INTRODUCTORY COLLEGE MATHEMATICS

HACKWORTH
and
HOWLAND

SAUNDERS SERIES IN MODULAR MATHEMATICS

Tables and Graphs

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ROBERT D. HACKWORTH, Ed.D.
Department of Mathematics
St. Petersburg Junior College at Clearwater
Clearwater, Florida

and

JOSEPH HOWLAND, M.A.T.
Department of Mathematics
St. Petersburg Junior College at Clearwater
Clearwater, Florida

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PREFACE

Tables and Graphs

This book is one of the sixteen content modules in the Saunders Series of Modular Mathematics. The modules can be divided into three levels, the first of which requires only a working knowledge of arithmetic. The second level needs some elementary skills of algebra and the third level, knowledge comparable to the first two levels. *Tables and Graphs* is in level 1. The groupings according to difficulty are shown below.

Level 1	Level 2	Level 3
Tables and Graphs	*Numeration*	*Real Number System*
Consumer Mathematics	*Metric Measure*	*History of Real Numbers*
Algebra 1	*Probability*	*Indirect Measurement*
Sets and Logic	*Geometric Measures*	*Algebra 2*
Geometry	*Statistics*	*Computers*
		Linear Programming

The modules have been class tested in a variety of situations: large and small discussion groups, lecture classes, and in individualized study programs. The emphasis of all modules is upon ideas and concepts.

Tables and Graphs is appropriate for all non-science majors especially social science students. The module is also well suited for math-science and technical students.

Tables and Graphs begins by explaining the common properties of tables and graphs. The emphasis is on skill in reading and constructing bar graphs, line graphs, circle graphs, and scattergrams. Also included is the use of graphing in selecting optimum opportunities in situations with a number of alternatives.

In preparing each module we have been greatly aided by the valuable suggestions of the following excellent reviewers: William Andrews, Triton College, Ken Goldstein, Miami-Dade Community College, Don Hostetler, Mesa Community College, Karl Klee, Queensboro Community College, Pamela Matthews, Chabot College, Robert Nowlan, Southern Connecticut State College, Ken Seydel, Skyline College, Ara Sullenberger, Tarrant County Junior College, and Ruth Wing, Palm Beach Junior College. We thank them and the staff at W. B. Saunders Company for their support.

Robert D. Hackworth
Joseph W. Howland

NOTE TO THE STUDENT

OBJECTIVES:

Upon completing this unit, the reader is expected to be able to demonstrate the following concepts and skills:

1. An ability to read and compare tabular data.

2. An ability to read and construct bar graphs, line graphs, and circle graphs.

3. An ability to use and construct problem solving graphs of business situations.

4. To evidence knowledge of trend lines and the correlation involved with scattergrams.

5. An ability to find the value of money invested at compound interest.

Three types of problem sets, with answers, are included in this module. Progress Tests are always short with four to six problems. The questions asked in Progress Tests come directly from the material of the section immediately preceding the test.

Exercise Sets appear less frequently in the module. More problems are in an Exercise Set than in a Progress Test. These problems arise from all sections of the module preceding the Exercise Set. Problems in Section I of each Exercise Set are specifically chosen to meet the objectives of the module. Problems in Section II of each Exercise Set are challenge problems.

A Self-Test is found at the end of the module. Self-Tests contain problems representative of the entire module.

In learning the material, the student is encouraged to try each problem set as it is encountered, check all answers, and restudy those sections where difficulties are discovered. This procedure is guaranteed to be both efficient and effective.


TABLES of PROFITS
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TIME
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CONTENTS

TABLES AND GRAPHS

INTRODUCTION

With the possible exception of basic arithmetic, no topic in mathematics is as important for the average citizen as an ability to read and interpret tables and graphs. The daily newspaper contains a variety of tables and graphs. Almost any non-fictional book uses tables and graphs. Even the visual displays on television frequently contain graphs and tables to better communicate their messages.

