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# FUNDAMENTALS OF MULTICOMPONENT DISTILLATION

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**Charles D. Holland**

*Texas A&M University*

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TO MY WIFE  
Eleanore

This book was set in Times Roman.  
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The production supervisor was Leroy A. Young.

## **FUNDAMENTALS OF MULTICOMPONENT DISTILLATION**

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7 8 9 10 11 12 BRBBRB 92 9 8 7 6 5 4 3 2 1 0

### **Library of Congress Cataloging in Publication Data**

Holland, Charles Donald.  
Fundamentals of multicomponent distillation.

(McGraw-Hill chemical engineering series)  
Includes bibliographical references and index.

1. Distillation. I. Title.

TP156.D5H64 660.2'8425 80-29238  
ISBN 0-07-029567-0

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# **FUNDAMENTALS OF MULTICOMPONENT DISTILLATION**

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# PREFACE

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This book constitutes an in-depth treatment of the subject of multicomponent distillation. It begins with first principles and goes to the frontiers of the subject. Each topic is introduced in an elementary and fundamental manner which makes the book suitable for the undergraduate student, the graduate student, and the practicing engineer. The subject matter is presented in the order of increasing difficulty and complexity.

The gap between the treatment of binary and multicomponent mixtures is closed in Chap. 1. This chapter is initiated by presenting the fundamental relationships and techniques needed for making bubble-point and dew-point calculations, and it is concluded by the presentation of techniques for solving a variety of special types of problems such as the separation of a multicomponent mixture by a single-stage flash process and the separation of a multicomponent mixture by use of multiple stages at the operating condition of total reflux.

In Chaps. 2 through 5, the theta methods and variations of the Newton-Raphson method are applied to all types of single columns and systems of columns in the service of separating both ideal and nonideal solutions. Applications of the techniques presented in Chaps. 2 through 5 to systems of azeotropic and extractive distillation columns are presented in Chap. 6. An extension of these same techniques as required for the solution of problems involving energy exchange between recycle streams is presented in Chap. 7. Special types of separations wherein the distillation process is accompanied by chemical reactions are treated in Chap. 8.

In Chap. 9, all of the techniques developed in Chaps. 1 through 8 are brought to bear in the design and operation of conventional and complex distillation columns. To complete the in-depth treatment of multicomponent distillation, the special topics of total reflux, minimum reflux, design of valve and sieve trays, plate efficiencies, design of packed columns, thermodynamic relationships, and selected numerical methods are presented in Chaps. 10 through 15. A Solutions Manual may be obtained (without cost) by Faculty members by writing directly to me or to McGraw-Hill.

Since the writings of any author are influenced by all that he has met, I am deeply indebted to all of my teachers, fellow faculty members, students, and the many past and present workers in the field of distillation. In particular, I wish to thank Professors P. T. Eubank and D. T. Hanson of the Department of Chemical Engineering as well as Professors Emeriti K. C. Klipple and H. A. Luther of the Department of Mathematics for their many helpful suggestions. The direction and advice generously offered by N. J. Tetlow and Ron McDaniel of Dow Chemical Company is appreciated immensely. I also want to thank Dr. Tetlow for his contributions to Chap. 14 (Thermodynamic Relationships for Multicomponent Mixtures) as coauthor. The helpful suggestions provided by W. L. Bolles of Monsanto are very much appreciated. For the support of the research (upon which much of this book is based) by David L. Rooke, Donald A. Rikard, and Holmes H. McClure (all of Dow Chemical Company), I am most thankful. Also, support of research provided by the Texas Engineering Experiment Station and the Center for Energy and Mineral Resources is appreciated. To both former and present graduate students (Najeh S. Al-Haj-Ali, G. W. Bentzen, Andy Feng, S. E. Gallun, Alejandro Gomez M., J. R. Haas, F. E. Hess, Alicia Izarraraz, P. E. Mommessin, and G. P. Pendon) who participated in this research, I salute you for your many contributions and I shall always be indebted to you. Finally, I want to pay special tribute to my Staff Assistant, Mrs. Wanda Greer, who helped make this book possible through her loyal service and effective assistance over the past eight years in the discharge of my administrative responsibilities.

