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ANALYSIS, MANIFOLDS AND PHYSICS

Part I: Basics

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分析流形和物理学

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Part I: Basics

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INTRODUCTION

All too often in physics familiarity is a substitute for understanding, and the beginner who lacks familiarity wonders which is at fault: physics or himself. Physical mathematics provides well defined concepts and techniques for the study of physical systems. It is more than mathematical techniques used in the solution of problems which have already been formulated; it helps in the very formulation of the laws of physical systems and brings a better understanding of physics. Thus physical mathematics includes mathematics which gives promise of being useful in our analysis of physical phenomena. Attempts to use mathematics for this purpose may fail because the mathematical tool is too crude; physics may then indicate along which lines it should be sharpened. In fact, the analysis of physical systems has spurred many a new mathematical development.

Considerations of relevance to physics underlie the choice of material included here. Any choice is necessarily arbitrary; we included first the topics which we enjoy most but we soon recognized that instant gratification is a short range criterion. We then included material which can be appreciated only after a great deal of intellectual asceticism but which may be farther reaching. Finally, this book gathers the starting points of some great currents of contemporary mathematics. It is intended for an advanced physical mathematics course.

Chapters I and II are two preliminary chapters included here to spare the reader the task of looking up in several specialized books the definitions and the basic theorems used in the subsequent chapters. Chapter I is merely a review of fundamental notions of algebra, topology, integration, and analysis. Chapter II treats the essentials of differential calculus and calculus of variations on Banach spaces. Each of the following chapters introduces a mathematical structure and exploits it until it is sufficiently familiar to become an "instrument de pensée"[1]: Chapter III, differentiable manifolds, tangent bundles and their use in Lie groups; Chapter IV, exterior derivation and the solutions of exterior differential systems; Chapter V, Riemannian structures which, together with the previous structures provide the basic geometric notions needed in physics; Chapter VI, distributions and the Sobolev spaces with recent applications to the theory of partial differential equations. The last

[1] "La Mathématique . . . instrument de pensée au service de notre effort d'intelligence du monde physique", A. Lichnerowicz.

chapter covers some selected topics in the theory of infinite dimensional manifolds.

At the end of each chapter, several problems are worked out. Most of them show how the concepts and the theorems introduced in the text can be used in physics. They should be of interest both to physicists and mathematicians. A sentence like "The Lagrangian is a function defined on the tangent bundle of the configuration space" helps explain to the physicist what a tangent bundle is and tells a mathematician what a Lagrangian is. A sentence like "The strain tensor is the Lie derivative of the metric with respect to the deformation" helps a physicist to understand the concept of Lie derivatives and defines the strain tensor to a mathematician. To both, they bring an added pleasure.

The pleasure of physical mathematics is well described by Hilbert: learning that some genetic laws of the fruit fly had been derived by the application of a certain set of axioms he exclaimed "So simple and precise and at the same time so miraculous that no daring fantasy could have imagined it"[2].

PREFACE TO THE SECOND EDITION

We are happy that the success of the first edition gave us a chance to prepare a revised edition. We have made numerous changes and added exercises with their solutions to ease the study of several chapters. The major addition is a chapter "Connections on principal fibre bundles" which includes sections on holonomy, characteristic classes, invariant curvature integrals and problems on the geometry of gauge fields, monopoles, instantons, spin structure and spin connections. Other additions include a section on the second fundamental form, a section on almost complex and kählerian manifolds, and a problem on the method of stationary phase. More than 150 entries have been added to the index.

Can this book, now polished by usage, serve as a text for an advanced physical mathematics course? This question raises another question: What is the function of a text book for graduate studies? In our times of rapidly expanding knowledge, a teacher looks for a book which will provide a broader base for future developments than can be covered in one or two semesters of lectures and a student hopes that his purchase will serve him for many years. A reference book which can be used as a text is an answer to their needs. This is what this book is intended to be, and thanks to a publishing company which keeps it moderately priced, this is what we hope it will be.

[2] "Hilbert" Constance Reid, Springer Verlag, 1970.

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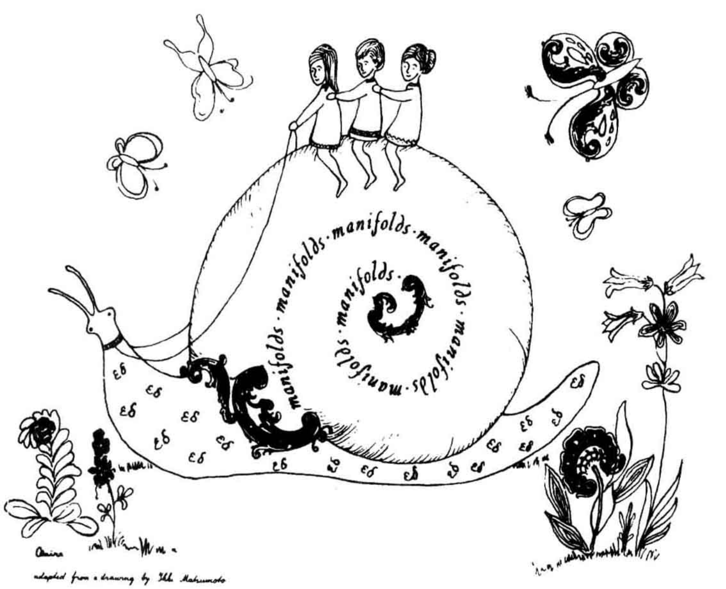
We regret that one of us who is now engaged in other pursuits could not contribute to the revised edition. The core of the book remains nevertheless under the trademark of our collaboration.

Y. Choquet-Bruhat
C. DeWitt-Morette



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