There are three major reasons for the importance of tables and graphs. First, materials in tables and graphs are organized and classified in a manner designed to promote understanding. Organizing and classifying data is a major step in any problem-solving or analysis situation. To the extent that a table or graph provides appropriate groupings to its data, the table or graph enables the reader to achieve a faster, easier understanding of its material.

A second reason for the importance of tables and graphs is the visual appeal they have for the reader. An old newspaper cliche is "a picture is worth a thousand words" and that quote is true in reporting numerical data as it is for any other newspaper subject.

The third reason for the importance of tables and graphs is the extent to which they are used in the evaluation, prediction, and decision-making processes. It is commonly believed that tables and graphs simply transmit information, but that view is unnecessarily narrow. In this module, an effort is made to broaden the uses or tables and graphs and show their value for disclosing similarities, contrasts, and future trends.

The content of the tables and graphs of this module came from current events with special efforts made to choose topics which are of broad general interest. Topics include the environment, the economy, the costs of a college education, the physical growth of adolescents, and future population projections. The tables in the latter half of this module show both business applications and computational tools for the mathematician.

THE BASIC CONCEPT OF A TABLE

The basic idea of most graphs and tables is to show a relationship between two or more categories of information. This is often accomplished by representing one type of information in a vertically written list and the other type of information in a horizontally written list. Each position in a table shows the relationship between its horizontal and vertical headings. The information or numbers shown in a table are called entries.

Table 1 illustrates the idea of horizontally and vertically listed categories of information. The table shows federally funded programs for the improvement of the environment and public health. The first item in the vertical listing is "Improved Public Transportation." The last item in the vertical listing is "Research on Pollutants." The first item in the horizontal listing is "Fiscal Year 1972" and the last item is "Percent Change."

TABLE 1

FEDERALLY FUNDED PROGRAMS IN DOMESTIC HEALTH AND IMPROVEMENT OF THE ENVIRONMENT

(All amounts in millions of dollars)

	Fiscal Year 1972	Fiscal Year 1973	Change 72-73	Percent Change
Improved Public Transportation	$456	$666	$+210	46
Cancer Research	337	430	+93	27
Production of Electricity without Pollution	392	480	+88	22
Research & Development in Education	142	197	+55	39
Safety Programs for Natural Disasters	93	136	+43	46
Poverty Programs	103	141	+38	31
Research on Pollutants	115	154	+39	34

The table of federal programs correlates some of the methods for improving the health and environment with the amounts of money provided for the separate programs. According to the table, 337 million dollars were spent in fiscal year 1972 for cancer research. This information is found using the horizontal row of "Cancer Research" and the vertical column of "Fiscal Year 1972." Where the row meets the column, is the entry "337" which means 337 million dollars. Notice that directly below the title of the table there is a statement that each amount is in millions of dollars.

The table shows that the fastest growing program in terms of money spent was "Improved Public Transportation." This fact is established by comparing the numbers in the column headed "Change 1972-73." In terms of percent, the fastest growing programs are "Improved Public Transportation" and "Safety Programs for Natural Disasters" because each shows 46 percent change.

The next table is taken from an article comparing the economics of the southern United States with the United States as a whole. This table is representative of the presentation of data in many newspapers, magazines, and textbooks. As in the last example, the table is read using its horizontal listings in conjunction with the vertical listings. There are four columns in the table; two columns contain information for 1960 and the other two columns contain information for 1970. There are also two sections of horizontal information in the table. The upper section shows amounts of money in billions of dollars. The lower section shows the percent of distribution for the dollar amounts of the upper section.

