MODERN COLLEGE ALGEBRA

With Applications

Ronald D. Jamison

MODERN COLLEGE ALGEBRA
With Applications

Ronald D. Jamison
Brigham Young University

HBJ

HARCOURT BRACE JOVANOVICH, Inc.
New York / Chicago / San Francisco / Atlanta

To my sweet Ann

Figures drawn by Rino Dussi
Cover adapted from photo by Norman Rothschild

ISBN: 0-15-560415-5

Library of Congress Catalog Card Number: 75-21841

Printed in the United States of America

MODERN COLLEGE ALGEBRA With Applications

Preface

This book is a result of nearly twenty years of teaching during which I have made a conscientious effort to understand the processes by which students learn. I have tried to reflect what I have learned in ways that provide a pleasant yet effective format for learning: The writing is informal yet precise and addresses the reader directly; a two-color treatment of text and illustrations focuses attention on key elements and procedures and stresses essential concepts and formulas; marginal notes (placed next to their referents) show the rationale for essential steps in proofs and solutions; and five categories of problem sets offer a wide variety of learning experiences.

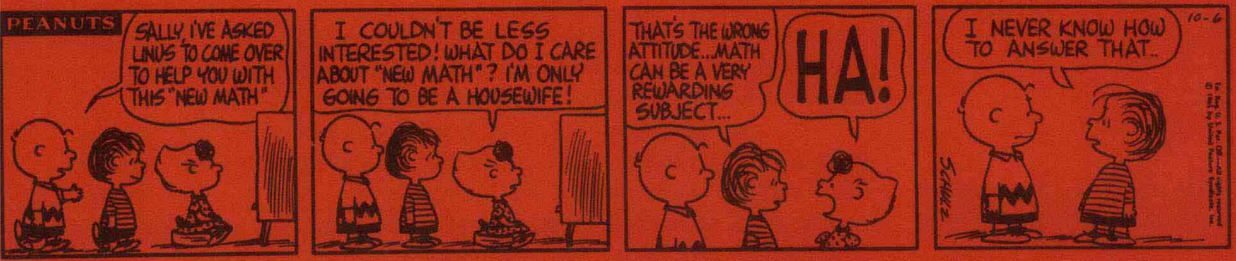
Because many students never see beyond algebraic manipulations to the real role of mathematics, Chapter 0 discusses the meaning, purpose, and role of mathematics in today's world. For most students, Chapter 1, on elementary logic and set theory, will be a review; in addition, it establishes certain notational forms that are used throughout the book. Chapter 2 treats basic properties of the real-number system, so essential to arithmetic and algebraic manipulation. This, too, will be a review and can be covered briefly, except for Sections 2.9 through 2.12, which examine more deeply the structure and properties of the real-number system. The remaining chapters may be studied in a sequence determined by curriculum requirements or the individual needs of the student. I suggest that the core chapters (0-4) be read in sequence. Mathematics or physical science majors may study the remaining chapters in any sequence, with the exception of Chapter 7, which should be preceded by Chapter 6. The same is true for business or social science majors, except that Chapter 12 may be omitted, as may Sections 2.9 through 2.12 and 7.10 through 7.11.

With respect to the problem sets, I recommend that all problems in the Reading Comprehension sets be used, since they cover basic principles, manipulation techniques, and frequently misunderstood concepts. A selection of problems from each of the other categories – Skills Development, Theoretical Developments, Applications, and Just For Fun – will provide a balanced learning experience. Because most readers of this book will be interested mainly in its applicability to non-mathematical training or employment, the problems are taken from the business world and the physical and social sciences – both for motivational reasons and to give first-hand experience in solving problems pertinent to student interests. Solutions, with appropriate comments, are provided for all Reading Comprehension problems. For all other problem sets except Just For Fun, answers to one-third (those numbered in color) are worked out in detail, and answers only are given for another third; Just For Fun problems are not answered in the book.

