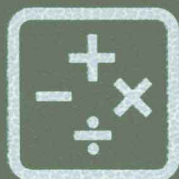

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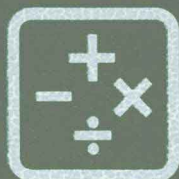
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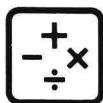


THE

WORLD BOOK OF

MATH

POWER



Volume 1

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Introduction

Math power is a term important for everyone. We use mathematics every day. Telling time from a clock or counting the change returned by the grocer requires math skills. So do a number of games and hobbies. Many people do not realize that managing a household or business successfully are directly related to their ability to use math properly. Math power can mean the difference between confidence and insecurity; vitality and boredom; goals and regrets.

What *Math Power* offers

Math Power was prepared with two goals in mind: first, to provide a close and thorough look at modern mathematics as a vital part of home, school, and work life; second, to provide a complete review of math coursework and show how best to take advantage of opportunities to study and use mathematics, in and out of school.

The two-volume set is divided into seven parts. Each part has a different focus. The first four parts comprise the *Math Skills Builder*.

Part I, “Relaxing with Math,” is designed to help you feel at ease with math by showing how math is a natural part of the way you think. Your first math challenge also comes in this part. You are shown how to do “quick math,” or mental arithmetic, to speed up the computation process and make your math more accurate.

Part II puts mathematics into perspective for students and parents alike. “School Math: Learning the Third *R*” talks about how children first start learning math as preschoolers and how the school mathematics curriculum builds on mathematics concepts the children bring with them to kindergarten or first grade. In this part, the history of mathematics education in the United States is also presented, giving some insights into what the future of mathematics instruction might be. How mathematics can influence career success is also covered in Part II.

“Tips and Tools for Math Students,” Part III, is a handy, well-organized aid for students who wish to excel in mathematics. Studying, taking tests, controlling math anxiety—all are covered here. Special features in this part are sections on calculators and computers: tools that are becoming increasingly important in the study and performance of mathematics.

For those who wish a sound review, Part IV, the “Complete Math Review Program,” provides dozens of math lessons and exercises that will recap the coursework in mathematics from kindergarten through grade 12. This part is not only useful to students but also to adults who have found that their math skills are inadequate to meet their needs.

Volume II, *Everyday Math*, puts your new math skills into practice. The volume opens with Part V, “Putting Math to Work.” This part shows how math is used in the workshop and kitchen, on the road, and in personal and business affairs. Then Part VI, “Math Marvels,” brings the extraordinary world of mathematical thought to you in the form of everyday entertainments. Have you ever thought of a jacket and vest as having entertainment potential? Would you like to make up words like *googolplex* or *maximillion*? Have you ever played Sprouts or Pentominoes? You will learn how to do so in Part VI.

Math Power ends with Part VII, the “Math Powerhouse.” It is a fingertip source for all sorts of mathematical information. Math history opens the part, and next comes an overview of U.S. math competitions that encourage mathematical excellence through individual and team effort. Following are math symbols, formulas, and tables, as well as summaries of mathematical operations. A section on math reading gives you an excellent list of math books for independent work. Each book entry is *annotated*, or described, for you.

How to use *Math Power*

Presented in these volumes are many aids to understanding math. Using them properly might determine how much you will learn. Make sure, for example, to read all the introductory material, both to each part and to each section. You will feel far more comfortable if you know the direction your study is about to take. And make sure you have a mathematics notebook and pencil or pen handy at all times. Some lessons call for more equipment, including compass, protractor, and ruler.

Precisely follow all explanations and directions. With them are hundreds of diagrams and other illustrations. Study them well. When appropriate, copy them.

Unless your math is strong, don't jump from one part to another. Start at the beginning of volume 1 and go straight through. Stop, relax, and reread when concepts seem difficult; math learning can't be hurried.

If you need to review a concept but can't remember where it was, use the index at the end of volume 2. It is a final aid in the mathematical journey these books hold in store.

