although great care has been taken to provide accurate and current information, neither the author(s) nor the publisher, nor anyone else associated with this publication, shall be liable for any loss, damage, or liability directly or indirectly caused or alleged to be caused by this book. The material contained herein is not intended to provide specific advice or recommendation for any specific situation.

Trademark notice: Product or corporate names may be trademarks or registered trademarks and are used only for identification and explanation without intent to infringe.

#### **Library of Congress Cataloging-in-Publication Data**

A catalog record for this book is available from the Library of Congress.

**ISBN: 0-8247-0965-9**

This book is printed on acid-free paper.

#### **Headquarters**

Marcel Dekker, Inc., 270 Madison Avenue, New York, NY 10016, U.S.A.  
tel: 212-696-9000; fax: 212-685-4540

#### **Distribution and Customer Service**

Marcel Dekker, Inc., Cimarron Road, Monticello, New York, 12701, U.S.A.  
tel: 800-228-1160; fax: 845-796-1772

#### **Eastern Hemisphere Distribution**

Marcel Dekker AG, Hutgasse 4, Postfach 812, CH-4001 Basel, Switzerland  
tel: 41-61-260-6300; fax: 41-61-260-6333

#### **World Wide Web**

<http://www.dekker.com>

The publisher offers discount on this book when ordered in bulk quantities. For more information, write to Special Sales/Professional Marketing at the headquarters address above.

**Copyright © 2004 by Marcel Dekker, Inc. All Rights Reserved.**

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage and retrieval system, without permission in writing from the publisher.

Current printing (last digit):

10 9 8 7 6 5 4 3 2 1

**PRINTED IN THE UNITED STATES OF AMERICA**

# **Nonlinear Optics**

# OPTICAL ENGINEERING

*Founding Editor*

**Brian J. Thompson**

*University of Rochester  
Rochester, New York*

1. Electron and Ion Microscopy and Microanalysis: Principles and Applications, *Lawrence E. Murr*
2. Acousto-Optic Signal Processing: Theory and Implementation, *edited by Norman J. Berg and John N. Lee*
3. Electro-Optic and Acousto-Optic Scanning and Deflection, *Milton Gottlieb, Clive L. M. Ireland, and John Martin Ley*
4. Single-Mode Fiber Optics: Principles and Applications, *Luc B. Jeunhomme*
5. Pulse Code Formats for Fiber Optical Data Communication: Basic Principles and Applications, *David J. Morris*
6. Optical Materials: An Introduction to Selection and Application, *Solomon Musikant*
7. Infrared Methods for Gaseous Measurements: Theory and Practice, *edited by Joda Wormhoudt*
8. Laser Beam Scanning: Opto-Mechanical Devices, Systems, and Data Storage Optics, *edited by Gerald F. Marshall*
9. Opto-Mechanical Systems Design, *Paul R. Yoder, Jr.*
10. Optical Fiber Splices and Connectors: Theory and Methods, *Calvin M. Miller with Stephen C. Mettler and Ian A. White*
11. Laser Spectroscopy and Its Applications, *edited by Leon J. Radziemski, Richard W. Solarz, and Jeffrey A. Paisner*
12. Infrared Optoelectronics: Devices and Applications, *William Nunley and J. Scott Bechtel*
13. Integrated Optical Circuits and Components: Design and Applications, *edited by Lynn D. Hutcheson*
14. Handbook of Molecular Lasers, *edited by Peter K. Cheo*
15. Handbook of Optical Fibers and Cables, *Hiroshi Murata*
16. Acousto-Optics, *Adrian Korpel*
17. Procedures in Applied Optics, *John Strong*
18. Handbook of Solid-State Lasers, *edited by Peter K. Cheo*
19. Optical Computing: Digital and Symbolic, *edited by Raymond Arrathoon*
20. Laser Applications in Physical Chemistry, *edited by D. K. Evans*
21. Laser-Induced Plasmas and Applications, *edited by Leon J. Radziemski and David A. Cremers*