My sincere thanks go to the following, who reviewed the original manuscript and offered many valuable suggestions that have significantly affected the final form of the text: Charles L. Murray and Kenneth Eberhard, both of Chabot Junior College; Robert A. Nowlan, Southern Connecticut State College; Janet Ray, Seattle Central Community College; and Richard Phillips, Michigan State University. I also thank my colleagues Steven Cottrell and Diana Armstrong for proofreading the final manuscript and galleys and for working out all the problems—a monumental task. And finally, my deepest thanks and love to my wife, Ann, who typed the manuscript from rough draft to completion, and without whose help and steady support this project would never have been realized.

Ronald D. Jamison

Contents



CHAPTER 0 Mathematics

0.1 Introduction

Today mathematics is one of the fastest growing and most radically changing sciences. Although new discoveries often render former theories obsolete in many scientific disciplines, nearly all the major mathematical developments of the past 4 to 5 thousand years are still valid and useful. Today, notwithstanding these countless contributions of the past, mathematics is experiencing a deluge of new, fresh ideas and ingenious discoveries unparalleled in all its history. In fact it has been estimated that if one could measure all the mathematical knowledge accumulated from the dawn of time to the year 1950, this amount would at least be equaled by that produced from 1950 to 1964. Moreover, if the present trend continues (and we have every reason to believe it will) we expect that within the next 10 to 15 years the total amount of mathematical knowledge accumulated through 1964 will be redoubled. Each year 18 to 20 thousand mathematical articles published in over a thousand scientific journals attest to this phenomenal growth.

This advancement has both inspired and been inspired by the increasingly widespread use of mathematical systems and logic in fields outside mathematics. For example, Boolean algebra • has recently been applied in designing telephone circuits and electronic computers. In biology we now find mathematical models of the nerve impulse and of energy transfer in predation. Social economist Richard Stone proclaims:

• Boolean algebra was developed by the English mathematician George Boole in the late 1840s.

> In every one of the social sciences it has become increasingly evident that an exclusively verbal description of complex systems and their interrelations results in generalizations that are difficult to analyze, compare, and apply. These difficulties are greatly reduced when *mathematical expressions are substituted for words*. For one thing, a number of problems that had seemed to be completely unrelated prove to be mathematically identical. For another, even in subjects whose concepts are rather vague and in which precise information is hard to find, mathematics can provide a means of obtaining valuable insights. •

• Richard Stone, "Mathematics in the Social Sciences," in *Mathematics in the Modern World,* W. H. Freeman, San Francisco, 1968, pp. 284–293.

Richard Bellman, commenting on the widespread application of mathematics, says, "In industry, control theory, implemented by the computer, is now widely used to regulate inventories, to schedule production lines and to improve the performance of power stations, steel mills, oil refineries and chemical plants." •

• Richard Bellman, "Control Theory," *Scientific American,* September 1964, Vol. 211, pp. 186–200.

With mathematical systems and techniques now being applied more extensively than ever before, one would expect most mathematical research to be taking place in applied mathematics. This, however, is not the case. That is, the vast majority of all mathematical advancements today are found in pure mathematics. The distinction between *applied* and *pure* mathematics is not always clear, however. One reason for this is that the two overlap considerably; another is that pure mathematics today often becomes applied tomorrow as more frequent and varied uses are encountered. In general, however, we may consider applied mathematics as that part that encompasses those systems and methods that are utilized in analyzing and solving physical problems—problems related to people and the physical world—whereas pure mathematics deals with abstract systems that may or may not have any immediate application or relevance to the physical world.