*Charles D. Holland*

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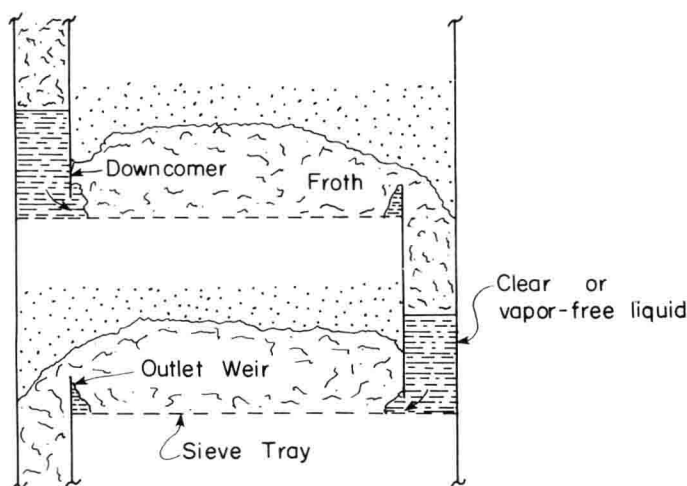
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## INTRODUCTION TO THE FUNDAMENTALS OF DISTILLATION

In this chapter, the fundamental principles and relationships involved in making multicomponent distillation calculations are developed from first principles. To enhance the visualization of the application of the fundamental principles to this separation process, a variety of special cases are considered which include the determination of bubble-point and dew-point temperatures, single-stage flash separations, multiple-stage separation of binary mixtures, and multiple-stage separation of multicomponent mixtures at the operating conditions of total reflux.

The general objective of distillation is the separation of compounds that have different vapor pressures at any given temperature. The word *distillation* as used herein refers to the physical separation of a mixture into two or more fractions that have different boiling points.

If a liquid mixture of two volatile materials is heated, the vapor that comes off will have a higher concentration of the lower boiling material than the liquid from which it was evolved. Conversely, if a warm vapor is cooled, the higher boiling material has a tendency to condense in a greater proportion than the lower boiling material. The early distillers of alcohol for beverages applied these fundamental principles. Although distillation was known and practiced in antiquity and a commercial still had been developed by Coffey in 1832, the theory of distillation was not studied until the work of Sorel<sup>14</sup> in 1893. Other early workers were Lord Rayleigh<sup>11</sup> and Lewis.<sup>8</sup> Present-day technology has permitted the large-scale separation by distillation of ethylbenzene and *p*-xylene, which have only a 3.9°F difference in boiling points (Ref. 1).



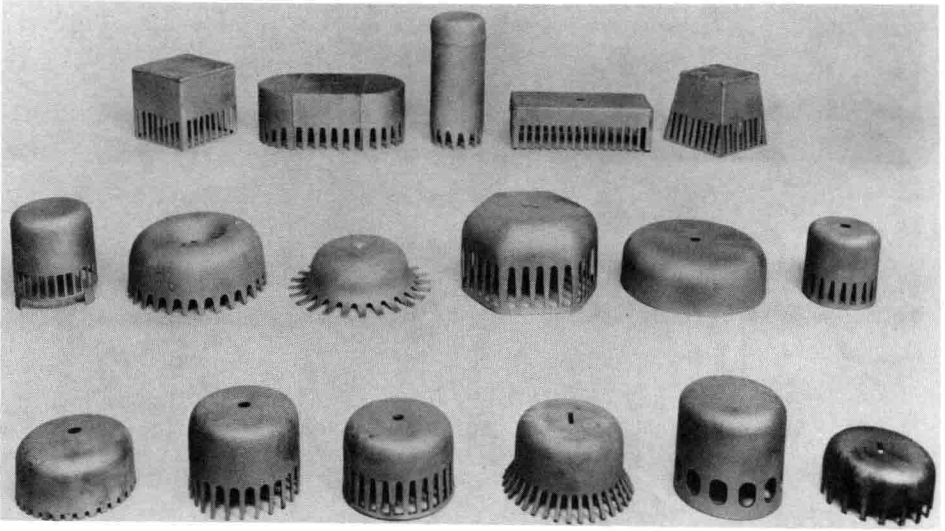
**Figure 1-1** Interior of a column equipped with sieve trays.

