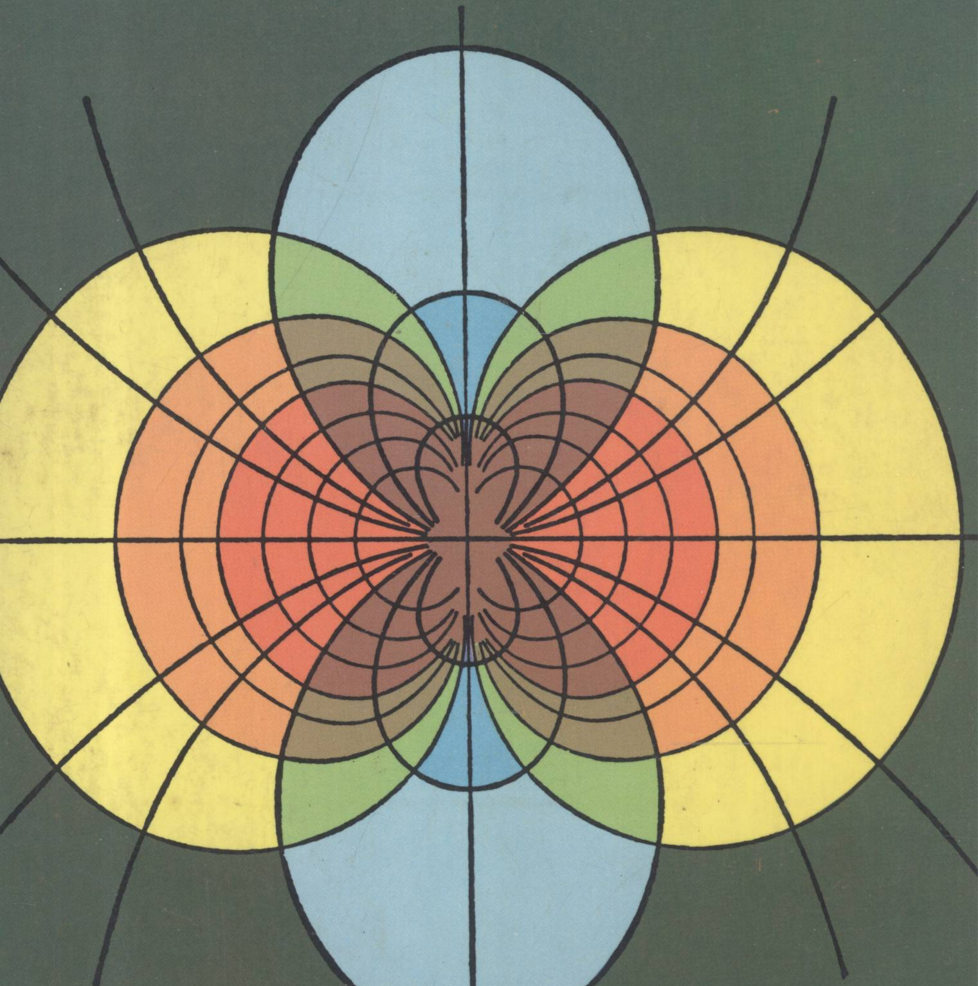
Richard Becker

ELECTROMAGNETIC FIELDS AND INTERACTIONS



With 148 Diagrams and Illustrations

ELECTROMAGNETIC FIELDS AND INTERACTIONS

RICHARD BECKER

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VOLUME I

Electromagnetic Theory and Relativity

FOREWORD

August Föppl's *Introduction to Maxwell's Theory* appeared in the year 1894. A completely revised second edition followed ten years later, this being the first volume of Max Abraham's *Theory of Electricity*. This, in turn, was followed a year later by a second volume on the electron theory. From the year 1930 onward, with the appearance of the eighth edition, Richard Becker took over the further editing of the work which, in succeeding years, underwent several basic changes.

With the sixteenth edition of the first volume and the eighth edition of the second, a thorough-going revision of the work, combined with an expansion to three volumes was planned by Becker. His sudden death, however, took him from the midst of his labours on this new revision.

In carrying on the work it was evident to me that Becker's plan should be continued. In remodelling—particularly in the first volume—there were many places in the previous first volume (Maxwell theory) and in the previous second volume (relativity theory) which were taken over practically unchanged. On the one hand, so as not to allow the new first volume to become too bulky, it was necessary in places to condense further and to make changes—on the other hand, however, it appeared necessary to write certain sections anew or to put them into a new form. Examples here are the sections on dipoles and quadrupoles and their radiation fields, and also the section on the forces on dipoles and quadrupoles in external fields. The sections on the energy relations and force effects in static fields were basically transformed, and with this in particular the Lorentz force, as an experimental formula, was placed at the forefront of the proceedings concerning force effects in the magnetic field.

As previously, so in the remodelling, the Gaussian CGS system of units is used. In remarks appended to certain sections, however, the more important formulae have been transcribed to the Giorgi MKSA system. In this way I hope to have suited in some measure the wishes of both physicists and engineers. The reason for preferring the Gaussian system to the volt-ampere system (which is employed more in practice) is that in the use of the first-mentioned system the formulae of both relativity theory and quantum mechanics can be written appreciably more simply. Moreoever, if the Giorgi system had been employed, the beautiful symmetry between the electric and magnetic-field quantities in the four-dimensional Minkowski formalism of special relativity would no longer be so evident.