Many mathematicians spend months, years, and even lifetimes expanding the frontiers of pure mathematics, and this has been the cause of some ill-conceived ridicule. Those who taunt pure mathematics either have forgotten or are not aware that history, particularly modern history, is replete with incidents of pure mathematical developments becoming the very cornerstones upon which have been built some of our most prestigious and honored accomplishments. Take, for example, the theory of functions of a complex variable (a variable containing the "imaginary" number i, sometimes denoted $\sqrt{-1}$). For many years these numbers were considered useless—unable to measure or describe anything quantitatively (hence the name, imaginary). Today the theory of complex functions is basic and virtually indispensable in the study of electricity and magnetism, radio and television, and the motion of satellites and planets—and these are but a few of the many areas where complex numbers are utilized.

Another very important example of a pure mathematical system becoming applied occurred early in the twentieth century when Albert Einstein (1870–1955) developed his famous general theory of relativity, using non-Euclidean geometry as an essential mathematical tool. The particular form of non-Euclidean geometry Einstein chose was originally developed by the famous German mathematician Georg Friedrich Bernhard Riemann (1826–1866). This geometry • does not possess lines of infinite length, although they are endless, nor lines that are parallel—a geometry quite distinct from that to which most of us are accustomed. You can get a feeling for the nature of this geometry by thinking of lines as being great circles on a sphere. (See Fig. 0.1.1.) Note that a spherical triangle of this space can have interior angles with a sum *greater* than 180°.

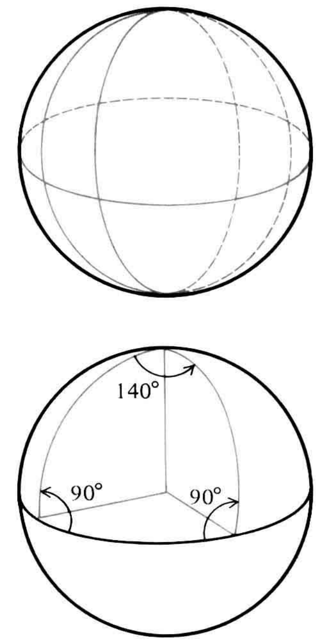


FIGURE 0.1.1

These are only two of an almost endless array of occurrences where pure mathematics has become applied. Thus, continued research in both pure and applied mathematics offers stimulating and exciting prospects.

In brief, modern mathematics, both pure and applied, is playing an increasingly vital role in nearly every facet of our eternal quest to understand the marvelous world that surrounds us.

• P. LeCorbeiller, "The Curvature of Space," in *Mathematics in the Modern World,* W. H. Freeman, San Francisco, 1968, pp. 128–133.

The Nature of Mathematics 0.2

The extent of mathematical discovery and application seems, as never before, to be boundless. Yet, surprisingly few people understand the real nature and purpose of mathematics. Many look upon the subject as just a study of various numbers and an attempt to skillfully perform endless calculations. Some envision mathematics as an enormous conglomeration of strange symbols and peculiar looking equations. Others view the mighty computing machines of today as the epitome of mathematical prowess. Mathematics is none of these, although each is used by the mathematician.

Admittedly any attempt to define or describe mathematics would surely fail—fail just as a purely verbal characterization of music or art would fall pitifully short of its intended mark. Mathematics, like music and art, is simply much broader than mere language can adequately describe. To understand these subjects, one must become personally involved with much of the substance of which they are composed.

Although the substance of mathematics fills volumes and can be very difficult, there are some basic and fundamental concepts underlying the very heart of the subject that most everyone can comprehend. Understanding these will add measurably to your vision and appreciation of mathematics. These concepts will also provide a suitable beginning from which we may move more naturally into the main body of this text.

Mathematics, despite its awesome reputation, is the simplest and clearest of studies dealing with the laws of thought and reasoning. We are all mathematicians of a sort. How often, when in a conversation with another, have you found yourself asserting that such and such is true and then following that assertion with a sequence of facts, each designed to help establish or prove your point? It may have gone something like this: "Sure I'm going to do better in algebra this year; I have not only attended every class but also completed every problem assignment." Or perhaps, "I feel confident that Mary Ann will win the election for Studentbody Vice-President — she is a very popular member of the Pep Club, a standout on the debate team, and simply adored by everyone."