Part I

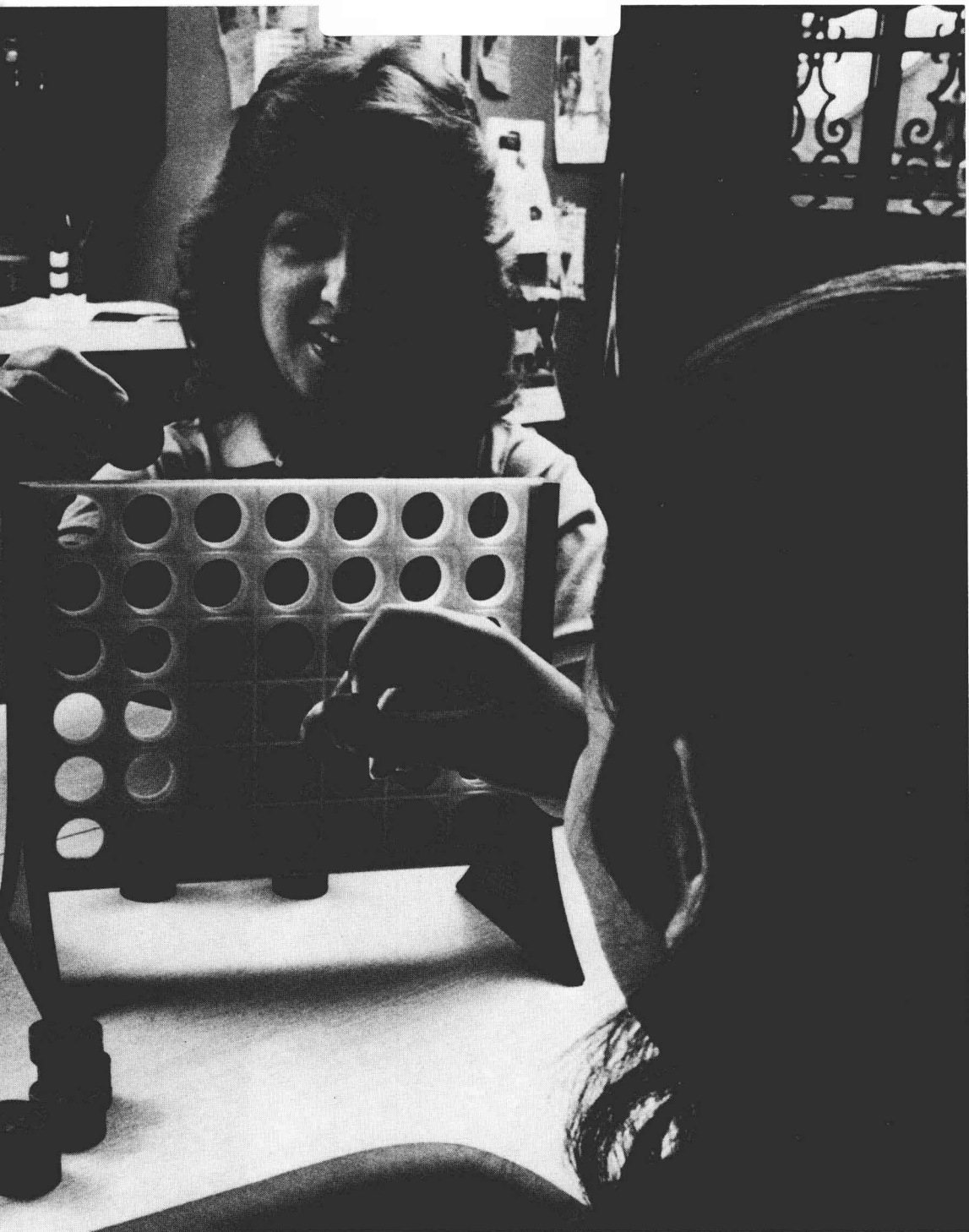
Relaxing with Math

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Many people, especially those who have “always had trouble with math,” think that it is impossible to relax with mathematics. What they probably don’t know is that math is a part of every life. People come by math understandings very early and very naturally. In other words, everyone relaxes with math quite often.