PERSONAL INCOME IN THE UNITED STATES
AND THE SOUTH, 1960 and 1970

	1960		1970	
	United States	South	United States	South
	(Billions)			
Farm	$ 15.0	$ 3.7	$ 19.0	$ 4.9
Nonfarm wage and salary	266.1	40.7	533.5	93.5
Other Labor income	11.0	1.6	30.8	5.1
Nonfarm Proprietor's income	34.2	5.6	51.0	8.7
Property Income	52.4	7.2	113.0	18.0
Transfer Payments	29.5	5.1	79.6	14.9
Less: Personal payments for Social Insurance	-9.2	-1.4	-28.0	-4.7
	$399.0	$62.3	$796.9	$140.4
	(Percent Distribution)			
Farm	3.8	5.9	2.4	3.5
Nonfarm wage and salary	66.7	65.2	66.8	66.6
Other Labor income	2.8	2.5	3.9	3.6
Nonfarm Proprietor's income	8.6	9.0	6.4	6.2
Property Income	13.1	11.5	14.1	12.8
Transfer Payments	7.4	8.2	10.0	10.6
Less: Personal payments for Social Insurance	-2.3	-2.3	-3.5	-3.4
TOTAL	100.00	100.00	100.00	100.00

Note: Detail may not add to total because of rounding.

Source: U.S. Department of Commerce, Office of Business Economics, Survey of Current Business, August, 1963 and August, 1971

The first horizontal line of the table shows that farm income in the United States in 1960 was 15 billion dollars, in the South in 1960 it was 3.7 billion dollars, in the United States in 1970 farm income was 19 billion dollars, and in the South in 1970, it was 4.9 billion dollars.

Property income is shown in the fifth horizontal line of the upper section of the table. That line shows that property income in the United States in 1970 was 113 billion dollars. Transfer payments were the sixth horizontal lines in both the upper and lower sections of the table. To find the percent of income attributable to transfer payments in the South in 1970, the reader should find the transfer payments line in the lower section of the table and read the entry in the last column on the right of the table. That entry is 10.6 and means that 10.6 percent of the personal income in the South in 1970 had been from transfer payments.

Progress Test 1

Below is shown a tabulated table in an advertisement. Use the table to answer the following questions.

1. What is the price of the machine that handles conversions?

2. How many models have the square roots feature?

3. If a customer wants a model only for adding, subtracting, multiplying, and dividing, which model is the most economical?

4. What two features are available on every model?

	Model No.	12R	21R	51R	61R	63R	80R	82R
Features	Price	$29.95	$49.95	$99.95	$79.95	$99.95	$139.95	$169.95
Adds, subtracts, multiplies, divides		√	√	√	√	√	√	√
Algebraic logic		√	√	√	√	√		
Floating decimal		√	√	√	√	√	√	√
Built-in rechargeability			√	√	√	√		
Memory			√	√√	√	√		√√
Percent key			√					√
Square roots		√			√	√		
Log and trig functions					√	√		
Scientific notation and parentheses						√		
Conversions—metric, fractions, etc.				√				
Printed tape							√	√

BAR GRAPHS

In a graph the horizontal and vertical listings are usually arranged along two perpendicular lines. These lines are often called axes. In figure 1 below, a pair of axes is shown. The horizontal axis is labeled by years and the vertical axis is labeled by dollars.