Proclaiming something true or valid under certain conditions, then proceeding to gather supporting facts and presenting them in an orderly, logical manner is probably the most fundamental characteristic of mathematics. Throughout history man has always welcomed and even sought out the kinds of problems that would challenge his keenest insights and produce clever feats of ingenuity. Many popular games require reasoning, concentration, and logic in order to consistantly win, for example, chess, bridge, three-dimensional tic-tac-toe, and Concentration.

Mathematics in its simplest form, deals with reasoning and the laws of thought, which are usually devoted to the proof of an assertion or to the solution of a problem. This thinking process often takes place without the use of numbers and extensive calculations. In fact, many mathematicians see very little relationship between mathematics and schoolbook arithmetic.

As an illustration, let us take the celebrated problem of the Koenigsberg bridges (Fig. 0.2.1). This problem was first solved by the most eminent of Switzerland's mathematicians, Leonhard Euler (pronounced oiler), in 1735. He wrote,

> In the town of Koenigsberg there is an island called Kneiphof, with two branches of the river Pregel flowing around it. There are seven bridges crossing the two branches. The question is whether a person

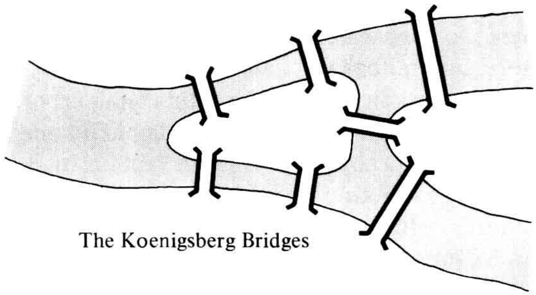


FIGURE 0.2.1

> can plan a walk in such a way that he will cross each of these bridges once, but not more than once. On the basis of the above, I formulated the following very general problem for myself: Given any configuration of the river and the branches into which it may divide, as well as any number of bridges, to determine whether or not it is possible to cross each bridge exactly once. ●

● Leonhard Euler, "The Koenigsberg Bridges," in *Mathematics in the Modern World*, W. H. Freeman, San Francisco, 1968, pp. 141–142.

Before reading further, see if you can determine for yourself the answer to Euler's original problem, using, if you wish, Fig. 0.2.1.

If you are like most people, your first impulse is to sketch several paths as you attempt to find one satisfying the required conditions. After each unsuccessful effort, you may begin to suspect that such a path does not exist. To be sure, one way to verify your suspicion would be to test all possible paths, but this manner of resolving the problem would not only be very tedious but would shed little light on how to solve a similar though more general problem, such as that suggested by Euler. One can often save time and effort by first carefully analyzing the problem. This would include identifying its defining characteristics, examining the effect or limitations these characteristics impose upon the situation, and then, by reason and logic, determining a solution, if possible. Let us apply this approach to the above problem (which you may have already done successfully).

Observe that there is an *odd* number of bridges adjoining each portion of land. From this we conclude that a walk could not end where it began. Why? Of course that observation itself poses no difficulty because the other three portions of land where the walk may terminate remain. However, where the path may end does present a problem. Note that if a path does not originate in an area joined by an *odd* number of bridges, then necessarily the path must terminate there. Why? Therefore, *if* a continuous (unbroken) path were to exist crossing each bridge once and only once, the path would have to end in all three remaining but separate portions of land – but that, of course, is *impossible*. Thus we assert that no path exists satisfying the required conditions.

Euler's argument was more refined and detailed than the one presented here; furthermore, it answered similar questions for considerably more general cases.