Math is much more than adding up long columns of figures and working algebra formulas. These exercises merely sharpen math skills that you already have, just as speaking and writing develop language skills. Mathematics includes so many different subjects that it is hard to define. One definition that fits most of the mathematics you learn in school is that mathematics is the study of quantities and relations through the use of numbers and symbols. Does that sound difficult? Then think of it this way. Whenever you rush to be on time, take two aspirins, or select the “biggest piece of fudge,” you are using math. See how mathematically inclined you are?

You are about to explore your everyday world with mathematics as a lively companion instead of a silent partner, as it may have seemed until now. First, you will learn how math is not only part of you, but of animals about you, as well. Then, you will see how you can “touch” math. Finally, you will find out how you can speed up math operations—or skip them entirely—and still arrive at correct or usable solutions to mathematical problems, perhaps without even using pencil and paper. Sound impossible? Relax, you’ve got what it takes. You see, the math is already in your mind.■

(Preceding photo)
Math games are a favorite pastime for many families.

The Math in Your Mind

Counting 13
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Patterns 19
Logic and Proof 20

From an early age, all human beings use some mathematics, even in cultures that have no written language. Other animals also use mathematics. Crows have been known to keep track of up to thirty persons. Bees can measure angles and lengths. And almost all animals learn to recognize shapes and sizes. (Yes, shapes and sizes are a part of math.) Rabbits must learn, for example, the shape of a flying hawk so that they can take cover. They must also learn the shapes of edible leaves. For animals, mathematics means survival.

Humans are probably born with some very basic mathematical abilities. With no teaching whatsoever, almost anyone can tell the difference between one object and two objects, know that one object is much larger than another, and recognize the difference between a circle and a triangle. But higher levels of mathematics require training. You must learn special techniques to tell the difference between 137 sheep and 141 sheep, or between a liter of water in a pail and a quart of water in a carton. The purpose of mathematics education is to build on inborn abilities and gradually take them to higher levels.

Counting

How can you be sure that crows count? In farming areas, crows can be a nuisance because they eat young plants. Scarecrows sometimes help keep crows away, but often the only way to get rid of crows is to shoot them. Crows are smart, however. If they see a person with a shotgun, they won't in-

vade the field until the person leaves. You see, crows recognize the shape of a shotgun. So to shoot crows, a farmer may build a hiding place in the field, called a *blind*. Even then the crows are hard to fool. If they see a farmer enter the blind, they won't attack the corn until the farmer leaves.

One farmer had an "easy" solution to this problem. Two people would enter the blind, but only one would come out. The person who was left would shoot the crows when they flew into the field. But when the plan was tried, the crows did not come into the field until the second person left the blind.

More help was needed. Three people went into the blind and two came out. The crows were not fooled. Four people going in and three coming out did not fool the crows either. At this point, everyone became very curious about how high the crows could count. So the farmer in charge asked more people to enter the blind. It was not until thirty people entered the blind and twenty-nine came out that the crows were fooled into the field. That is, the crows had finally "lost count."

What is counting? Adult human beings can usually count up to five objects without any special technique. A person can look at a stack of four or five books and tell how many there are without actually counting. If a stack contains six or seven books, however, the person must count in order to tell their exact number. Counting is done by matching each book in the stack with a number name. People learn different number names and rules for combining the names to form numbers in order from one upward. A person may count the books by saying, "One, two, three, four, five, six, seven." The person matches each number name with one of the books in the stack. If seven is matched with the last book in the stack, it tells how many books there are.