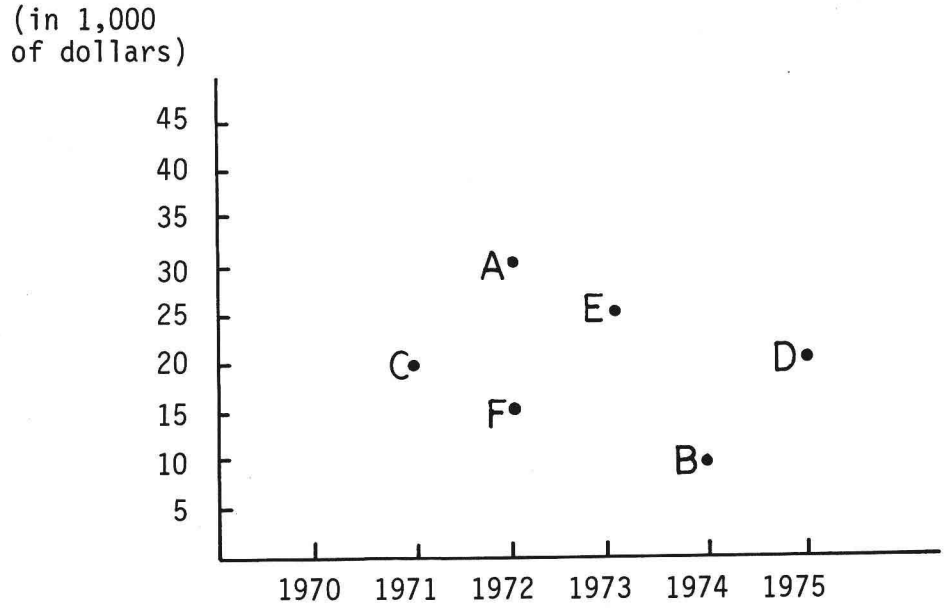

Figure 1

The point A on the graph represents two types of information. Because point A is directly above 1972 on the horizontal axis and directly across from 30 on the vertical axis, point A represents both 1969 and $30,000. (Note: numbers on the vertical axis represent thousands of dollars.) Point A therefore correlates or represents a relationship between the year 1972 and the dollar amount $30,000.

Similarly, point B in figure 1 represents both 1974 and $10,000. Each point of the graph shows a dual relationship between the years and the dollar amounts. Point B shows that for the year 1974 there is a matching dollar amount of $10,000 , but it also shows that for a dollar amount of $10,000 there is a matching

year of 1974. The relationship is reversible because the year can be matched to the dollar amount or the dollar amount matched to the year.

Points C and D lie on the same horizontal line. Consequently, both points are associated with $20,000; C matches $20,000 to 1971 and D matches $20,000 to 1975. Notice that the points C and D match $20,000 to two different years, while the years 1971 and 1975 match to a single dollar amount.

Figure 2 is shown below and illustrates a simple bar graph.

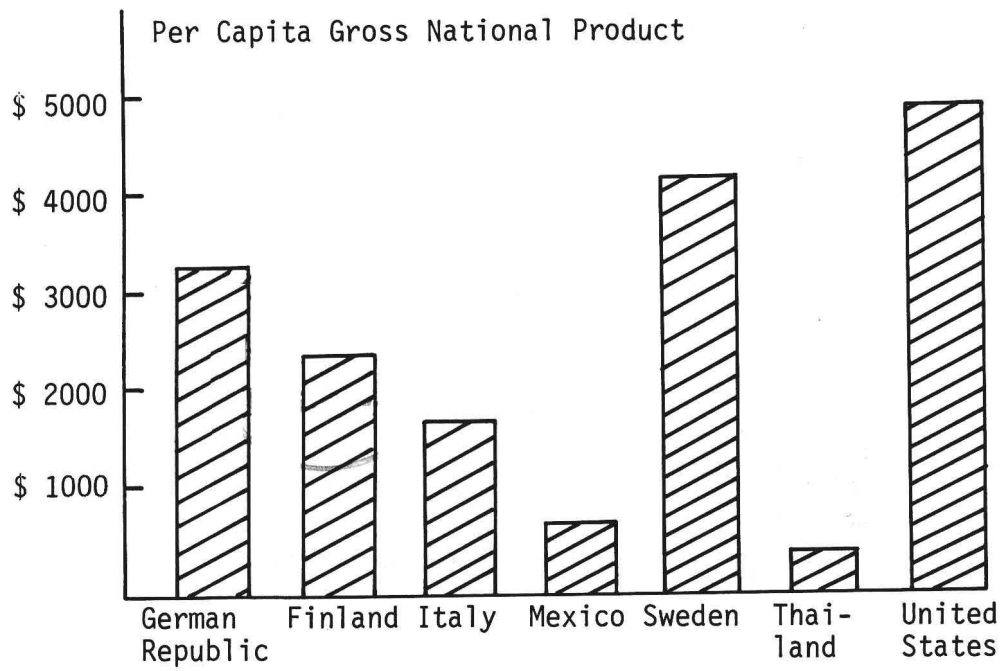

Figure 2

In Figure 2, the vertical axis is labeled in thousands of dollars and the horizontal axis lists seven selected countries. The heading for the graph is "Per Capita Gross National Product" which means the average economic production of the individual citizen.