A distillation column consists of a space for contacting vapor and liquid streams for the purpose of effecting mass transfer between the two phases. Although the contacting of two phases is generally effected by a series of plates (or trays), packed columns are becoming more widely used as discussed in Chap. 13. However, in the development of the fundamentals of the various calculational procedures in this and subsequent chapters, it is supposed that the column is equipped with plates.

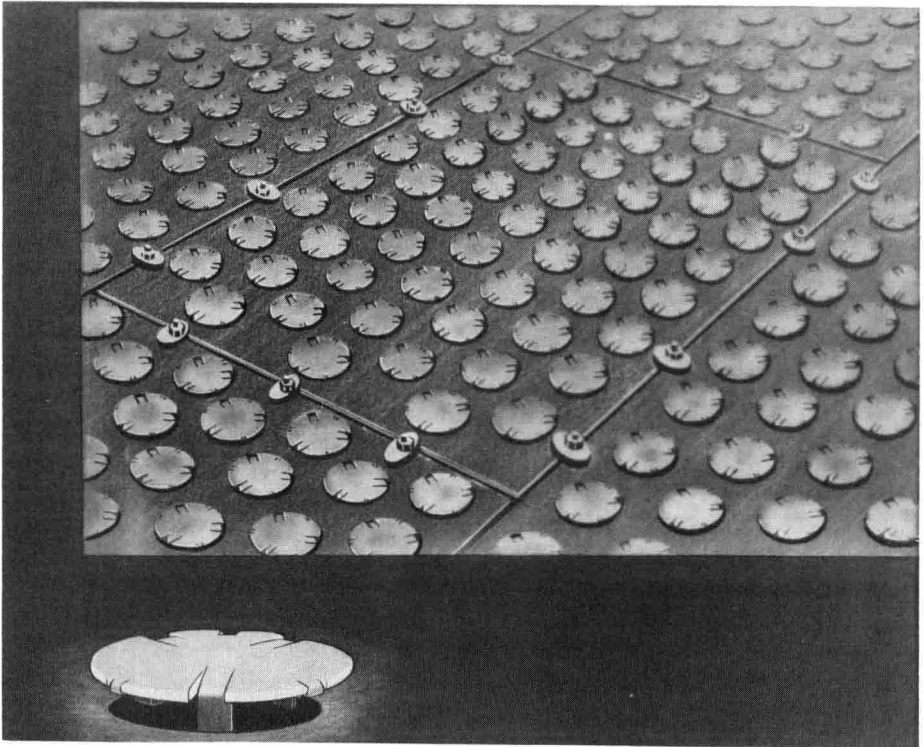
In normal operation, there is a certain amount of liquid on each plate, and some arrangement is made for ascending vapors to pass through the liquid and make contact with it. The descending liquid flows down from the plate above through a downcomer, across the next plate, and then over a weir and into another downcomer to the next lower plate as shown in Fig. 1-1. For many years, *bubble caps* were used for contacting the vapor with the liquid. A variety of designs of bubble caps are shown in Fig. 1-2. These contacting devices promote the production of small bubbles of vapor with relatively large surface areas.

Over the past 20 years, most of the bubble-cap trays have been replaced by other types of contacting devices. New columns are usually equipped with either *valve trays* (see Fig. 1-3) or *sieve trays* (see Fig. 1-1), sometimes called *perforated trays*. In valve trays, the valve opens wider as the vapor velocity increases and closes as the vapor velocity decreases. This feature of opening and closing allows the valve to remain immersed in liquid and thereby preserve a liquid seal over wide ranges of liquid and vapor flow rates.