22. Infrared Technology Fundamentals, *Irving J. Spiro and Monroe Schlessinger*
23. Single-Mode Fiber Optics: Principles and Applications, Second Edition, Revised and Expanded, *Luc B. Jeunhomme*
24. Image Analysis Applications, *edited by Rangachar Kasturi and Mohan M. Trivedi*
25. Photoconductivity: Art, Science, and Technology, *N. V. Joshi*
26. Principles of Optical Circuit Engineering, *Mark A. Mentzer*
27. Lens Design, *Milton Laikin*
28. Optical Components, Systems, and Measurement Techniques, *Rajpal S. Sirohi and M. P. Kothiyal*
29. Electron and Ion Microscopy and Microanalysis: Principles and Applications, Second Edition, Revised and Expanded, *Lawrence E. Murr*
30. Handbook of Infrared Optical Materials, *edited by Paul Klocek*
31. Optical Scanning, *edited by Gerald F. Marshall*
32. Polymers for Lightwave and Integrated Optics: Technology and Applications, *edited by Lawrence A. Hornak*
33. Electro-Optical Displays, *edited by Mohammad A. Karim*
34. Mathematical Morphology in Image Processing, *edited by Edward R. Dougherty*
35. Opto-Mechanical Systems Design: Second Edition, Revised and Expanded, *Paul R. Yoder, Jr.*
36. Polarized Light: Fundamentals and Applications, *Edward Collett*
37. Rare Earth Doped Fiber Lasers and Amplifiers, *edited by Michel J. F. Digonnet*
38. Speckle Metrology, *edited by Rajpal S. Sirohi*
39. Organic Photoreceptors for Imaging Systems, *Paul M. Borsenberger and David S. Weiss*
40. Photonic Switching and Interconnects, *edited by Abdellatif Marrakchi*
41. Design and Fabrication of Acousto-Optic Devices, *edited by Akis P. Goutzoulis and Dennis R. Pape*
42. Digital Image Processing Methods, *edited by Edward R. Dougherty*
43. Visual Science and Engineering: Models and Applications, *edited by D. H. Kelly*
44. Handbook of Lens Design, *Daniel Malacara and Zacarias Malacara*
45. Photonic Devices and Systems, *edited by Robert G. Hunsperger*
46. Infrared Technology Fundamentals: Second Edition, Revised and Expanded, *edited by Monroe Schlessinger*
47. Spatial Light Modulator Technology: Materials, Devices, and Applications, *edited by Uzi Efron*
48. Lens Design: Second Edition, Revised and Expanded, *Milton Laikin*
49. Thin Films for Optical Systems, *edited by François R. Flory*
50. Tunable Laser Applications, *edited by F. J. Duarte*
51. Acousto-Optic Signal Processing: Theory and Implementation, Second Edition, *edited by Norman J. Berg and John M. Pellegrino*
52. Handbook of Nonlinear Optics, *Richard L. Sutherland*
53. Handbook of Optical Fibers and Cables: Second Edition, *Hiroshi Murata*