The fourth vertical bar from the left represents Mexico. It is approximately two-thirds of the height of $1000 on the vertical axis and means that the per capita gross national product in Mexico is approximately $650.

By comparing the heights of the bars in Figure 2, the greatest per capita income is found in the United States because it has the longest bar,and the smallest per capita income is found in Thailand because it has the shortest bar.

How does the average production of the citizens of the German Republic and Sweden compare? The average citizen in the German Republic produces about \$3,300 and the average citizen in Sweden produces about \$4,200 or \$900 more per person. \$900 is $\frac{900}{3,300}$ = 27% rounded to the nearest percent. The average citizen in Sweden produces about 27% more than the average citizen in the German Republic.

Figure 3 is another bar graph in which the horizontal axis is marked in years and each separate bar is labeled by a dollar amount. The graph shows the annual tuition costs in the public universities of Florida. Most of the bars in the table are darkened to indicate that figures for those years have been established. The bar for 1975-76 is indicated by an undarkened area with a dotted line. The meaning for the year 1975-76 is that this is a predicted amount. The table shows both data of the past and prediction for the near future.

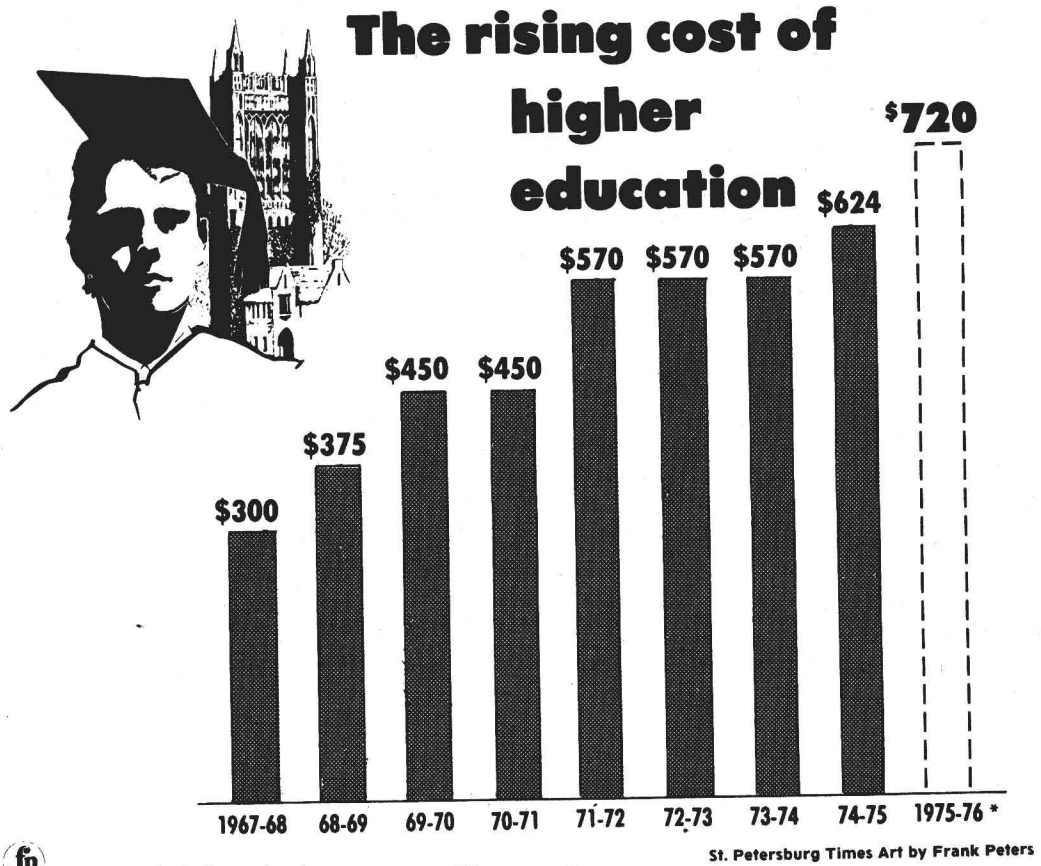

Figure 3

Progress Test 2

1. Use Figure 2 to rank in order, highest to lowest, the countries in terms of their per capita gross national products.

2. Use Figure 3 to find the year in which the greatest dollar increase in tuition costs was experienced.

3. Use the following information on per capita gross national product to make a bar graph like that in Figure 2.

   Canada $3676; France $2901; Israel $1836; Kenya $140;

   Japan $1911; Switzerland $3135; and Venezuela $979.

Exercise Set 1

1. 1. The table below shows the premiums on liability insurance for an accountant. The size of his total office staff affects the size of the premium.

ACCOUNTANT'S PROFESSIONAL LIABILITY

Rates for all states excepting New York

Total Staff	Deductible	$25,000	$50,000	$100,000	$250,000	$500,000	$1 Mill.