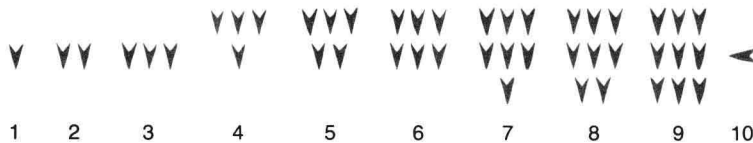
Crows, however, probably "count" by the mental technique that humans use for five or fewer objects. Since crows cannot use language, they have developed the ability to judge larger quantities by sight.

The number-name method was probably not the first way that humans used to count. Long ago, humans probably used sets of objects to match things they wanted to count. For example, a shepherd who wanted to make sure that all the sheep were safe for the night could match each sheep with a pebble and keep the pebbles in a bag. Each night, the shepherd could check to see if there was a sheep for each pebble and a pebble for each sheep. In that way, the sheep were counted, even though no number name was used. The matching process was more important than the use of number names.

Along trade routes in the Middle East, archeologists have found hollow clay balls filled with markers. The archeologists

believe that ancient merchants used these balls to tell buyers how many items they had sent. For example, if seventeen bars of copper were shipped from Cyprus to Turkey, a ball containing seventeen markers would be shipped, too. When the shipment got to Turkey, the buyer could break open the ball, match the markers with the bars of copper, and know if the proper amount had arrived safely.

Eventually, the markers were shown as dents on the outside of the ball, so people could check the number along the way without breaking the ball open. The clay was baked hard after the dents were made so no new dents could be added or old dents removed. The dents became the first system of writing numbers, the *cuneiform* system. In fact, people developed ways to write numerals before they developed ways to write words. About five thousand years ago, Babylonians used numerals that looked like this



Measurement

The counting process results in a whole set of numbers—1, 2, 3, 4, 5, and so forth—that can go on indefinitely. These numbers are often called the *counting numbers* or the *natural numbers*. The counting numbers are the basis of all numbers, but they are not enough to solve all the mathematical problems that might arise.

Suppose, for example, that the merchant in Cyprus had more than enough copper to make 16 bars but not enough to make 17 bars. If the merchant wanted to ship all the copper, he would need a way to show that he was sending sixteen whole bars and one partial bar. How could he relate this information to the buyer? The answer is to use what today are called fractions.

Fractions are numbers but they are different from the counting numbers. If two partial bars make one whole bar, then each bar is a half, or $\frac{1}{2}$, of a whole bar. If three partial bars make one whole, then the size of each partial bar is a

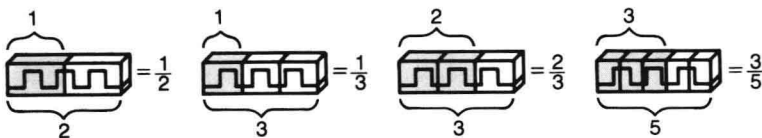
third, or $\frac{1}{3}$. In each case, a *measurement* takes place. The merchant is *measuring* the size of the partial bar in terms of the whole number 1. Fractions thus allow the merchant to measure the partial quantity against the whole quantity.

Things may become a bit more complicated for the merchant. Perhaps the amount of leftover copper he wishes to send will not “go evenly” into one bar. For example, it will take three partial copper bars to make up two (not one) whole bars. The easy solution is to use the fraction $\frac{2}{3}$.



A fraction is always a way of showing a relationship between two numbers—the number of parts and the number of wholes. If you had any difficulty following the example of the merchant, try this: You divide a candy bar and give your friend one-half. He has half of the whole bar. You just split the whole down the middle, in other words. Neither one of you is confused because fractions are a natural part of the way you think. You share via fractions.

One number in a fraction tells how many parts the whole was divided into. The other number tells how many parts are in the piece being measured. Here is a new way of thinking about the merchant’s copper bars using fractions to split the bars mentally.



Can you split a candy bar into halves? thirds? fifths? Then you can use fractions correctly, and you understand the mathematical concept ratio. A *ratio* between two quantities is the number of times one contains the other. Since fractions show a ratio of two numbers, mathematicians call fractions the *rational numbers*. You don’t need mathematicians to explain this concept to you. You’ve been using it for years.