54. Optical Storage and Retrieval: Memory, Neural Networks, and Fractals, *edited by Francis T. S. Yu and Suganda Jutamulia*
55. Devices for Optoelectronics, *Wallace B. Leigh*
56. Practical Design and Production of Optical Thin Films, *Ronald R. Willey*
57. Acousto-Optics: Second Edition, *Adrian Korpel*
58. Diffraction Gratings and Applications, *Erwin G. Loewen and Evgeny Popov*
59. Organic Photoreceptors for Xerography, *Paul M. Borsenberger and David S. Weiss*
60. Characterization Techniques and Tabulations for Organic Nonlinear Optical Materials, *edited by Mark Kuzyk and Carl Dirk*
61. Interferogram Analysis for Optical Testing, *Daniel Malacara, Manuel Servín, and Zacarias Malacara*
62. Computational Modeling of Vision: The Role of Combination, *William R. Uttal, Ramakrishna Kakarala, Sriram Dayanand, Thomas Shepherd, Jagadeesh Kalki, Charles F. Lunskis, Jr., and Ning Liu*
63. Microoptics Technology: Fabrication and Applications of Lens Arrays and Devices, *Nicholas F. Borrelli*
64. Visual Information Representation, Communication, and Image Processing, *Chang Wen Chen and Ya-Qin Zhang*
65. Optical Methods of Measurement: Wholefield Techniques, *Rajpal S. Sirohi and Fook Siong Chau*
66. Integrated Optical Circuits and Components: Design and Applications, *edited by Edmond J. Murphy*
67. Adaptive Optics Engineering Handbook, *edited by Robert K. Tyson*
68. Entropy and Information Optics, *Francis T. S. Yu*
69. Computational Methods for Electromagnetic and Optical Systems, *John M. Jarem and Partha P. Banerjee*
70. Laser Beam Shaping: Theory and Techniques, *edited by Fred M. Dickey and Scott C. Holswade*
71. Rare-Earth-Doped Fiber Lasers and Amplifiers: Second Edition, Revised and Expanded, *edited by Michel J. F. Digonnet*
72. Lens Design: Third Edition, Revised and Expanded, *Milton Laikin*
73. Handbook of Optical Engineering, *edited by Daniel Malacara and Brian J. Thompson*
74. Handbook of Imaging Materials, *edited by Arthur S. Diamond and David S. Weiss*
75. Handbook of Image Quality: Characterization and Prediction, *Brian W. Keelan*
76. Fiber Optic Sensors, *edited by Francis T. S. Yu and Shizhuo Yin*
77. Optical Switching/Networking and Computing for Multimedia Systems, *edited by Mohsen Guizani and Abdella Battou*
78. Image Recognition and Classification: Algorithms, Systems, and Applications, *edited by Bahram Javidi*
79. Practical Design and Production of Optical Thin Films: Second Edition, Revised and Expanded, *Ronald R. Willey*

80. Ultrafast Lasers: Technology and Applications, *edited by Martin E. Fermann, Almantas Galvanauskas, and Gregg Sucha*
81. Light Propagation in Periodic Media: Differential Theory and Design, *Michel Nevière and Evgeny Popov*
82. Handbook of Nonlinear Optics: Second Edition, Revised and Expanded, *Richard L. Sutherland*
83. Polarized Light: Second Edition, Revised and Expanded, *Dennis Goldstein*
84. Optical Remote Sensing: Science and Technology, *Walter G. Egan*
85. Handbook of Optical Design: Second Edition, *Daniel Malacara and Zacarias Malacara*
86. Nonlinear Optics: Theory, Numerical Modeling, and Applications, *Partha P. Banerjee*

**Additional Volumes in Preparation**



# Preface

*Nonlinear Optics: Theory, Numerical Modeling, and Applications* is a self-explanatory book in a rather new and changing area, and is geared toward advanced senior or first-year graduate students in electrical engineering and physics. It is assumed that the students taking the course have had exposure to Fourier optics and electro-optics. This book is the culmination of a course on nonlinear optics that I have taught several times at the graduate level over the last ten years, and has also introduced some of the topics of senior-level classes on laser systems. It is also based on my research in the area over the last 20 years.

The unique features of the book are as follows. Students are first reacquainted with pertinent topics from linear optics that are useful in understanding some of the concepts used later on in the book. Thereafter, rigorous treatment of nonlinear optics is developed alongside a heuristic treatment to enable the reader to understand the underlying essential physics, instead of being overwhelmed with extensive tensor calculus. Recent topics of interests, applications, and measurement and calculation techniques are discussed. While the plane wave approach to harmonic generation is first explained, more recent developments such as the effect of beam profile on second harmonic generation, second generation during guided wave propagation, and the combined role of quadratic and cubic nonlinearities are also examined. Cubic nonlinearities are discussed at length along with their effects such as self-focusing and defocusing, self-bending of beams, and spatial solitons. The role of cascaded second-order nonlinearities is also examined. The  $z$ -scan technique and its modification are described in detail as a means of character-