An important lesson to learn from this analysis of the Koenigsberg bridges problem is the essential role played by one's own ability to reason carefully in problem solving. In fact, as mentioned earlier, logical and orderly thinking lie at the very heart of mathematics. One may inquire, "How do you develop, improve, and best employ these human virtues?" The answer is—*use them.* Face up to the challenge of a wide variety of problems. For this reason, a plentiful supply of thought provoking and relevant problems is provided in the exercises throughout this book. In addition, many sections and even some chapters exist primarily to provide certain guidelines and ground rules associated with logical thought patterns and carefully organized reasoning sequences.

Before leaving this perspective of mathematics, an additional comment is in order. Despite our best efforts, we all err in our reasoning from time to time. This brings to mind a story of an energetic young man who, after digging a deep hole, decided to fill it back up. Upon finishing the project he was amazed to find dirt left over—dirt that originally occupied space in the hole before any digging took place. Scratching his head and wondering for a moment how this could be, he finally concluded that he simply had not dug the hole deep enough! Well, perhaps his back was a bit stronger than his brain. Yet we are all susceptible to various forms of illogical thinking. A primary objective of this text is to assist you in becoming more adept in the reasoning processes, particularly as they relate to problem solving—and, after all, the primary reason that most people study mathematics is to improve their ability to solve problems.

0.3 The Language of Mathematics

It would be convenient indeed if all problems could be resolved by simply thinking them through carefully as we did with the Koenigsberg bridges problem. Of course, this is not the case. Most of the really significant and challenging problems we face today are difficult even to understand, let alone to solve. We need some kind of device first to help us comprehend the nature of some problems and second to assist us in discovering their solutions. Herein lies the major role of the language of mathematics. As languages go, mathematics would certainly rank among the simplest—there are no conjugations, declensions, tenses, nor genders, and there is only a minimum of grammar. Yet it is a language, a means of communication, with some rather unique characteristics not enjoyed by most other languages.

The essential function of nearly all languages is to communicate in either the spoken or written form. Mathematics, however, is used almost exclusively as a written language. Moreover, rather than providing a means of communication between people, it is generally a vehicle by which problems of various sorts show more clearly and precisely their character and solution to those investigating them.

To illustrate, let us take the following example. Suppose the liquid in the radiator of a car were 18 percent antifreeze and 82 percent water, giving protection, say, down to 18° (Fahrenheit). Further, suppose with winter coming on, it was necessary to have protection down to −10°, which requires a solution that is 36 percent antifreeze. If the capacity of the cooling system of the car were 20 quarts, how much liquid should be drained off and replaced by antifreeze in order to obtain the desired 36 percent concentration? (See Fig. 0.3.1.)

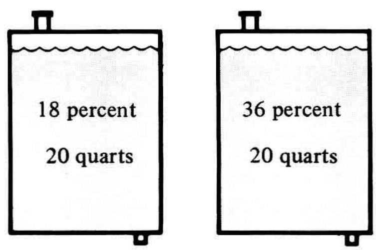


FIGURE 0.3.1

To solve this problem, we must first be certain the meaning of the given information is clearly understood. That is, a concentration of 18 percent means 18 one-hundredths or eighteen parts in each one hundred. Thus, the amount of antifreeze initially in the radiator would be 18 percent of 20 quarts (0.18 times 20) or 3.6 quarts. Of course, a portion of that amount will be lost in the liquid drained off. Because we do not know, for the moment, how much liquid to release, let us denote that unknown quantity by the letter x. Observe, as you analyze the problem, that the amount of antifreeze remaining in the radiator after draining off x quarts of the original coolant is $0.18(20-x)$. Further observe, if to this amount we add x quarts of antifreeze, we obtain the desired 36 percent solution. We now *translate* this observation into the language of mathematics. It would read

$$\overbrace{0.18(20-x)}^{\substack{\text{the amount of antifreeze} \\ \text{left in the tank}}} + \overbrace{x}^{\substack{\text{the amount} \\ \text{added}}} = \overbrace{0.36(20)}^{\substack{\text{the amount} \\ \text{required}}} \qquad (1)$$

This is an equation. Such an expression is often called a mathematical model of the problem. One advantage of this form is the