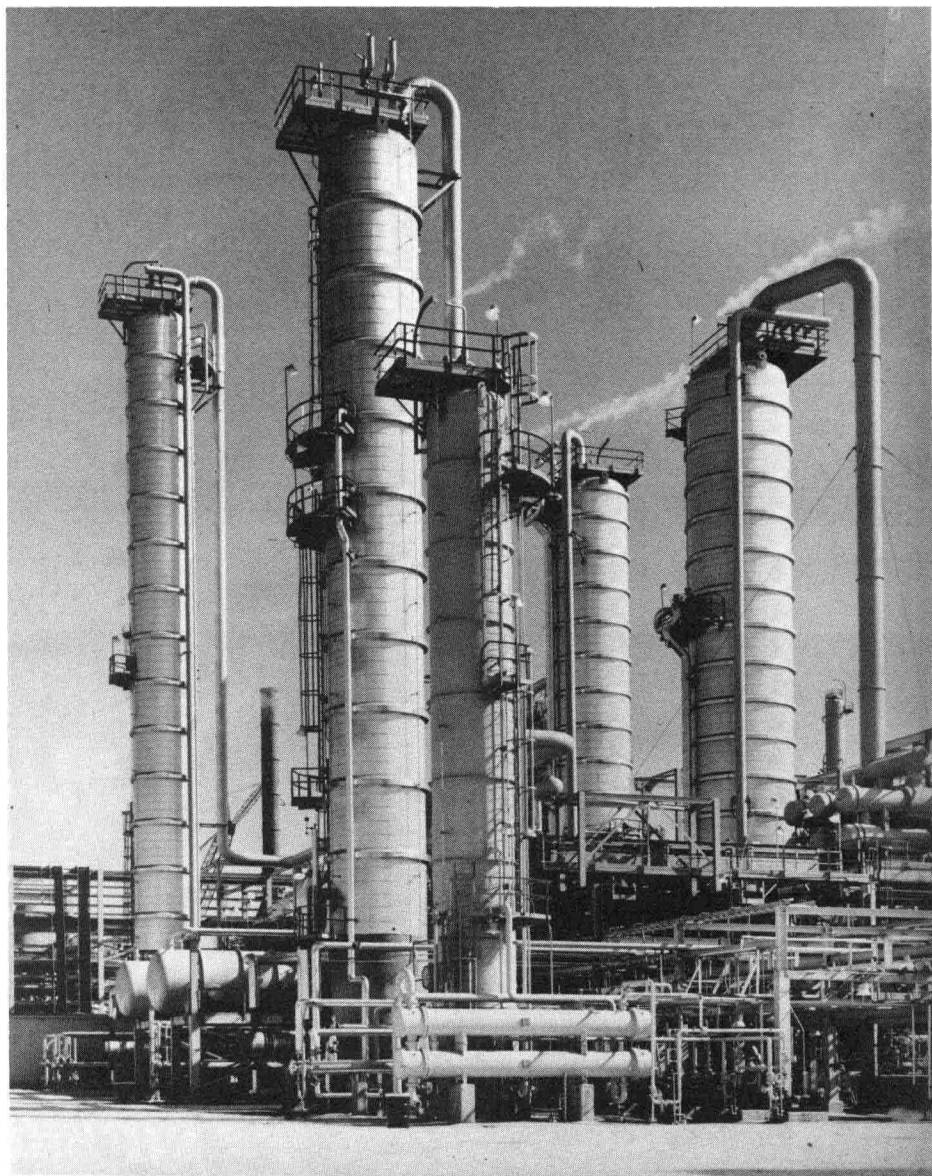
Distillation columns have been built as high as 338 feet. Diameters as large as 50 feet have been used. Operating pressures for distillation columns have been reported which range from 15 mmHg to 500 lb/in<sup>2</sup> abs. A typical commercial installation is shown in Fig. 1-4.



**Figure 1-2** Various types of bubble caps used in distillation columns. (*By courtesy of Glitsch, Inc.*)



**Figure 1-3** Portion of a Glitsch V-1 ballast tray. (*By courtesy of Glitsch, Inc.*)



**Figure 1-4** Typical view of distillation columns at the Gulf refinery at Alliance, Louisiana. (By courtesy Gulf Oil Corporation and Glitsch, Inc.)