ization of optical nonlinearities. We also discuss other cubic nonlinearity effects such as soliton propagation through nonlinear fibers, with some attention to the recent development of dispersion management in nonlinear optical fibers. Optical bistability and switching in a nonlinear ring cavity as well as during optical propagation across a linear/nonlinear interface are treated at length. Traditional topics such as stimulated Brillouin and Raman scattering are summarized. Also, phase conjugation in a cubically nonlinear material and dynamic holography are introduced. A simple  $k$ -space picture is used to explain phase conjugation of beams and pulses. Thereafter, the nonlinear optics of photorefractive materials is discussed in detail, including applications to dynamic holography, two-wave mixing, phase conjugation, and image processing. Photorefractive crystals as well as organic thin-film photorefractive materials are discussed. Examples of image processing such as edge enhancement using these materials are introduced. The nonlinear optics of liquid crystals is discussed at length, including the effects of applied electric and optical fields (including beams) on the nonlinearity. The effective nonlinearity of liquid crystals is determined from a careful evaluation of the position-dependent nonlinearity in the material. Self-organization plays a vital role in human behavioral system, in the brain, in fluid mechanics, in chemical reactions, etc.—in any system that has nonlinearity and feedback. It is therefore not unnatural to expect self-organization in optical systems as well. In this book, we therefore discuss spatiotemporal effects in nonlinear optical materials, leading to self-organization and spatial pattern formation, using photorefractives as a representative nonlinear medium. Innovative potential applications of self-organization are also presented. Finally, we treat photonic crystals or photonic bandgap structures that can be engineered to yield specific stop-bands for propagating waves, and demonstrate their application in optical bistability and hysteresis, soliton formation, and phase matching during second harmonic generation. Pertinent numerical methods, often used to analyze beam and pulse propagation in nonlinear materials, such as the split-step beam propagation method and the fully adaptive wavelet transform technique, are presented in the Appendices. Also, illustrative problems at the end of each of chapter are intended to aid the student in grasping the fundamentals and applying them to other interesting problems in nonlinear optics. In short, the book extends the concepts of nonlinear optics to areas of recent interest and, in a sense, brings contemporary and ongoing research areas not usually covered in many nonlinear optics books to the attention of readers.

The emphasis of this book is on the understanding of physical principles and potential applications. Students interested in further in-depth coverage of basics are referred to more comprehensive treatments such as the *Handbook of Nonlinear Optics* (Richard L. Sutherland, ed., Marcel Dekker, 2003).

I would like to thank Ms. Cheryl McKay from the University of Alabama in Huntsville for typing parts of the manuscript, my graduate student Ms. Jia Zhang for assistance with most of the figures, all of my graduate students whose work appears in the text who have worked with me through the years, and several students who took the course during the preparation of the manuscript for their helpful comments. Finally, I would like to thank my family and friends for their moral support.

*Partha P. Banerjee*

# Contents

<i>Preface</i>	<i>iii</i>
1 Optical Propagation in Linear Media	1
1.1 Maxwell's Equations	1
1.2 Linear Wave Propagation in Homogeneous, Linear Isotropic Media	4
1.3 Wave Propagation in Anisotropic Media	12
1.4 Diffraction	25
1.5 Dispersion	31
1.6 Problems	34
References	35
2 The Origin and Modeling of Optical Nonlinearity	37
2.1 A Simple Physical Model for Optical Nonlinearity	38
2.2 Physical Effects of Nonlinear Polarization	42
2.3 Mathematical Modeling of Optical Nonlinearities	43
2.4 An Alternative Approach	54
2.5 Summary	56
2.6 Problems	57
References	57
3 Second Harmonic Generation	59
3.1 The Physics of Second Harmonic Generation	59
3.2 SHG in Crystals—Mathematical Formulation	62
3.3 Phase Matching in Anisotropic Crystal	70
3.4 Nonlinear Transverse Effects in Second Harmonic Generation	71
3.5 SHG in a Medium with Second-Order and Third-Order Nonlinear Susceptibilities	73
	<i>vii</i>