I have attempted to bring out clearly the mutual relationship of the two measure systems. The transition from the four basic units of the MKSA system—units which at first sight seem to appear naturally—to the three basic units of the Gaussian CGS system, requires that Coulomb's law be not only considered as an experimental law, but that at the same time it be regarded as the defining equation for the unit of charge in the Gaussian system. This is because the resulting constant of proportionality, having the dimensions of force times length squared, divided by charge squared, is arbitrarily assumed to be dimensionless and equal to 1. This procedure, violently criticized by certain advocates of the MKSA system, corresponds exactly with today's custom in high-energy physics of combining the length and time dimensions with one another through the arbitrary assumption that the velocity of light in a vacuum is dimensionless and equal to 1.

Since, in the following, the equations of electrodynamics are on the one hand to be understood as quantity equations and not as relationships between the numerical values of physical quantities in special units, and since on the other hand these equations are transcribed part by part in different ways in the two unit systems employed here, only a few of all the symbols employed represent the same physical quantity in both measure systems. Examples are the symbols of kinematic and dynamic quantities, and of electric charge, electric polarization, and electric field strength. In opposition to this (though a fact often overlooked) the symbols of electric displacement as well as those of total magnetic-field quantities, have different meanings in the two systems. Table 6 gives a summary of the relationships between such quantities customarily described by the same symbol, and it also gives the conversion factors between them. I hope that by means of this table, and also through the presentation of the corresponding relationships in the text, my readers—and in particular those who are students—will be helped through the customarily troublesome process of going over from one unit system to the other.

The translation is the work of Mr. A. W. Knudsen.

F. SAUTER

COLOGNE, 1964

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Introduction to vector and tensor calculus

CHAPTER A I

Vectors

§ 1. Definition of a vector

The science of electric and magnetic phenomena prior to the appearance of Maxwell's theory was based on the concept of action at a distance between bodies which are electrified, magnetized, or traversed by electric currents. Only the ideas of Faraday differed in this respect from those of other physicists in that he conceived all electric and magnetic actions of one body upon another separated from it as the effect of an electric and/or magnetic field existing between the bodies. Although his manner of interpreting and describing the phenomena was basically a mathematical one, Faraday was nevertheless unable to give his interpretation sufficient completeness and freedom from contradiction to have it raised to the rank of a theory. In this, success was first achieved by Maxwell, who gave Faraday's ideas rigorous mathematical form and thereby created a theoretical structure which, as a *field-action theory*, is essentially different in conception from the *action-at-a-distance theory*.

Maxwell himself formulated his equations in Cartesian coordinates and only incidentally made use of quaternion theory. A general overall view of the interconnection of all formulae is however considerably facilitated by the use of *vector calculus*. This method of calculating would appear as if created for the task of representing in the best way possible the ideas of Faraday. The labour expended in becoming familiar with the method is certainly outweighed by the advantages gained. At the beginning of this work, therefore, we give an account of the theory of vectors and vector fields, together with a short section on tensors which occasionally appear in Maxwell's theory.

The simplest physical quantities are those which are completely determined in known units by specifying a single measure-number. They will be called *scalars*. Examples are mass and temperature. In general we shall represent them by Latin or Greek letters.

In addition there are physical quantities whose establishment requires the employment in a known way of three specifying numbers, as for example the displacement of a point from a given initial position. We could characterize this displacement by specifying the three Cartesian coordinates of the end-position with respect to an origin placed at the initial point, from there on calculating with these scalar displacement components. If we did this, however, we should in the first place be neglecting the fact that, physically speaking, a displacement is a single idea; and secondly we should be bringing a foreign element into the question, namely the coordinate system, which has nothing to do with the displacement itself. For the description of displacements, therefore, we shall with advantage introduce quantities of a new type, definable without reference to a coordinate system, and we shall establish appropriate rules for their use. Only in the numerical evaluation of formulae will it be necessary to introduce a definite coordinate system.

The rectilinear displacement of a point, as well as all other physical quantities which, like displacements, are uniquely established by specifying their direction in space and their magnitude, and which obey the same addition rule as displacements, we call *vectors*. In the following we shall always designate vectors by bold-face letters. Thus **A** stands for a vector, graphically representable as an arrow, and $|\mathbf{A}| = A$ its magnitude, designated by the length of the arrow.

§2. Addition and subtraction of vectors

For adding two displacements **A** and **B** we imagine a movable point which is initially at position 1 (figure 1). To this point is first assigned the

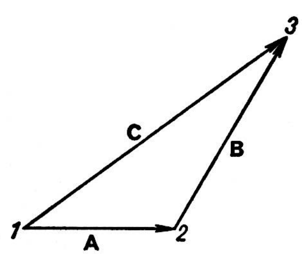

Fig. 1.—Vector addition

displacement **A** from 1 to 2; then the point is moved through displacement **B** corresponding to the interval from 2 to 3. Now the rectilinear