1	250.	80.	90.	100.	125.	160.	200.
2	250.	100.	112.	125.	156.	200.	250.
3	250.	120.	135.	150.	188.	240.	300
4	250.	140.	155.	175.	219.	280.	350.
5	250.	160.	180.	200.	250.	320.	400.
6	500.	—	194.	215.	269.	344.	430.
7	500.	—	207.	230.	288.	368.	460.

   a. What is the premium for $1,000,000 liability insurance for an accountant with a total office staff of 7?

   b. How much liability insurance can an accountant buy for $219 if there is a total of four people on his staff?

   c. Ms. Goldrock had $500,000 liability coverage when she had a total staff of four. After hiring three more people she raises the coverage to $1,000,000. How much more will she pay for the increased coverage?

2. Use the table below to answer the questions following.

FAMILY INCOME DISTRIBUTION BY PERCENTAGE IN RACIAL MINORITY, FOR THE NORTH AND WEST AND FOR THE SOUTH, 1959 AND 1969.*

	Percentage of Families with Low Incomes				Percentage of Families with High Incomes			
	1959 (below $3,000)		1969 (below $5,000)		1959 ($10,000 and over)		1969 ($15,000 and over)	
Percentage Minority	Whites	All Races	Non-blacks	All Races	Whites	All Races	Non-blacks	All Races
North and West								
0.0-9.9	11.9	12.7	12.3	12.9	19.3	18.0	23.7	23.1
10.0 and over	10.3	12.6	10.1	12.1	23.9	21.8	32.1	29.1
South								
0.0-24.9	18.7	23.1	15.7	19.1	15.6	13.6	21.1	18.7
25.0 and over	14.2	24.5	14.2	21.6	17.7	13.5	20.3	16.0

* 1959 units of analysis are Urbanized Areas; 1969 units are Standard Metropolitan Statistical Areas. Source of 1959 data: Glenn (1966), Table 2.

a. What percent of the white families in the North and West in the 0.0 - 9.9% minority group made less than $3,000 in 1959?

b. What percent of the families of all races made more than $10,000 is 1959 that were in the 25.0% and over minority in the South?

c. By how much percent did the all race family income percent change from 1959 to 1969 of the people that made high incomes in the North and West in the 10.0 percentage minority and over?

d. What percent of the non-black families in the 0.0 - 24.9 percent minority in the South made $15,000 or more in 1969?

e. Find the percent minority group and area that showed the greatest gain in percent of family income distribution in the all race, high income group from 1959 to 1969.