3.6	Summary	76
3.7	Problems	77
	References	77
4	Self-Refraction of Optical Beams	79
4.1	The Nonlinear Refractive Index Coefficient $n_2$	80
4.2	Self-Refraction of Gaussian Beams	81
4.3	Focused Gaussian Beam in a Kerr-Slice: The $z$ -Scan Method	90
4.4	Self-Bending of Optical Beams	96
4.5	Self-Phase Modulation Due to $\chi^{(2)}$ : Effective $\chi^{(3)}$	97
4.6	Summary	98
4.7	Problems	99
	References	100
5	Optical Bistability	101
5.1	The Nonlinear Ring Cavity	101
5.2	Transverse Optical Bistability	109
5.3	Linear–Nonlinear Interface	110
5.4	Conclusion	119
5.5	Problems	120
	References	120
6	Optical Phase Conjugation	123
6.1	Comparison with Holography	124
6.2	The $k$ -Space Formalism	125
6.3	Semiclassical Analysis: CW Case	132
6.4	Phase Conjugation of Pulses and Transient Phase Conjugate Response	137
6.5	Discussion	142
6.6	Problems	142
	References	144
7	Stimulated Brillouin and Raman Scattering	145
7.1	Stimulated Brillouin Scattering	145
7.2	Stimulated Raman Scattering	151
7.3	Problems	153
	References	154
8	Solitons in Optical Fibers	155
8.1	Background on Linear Optical Fibers	156
8.2	Fiber Dispersion and Nonlinearity	158

8.3	Fiber-Optic Solitons and the NLS Equation	161
8.4	Dispersion Managed Soliton Communication Systems	164
8.5	Dark Solitons	166
8.6	Optical Shocks and Self-Steepening of Pulses	171
8.7	Problems	173
	References	173
9	Photorefractive Nonlinear Optics	175
9.1	Introduction	175
9.2	The Kukhtarev Equations and Their Simplification	177
9.3	Beam Fanning and Distortion in Photorefractive Materials	181
9.4	Two-Wave Mixing in Photorefractive Materials	194
9.5	Four-Wave Mixing and Phase Conjugation in Photorefractive Materials	196
9.6	Self-Phase Conjugation and Edge Enhancement	200
9.7	Organic Photorefractive Materials	202
9.8	Problems	206
	References	207
10	Nonlinear Optical Properties of Nematic Liquid Crystals	209
10.1	The Liquid Crystalline State of Matter	210
10.2	Classification of Liquid Crystals	210
10.3	Liquid Crystal Alignment	211
10.4	Principles of the Continuum Theory	213
10.5	Director Distribution of Homogeneously Aligned Nematic Liquid Crystal Under an External Electrical Field	215
10.6	Nonlinear Optical Properties from Optically Induced Molecular Reorientation	224
10.7	Optically Induced Reorientational Nonlinearity with External Voltage	228
10.8	Analysis of Beam Propagation in Liquid Crystals and $z$ -Scan	235
10.9	Problems	240
	References	241
11	Self-Organization in Nonlinear Optical Systems	243
11.1	Introduction	243
11.2	Nonlinear Two-Dimensional Systems with Feedback	245

11.3	Self-Organization in Photorefractive Materials	249
11.4	Theory of Self-Organization in Photorefractive Materials	255
11.5	Instability Criterion and the Dispersion Relation	257
11.6	Nonlinear Eigenmodes in the Steady State	259
11.7	Model of Hexagonal Formation Based on Transverse Electrical Instability	264
11.8	Potential Applications	265
11.9	Problems	267
	References	268
12	Nonlinear Optics of Photonic Bandgap Structures	271
12.1	Introduction	271
12.2	The Linear Dispersion Relation	272
12.3	The Effect of Cubic Nonlinearity	276
12.4	Second Harmonic Generation in Photonic Bandgap Structures	282
12.5	Problems	287
	References	288
<i>Appendix A</i>	<i>The Split Step Beam Propagation Method</i>	289
	<i>A.1 Examples</i>	290
	<i>References</i>	294
<i>Appendix B</i>	<i>Wavelet Transforms and Application to Solution of Partial Differential Equations</i>	295
	<i>B.1 Introduction to Wavelets</i>	295
	<i>B.2 Wavelet Properties and Scaling Functions</i>	296
	<i>B.3 Digital Filters and Multi-Resolution Analysis</i>	298
	<i>B.4 Application of Wavelet Transform to the NLS Equation</i>	302
	<i>References</i>	306
<i>Index</i>		307

# 1

## Optical Propagation in Linear Media

In this chapter, we will review some of the properties of optical waves propagating through an unbounded linear medium. We believe that this review will serve as an adequate foundation for the topics in nonlinear optics, to which the entire book is devoted. To this end, we enunciate Maxwell's equations and derive the wave equation in a linear homogeneous isotropic medium. We define intrinsic impedance, the Poynting vector and irradiance, as well as introduce the concept of polarization. We then expose readers to concepts of plane-wave propagation through anisotropic media, introduce the index ellipsoid, and show an application of electro-optic materials. We also summarize concepts of Fresnel and Fraunhofer diffraction, starting from the paraxial wave equation, and examine the linear propagation of a Gaussian beam. Finally, we expose readers to the important topic of dispersion, which governs the spreading of pulses during propagation in a medium. More importantly, we show how by knowing the dispersion relation, one can deduce the underlying partial differential equation that needs to be solved to find the pulse shapes during propagation. We hope this chapter presents readers with most of the background material required for starting on the rigors of nonlinear optics, which will be formally introduced in Chap. 2. For further reading on related topics, the reader is referred to Cheng (1983), Banerjee and Poon (1991), Goodman (1996), Yariv (1997), and Poon and Banerjee (2001).

### 1 MAXWELL'S EQUATIONS

In the study of optics, we are concerned with four vector quantities called electromagnetic fields: the electric field strength  $\mathbf{E}$  (V/m); the electric flux density  $\mathbf{D}$  (C/m<sup>2</sup>); the magnetic field strength  $\mathbf{H}$  (A/m); and the magnetic flux



density  $\mathbf{B}$  ( $\text{Wb}/\text{m}^2$ ). The fundamental theory of electromagnetic fields is based on Maxwell's equations. In differential form, these are expressed as

$$\nabla \cdot \mathbf{D} = \rho, \quad (1-1)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (1-2)$$

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}, \quad (1-3)$$

$$\nabla \times \mathbf{H} = \mathbf{J} = \mathbf{J}_C + \frac{\partial \mathbf{D}}{\partial t}, \quad (1-4)$$

where  $\mathbf{J}$  is the current density ( $\text{A}/\text{m}^2$ ) and  $\rho$  denotes the electric charge density ( $\text{C}/\text{m}^3$ ).  $\mathbf{J}_C$  and  $\rho$  are the sources generating the electromagnetic fields.

We can summarize the physical interpretation of Maxwell's equations as follows: Equation (1-1) is the differential representation of *Gauss' law for electric fields*. To convert this to an integral form, which is more physically transparent, we integrate Eq. (1-1) over a volume  $V$  bounded by a surface  $S$  and use the *divergence theorem* (or *Gauss' theorem*),

$$\int_V \nabla \cdot \mathbf{D} dV = \oint_S \mathbf{D} \cdot d\mathbf{S} \quad (1-5)$$

to obtain

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho dV. \quad (1-6)$$

This states that the electric flux  $\oint_S \mathbf{D} \cdot d\mathbf{S}$  flowing out of a surface  $S$  enclosing  $V$  equals the total charge enclosed in the volume.

Equation (1-2) is the magnetic analog of Eq. (1-1) and can be converted to an integral form similar to Eq. (1-6) by using the divergence theorem once again:

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0. \quad (1-7)$$

The right-hand sides (RHSs) of Eqs. (1-2) and (1-7) are zero because, in the classical sense, magnetic monopoles do not exist. Thus the magnetic flux is always conserved.

Equation (1-3) enunciates *Faraday's law of induction*. To convert this to an integral form, we integrate over an open surface  $S$  bounded by a line  $C$  and use Stokes' theorem,

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = \int_C \mathbf{E} \cdot d\mathbf{l}, \quad (1-8)$$

to obtain

$$\int_C \mathbf{E} \cdot d\mathbf{l} = - \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}. \quad (1-9)$$